Evolution of spectral lines under time-dependent illumination of an absorbing and scattering atmosphere

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Abstract

This is a continuation of the author's research on the temporal variations in the spectral line profiles formed in absorbing and scattering atmospheres illuminated by non-stationary energy sources. The line radiation transfer in an atmosphere of finite optical thickness is considered under assumption of complete frequency redistribution. We treat two types of illumination of the medium: the sources of the form of Dirac $\delta(t)$ and Heaviside $H(t)$ unit jump functions. The goal of the work is to identify the dependence of the dynamics of changes in the spectral line profiles on frequency redistribution and the values of various optical parameters such as the optical thickness of the medium and the value of scattering coefficient. It is shown that in some cases this dependence allows to use observational material on dynamics of the burst phenomena to estimate some of these characteristics.

1. Introduction

In previous works of this research (Nikoghossian, 2021a,b) we limited ourselves to considering time variations of spectral line profiles under the simple assumption of invariance of the quantum frequency in an elementary act of scattering. In this connection, there is a need to handle the problems of the evolution of the observed spectra in their more general formulation, in which the role of redistribution of radiation by frequency is taken into account. In this paper we find natural to turn to the case important from the point of view of numerous astrophysical applications, considering that diffusion of the line radiation occurs with complete frequency redistribution. As we know, this widely accepted assumption leads to rather good results for opaque usually resonant lines formed at relatively high temperatures and densities.

The medium is assumed to be of finite optical thickness one of boundaries of which is illuminated by non-stationary energy sources. Two types of these sources are considered: those of the form of Dirac $\delta(t)$ and Heaviside $H(t)$ unit jump functions. Solution of the transfer problem for finite medium allows, in particular, to trace the asymptotical behavior of the studied time-dependent phenomenon in passing to the semi-infinite limit. The speed and duration of each evolution phase of profiles besides the redistribution law essentially depend also on the optical parameters such as the scattering (or destruction) coefficient which control the level of the diffusion process in the medium. All of these relationships will be discussed at length.

The outline of the paper is as follows: we begin in the opening Sect.2 by constructing the Neumann series for the stationary problem of diffuse reflection and transmission of a medium with finite optical thickness. Next section treats the corresponding time-dependent version of the problem under assumption that the medium is illuminated by non-stationary fluxes of radiation. The proper PDF (probability density function) and CDF (cumulative distribution function) are derived. The results of numerical calculations for a number of cases of interest are given. A qualitative analysis of the numerical calculations carried out is given. The final section discusses the influence of the frequency redistribution law as well as some physical parameters, such as the thickness of the medium and the scattering coefficient, on the evolution curve of the spectral line profiles.

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2. Neumann series

Central to our theory is the representation of the desired physical quantities in the form of expansions in powers of the scattering coefficient λ. In this case, for reflection and transmission coefficients $\rho(x', x, \tau_0)$ and $q(x', x, \tau_0)$, they are

$$\rho\left(x', x, \tau_0\right) = \sum_{n=1}^{\infty} \rho_{\mathrm{n}}\left(x', x, \tau_0\right) \lambda^n, \qquad q\left(x', x, \tau_0\right) = \sum_{n=1}^{\infty} q_{\mathrm{n}}\left(x', x, \tau_0\right) \lambda^n. \tag{1}$$

where x' and x are the dimensionless frequencies of incident quantum and reflected or transmitted quanta. The terms in the expansions obviously have a probabilistic meaning similar to that of reflection and transmission coefficients, relating however to a certain number of scattering events n. The latter satisfy the functional equations obtained by the well-known invariant imbedding technique (Bellman & Wing, 1973, Casti & Kalaba, 1976)

$$\frac{d\rho}{d\tau_0} = -\left[v(x) + v(x')\right] \rho\left(x', x, \tau_0\right) + \frac{\lambda}{2}\left[r\left(x', x\right) + \int_{-\infty}^{\infty} r\left(x', x''\right) \rho\left(x', x, \tau_0\right) dx'' + \right.$$
$$\left. \int_{-\infty}^{\infty} \rho\left(x', x'', \tau_0\right) r\left(x'', x\right) dx'' + \int_{-\infty}^{\infty} \rho\left(x', x'', \tau_0\right) dx'' \int_{-\infty}^{\infty} r\left(x'', x'''\right) \rho\left(x''', x, \tau_0\right) dx''' \right] \tag{2}$$

and

$$\frac{dq}{d\tau_0} = -v(x)\, q\left(x', x, \tau_0\right) + \frac{\lambda}{2}\left[\int_{-\infty}^{\infty} r\left(x', x''\right) q\left(x', x, \tau_0\right) dx'' + \right.$$
$$\left. \int_{-\infty}^{\infty} q\left(x', x'', \tau_0\right) dx'' \int_{-\infty}^{\infty} r\left(x'', x'''\right) \rho\left(x''', x, \tau_0\right) dx''' \right], \tag{3}$$

where the following notations are adopted: $r(x', x)$ is the frequency redistribution function, $v(x) = \alpha(x) + \beta$. We retain the commonly accepted designation the $\alpha(x)$ for the profile of absorption coefficient and β for the ratio of absorption coefficient in continuum to that in the center of the spectral line. The equations are solved under initial conditions $\rho(0, x', x) = 0$ and $q(0, x', x) = \delta(x - x')$. The above equations allow, in principle, the construction of a calculation scheme for determining the required components of the reflection and transmittance functions. The task is greatly simplified if one resorts to the approach proposed in our previous papers. It consists in pre-determining the first two simplest components of the reflection and transmittance functions associated with either the absence or the least number of scattering events

$$\rho_1\left(x', x, \tau_0\right) = \frac{\lambda}{2} \frac{r\left(x', x\right)}{v(x) + v(x')}\left\{1 - \mathrm{e}^{-\left[v(x) + v(x')\right]\tau_0}\right\}, \qquad q_0\left(x', x, \tau_0\right) = \delta\left(x - x'\right) \mathrm{e}^{-\mathrm{v(x)}\tau_0}, \tag{4}$$

The remaining required components are found by solving the following first-order linear inhomogeneous differential equations

$$\frac{d\rho_n}{d\tau_0} = -\left[v(x) + v(x')\right] \rho_n\left(x', x, \tau_0\right) + \Phi_{\mathrm{n}}\left(x', x, \tau_0\right), \tag{5}$$

$$\frac{dq_n}{d\tau_0} = -v(x)\, q_n\left(x', x, \tau_0\right) + \Psi_{\mathrm{n}}\left(x', x, \tau_0\right), \tag{6}$$

where

$$\Phi_2\left(x', x, \tau_0\right) = \frac{\lambda}{2}\left(\int_{-\infty}^{\infty} r\left(x', x'',\right) \rho_1\left(x'', x, \tau_0\right) dx'' + \int_{-\infty}^{\infty} \rho_1\left(x', x'', \tau_0\right) r\left(x'', x\right) dx''\right), \tag{7}$$

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$$\Psi_1\left(x', x, \tau_0\right) = \frac{\lambda}{2} \int\limits_{-\infty}^{\infty} q_0\left(x', x'', \tau_0\right) r\left(x'', x\right) dx''. \tag{8}$$

and for higher values of n

$$\Phi_n\left(x', x, \tau_0\right) = \Phi_2\left(x', x, \tau_0\right) +$$

$$\frac{\lambda}{2} \sum_{k=1}^{n-2} \int\limits_{-\infty}^{\infty} \rho_k\left(x', x'', \tau_0\right) dx'' \int\limits_{-\infty}^{\infty} r\left(x'', x'''\right) \rho_{n-k-1}\left(x''', x, \tau_0\right) dx''', \tag{9}$$

$$\Psi_\mathrm{n}\left(x', x'', \tau_0\right) = \Psi_1\left(x', x'', \tau_0\right) +$$

$$\frac{\lambda}{2} \sum_{k=0}^{n-1} \int\limits_{-\infty}^{\infty} q_k\left(x', x'', \tau_0\right) dx'' \int\limits_{-\infty}^{\infty} r\left(x'', x'''\right) \rho_{n-k-1}\left(x''', x, \tau_0\right) dx'''. \tag{10}$$

Thus, each of the components of the reflection and transmittance functions is found as a result of solving differential equations (5) and (6), which assume knowledge of all previous such components. As is not difficult to see, implementing a numerical solution to the problem is generally very time consuming. Nevertheless, it can be somewhat simplified due to the following considerations. The first one concerns the reflection function and its components and is that they are determined separately as it follows from Eq.(2). The number of determinable coefficients in the Neumann series needed to achieve satisfactory accuracy depends on the optical thickness of the medium and the value of the scattering coefficient λ. It is evident that closer to the conservative state $\lambda = 1$ the convergence of the series slows down. However, this fact starts to play a role in the line profiles at relatively high optical thicknesses, e.g. starting from a $\tau_0 = 3$ where the required accuracy still does not exceed 1 per cent. The volume of numerical calculations is considerably reduced by limiting the consideration to complete frequency redistribution, when the frequency redistribution law may be written in the form

Table 1.

x_i	R_1	R_2	R_3	R_4	R_5	R_6
0.0000	0.295840	0.132594	0.030205	0.015939	0.010014	0.006790
0.3158	0.283697	0.130920	0.031725	0.017206	0.010925	0, 007428
0.6319	0, 245898	0.124040	0, 035821	0.021156	0, 013923	0, 009583
0.9486	0, 182869	0.107053	0, 039881	0.027230	0, 019237	0, 013732
1.2664	0, 108693	0.077409	0, 037955	0.031127	0, 024526	0, 018936
1.5854	0, 049275	0.043453	0, 026935	0.026202	0, 023214	0, 019847
1.9061	0, 017022	0.018343	0, 013416	0.014718	0, 014207	0, 013159
2.2289	0, 004575	0.005815	0, 004706	0.005531	0, 005592	0, 005409
2.5542	0, 000971	0.001375	0, 001172	0.001423	0, 001467	0, 001445
2.8824	0, 000163	0.000241	0, 000210	0.000257	0, 000267	0, 000264
3, 2141	0.000022	0.000032	0.000028	0.000035	0.000036	0.000036
3.5499	0.000002	0.000003	0.000003	0.000004	0.000004	0.000004

$$r(x', x) = \alpha_0\left(x'\right) \alpha_0\left(x\right), \tag{11}$$

and $\alpha_0\left(x\right) = \pi^{-1/4}\alpha\left(x\right)$.

In this case, the calculations include the moments of the quantities being sought, thereby reducing the number of independent variables. On the other hand, since we are interested in spectral line profiles, instead of reflection and transmission coefficients it is sufficient to know the integrals

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Table 2.

x_i	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
0.0000	0.049638	0.073712	0.113676	0.061489	0.037741	0.026191
0.3158	0.065989	0.088693	0.11706	0.061317	0.037995	0, 026424
0.6319	0, 133268	0.132753	0, 106953	0.059244	0, 038067	0, 026750
0.9486	0, 294402	0.177762	0, 089281	0.051058	0, 035582	0, 025830
1.2664	0, 545273	0.162882	0, 058524	0.034976	0, 027701	0, 021596
1.5854	0, 781959	0.094040	0, 028157	0.017593	0, 015907	0, 013622
1.9061	0, 921009	0.036146	0, 009996	0.006443	0, 006374	0, 005876
2.2289	0, 976413	0.010087	0, 002713	0.001773	0, 001833	0, 001754
2.5542	0, 992624	0.002164	0, 000577	0.000380	0, 000398	0, 000386
2.8824	0, 996268	0.000365	0, 000097	0.000064	0, 000067	0, 000065
3, 2141	0.996907	0.000048	0.000013	0.000008	0.000009	0.000009
3.5499	0.996994	0.000005	0.000001	0.000001	0.000001	0.000001

$$R_n(x,\tau_0) = \int_{-\infty}^{\infty} \rho_n\left(x', x, \tau_0\right) dx', \qquad Q_n(x,\tau_0) = \int_{-\infty}^{\infty} q_n\left(x', x, \tau_0\right) dx'. \tag{12}$$

Tables 1 and 2 demonstrate the values of the first six quantities $R_n(x,\tau_0)$ and $Q_n(x,\tau_0)$ for a medium with of optical thickness $\tau_0 = 3$ as a function of the frequency in the line. The values of the latter are the nodes of the 49th order Hermite polynomial.

3. Time dependent problem

As we have shown in recent papers (Nikoghossian, 2021a,b) the probability density function (PDF) of the total time taken by a quantum at a certain number of n scattering events has the form

$$F_n\left(z\right) = \frac{z^{2n-1}}{(2n-1)!} \mathrm{e}^{-z}, \qquad n = 1, 2, ... \tag{13}$$

where the dimensionless time z is read in units of the mean time $\bar{t} = t_1 t_2 / \left(t_1 + t_2\right)$ expressed in terms of t_1 and t_2 representing respectively the frequency-averaged mean time taken by a quantum to fly between two successive acts of scattering and the mean time for an atom to be in an excited state. A remarkable property of the family of functions $F_n(z)$, which is a special case of the $Erlang - n$ distribution, their extremum (maximum) taken at $z = 1$. It is also important to emphasize that the value of t_1 depends on the density of scattering centres, which in turn is determined by the physical conditions in the medium.

Let us now introduce the PDF functions $\bar{R}(x,\tau_0,z)$, $\bar{Q}(x,\tau_0,z)$ such that $\bar{R}dz$ and $\bar{Q}dz$ are respectively the probabilities of observing the given reflected and transmitted lines profiles in the time interval $(z, z+dz)$. They are obviously represented as

$$\bar{R}\left(x,\tau_0,z\right) = \sum_{n=1}^{\infty} R\left(x,\tau_0\right) F_n\left(z\right) \lambda^n \qquad \bar{Q}\left(x,\tau_0,z\right) = \sum_{n=1}^{\infty} Q\left(x,\tau_0\right) F_n\left(z\right) \lambda^n \tag{14}$$

These functions describe the evolution of the spectral line profiles formed at the boundaries of the medium when it is illuminated by a δ-function shaped pulse. We see that all components of the Neumann series, and hence the profiles themselves, take their maximal values, as it was said, at $z = 1$ to which some real time $t = \bar{t}$ corresponds. Knowledge of $\bar{t}$ allows to estimate the value of t_1 and making then an idea about some physical characteristics of the medium such as the level of ionization of some atoms or ions.

In addition, we consider also the establishment of the equilibrium state regime under prolonged illumination of the medium by a source of the unit jump form given by the Heaviside H-function. The

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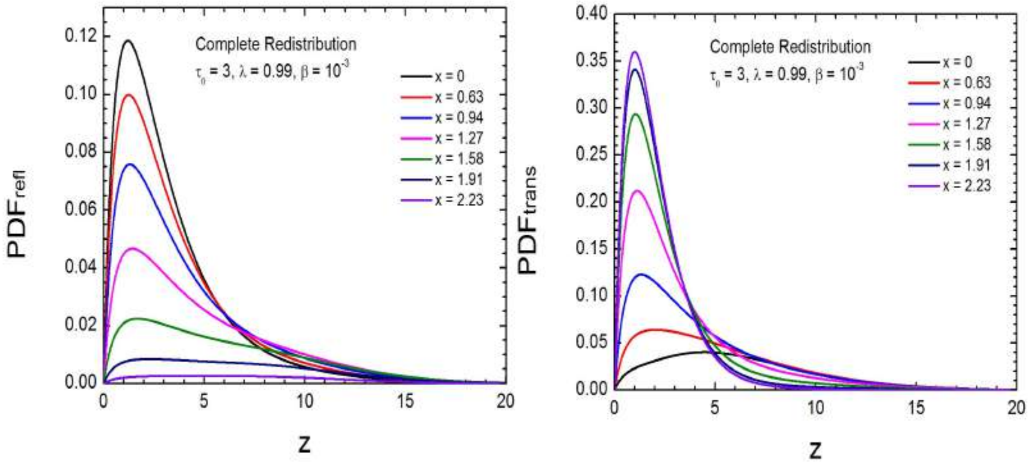

Figure 1. Probability density functions for radiation reflected from the medium (left panel) and transmitted radiation (right panel) for the medium of optical thickness $\tau_0 = 3$ and $\lambda = 0,99$

evolution of the line profiles in this case is described by the cumulative distribution function (CDF) given by

$$C_R(x, \tau_0, z_0) = \mathrm{e}^{-z_0} \sum_{n=1}^{\infty} R_n (x, \tau_0) \lambda^n \sum_{k=0}^{\infty} \frac{z^{2n+k}}{(2n+k)!}, \tag{15}$$

$$C_Q(x, \tau_0, z_0) = \mathrm{e}^{-z_0} \sum_{n=1}^{\infty} Q_n (x, \tau_0) \lambda^n \sum_{k=0}^{\infty} \frac{z^{2n+k}}{(2n+k)!} \tag{16}$$

In both illumination cases it is assumed that the incident radiation has a unit intensity at all frequencies within the line.

4. The evolution of the line profiles

Now we consider in detail changes of spectral line profiles separately for two following cases: lines generated when the medium is illuminated by a $\delta(z)$ pulse as a result of a. reflection from the medium and b. due to transmission through it. In both cases, the intensity of incident radiation is assumed equaled to unity. Figs.1-3 show the probability density functions for the radiation reflected from the medium (left panels) and for radiation transmitted through it (right panels). Here and below, the values of the optical medium parameters are chosen so as to show as clearly as possible the differences in evolution of the strong and weak lines.

Obviously, the picture of the processes in both reflection and transmission cases depends on both the optical thickness of the medium and the value of the scattering coefficient. In particular, the duration of changes in the observed spectral lines essentially depends on the value of the scattering coefficient. As one would expect, the greater the role of multiple scattering in the medium, the longer duration of the line evolution (cf. Fig.1,2). For the same reason the process is comparatively short in a medium of smaller optical thickness. (cf. Fig.3). The absorption lines produced by the passage of radiation through a medium also have a faster time course, which is due to the possibility of transmittance without scattering.

It is easy to note the independence of the maximum of PDF values of these lines attained at $z = 1$ from the value of the scattering factor. At the media thicknesses we are considering ($\tau_0 \leq 3$), it is also weakly dependent on the optical thickness (cf. Fig.3). The physically weak dependence of the mentioned value on the parameters λ and τ_0 is explained by the asymptotic tendency of the line profile in the far wings to unity. As a result, the maximum value of the PDF becomes approximately $\mathrm{e}^{-1} = 0.35$. It follows that the numerical value of this quantity depends only on the intensity of the primary radiation incident on the medium in the continuum, which is taken as one in the paper.

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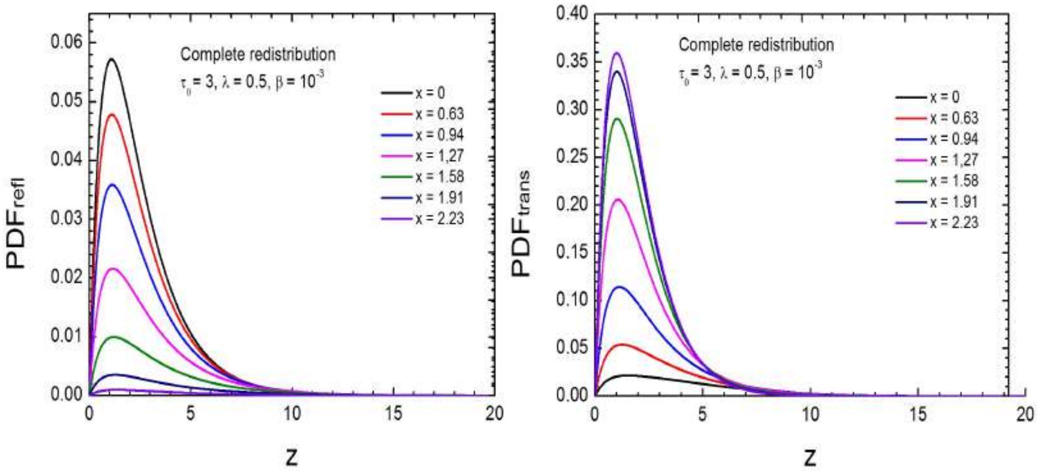

Figure 2. The same as in Fig.1 for $\lambda = 0,5$.

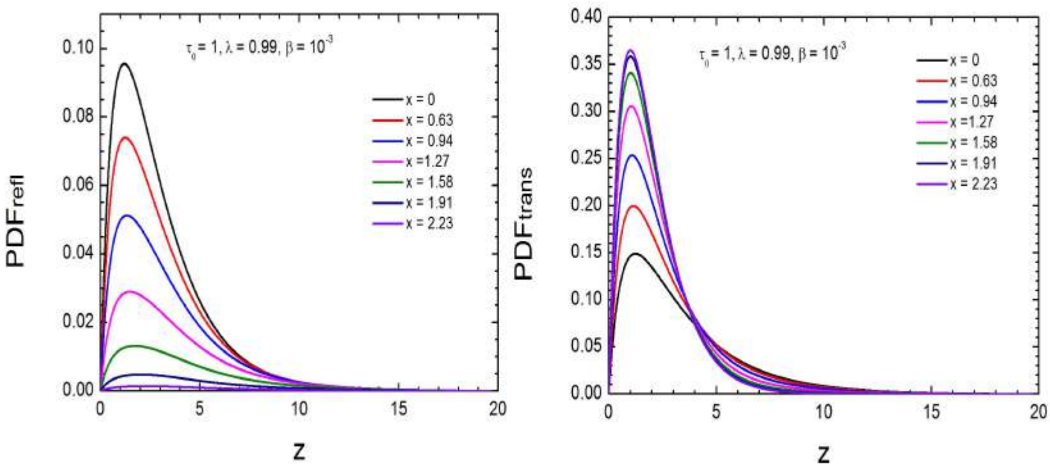

Figure 3. The same as in Fig.1 for $\tau_0 = 1$ and $\lambda = 0,99$.

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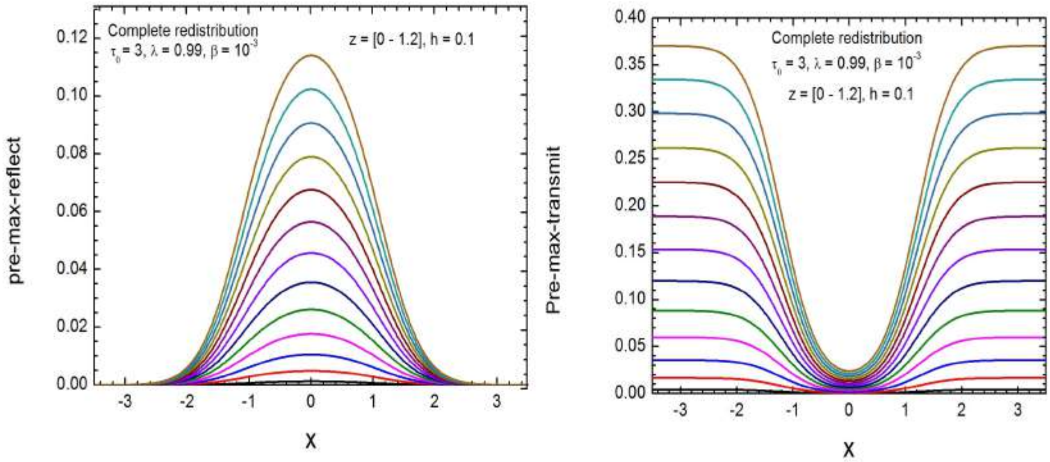

Figure 4. Evolution of the reflected (left) and transmitted spectral lines profiles formed as a result of reflection (left) and transmission (right) before PDF maximum. The value of the time variable z grows from the bottom to top in increment $h = 0.1$

Therefore, a comparison of the theoretical and observed values of the maximum allows us to judge about the intensity of the radiation falling on the medium.

Then, by following the evolution of the absorption line profiles arising as the result of transmittance through the medium, it is possible to gain insight into both the optical thickness of the medium and the role of diffusion in the medium in the observed spectra. This can be done by comparing the different parameters of the theoretically derived distributions and the profiles of the observed lines. For example, the relative fraction of the total energy absorbed in the line during the period up to the maximum is calculated to be weakly dependent on the scattering coefficient over the whole interval of $0 < \lambda \leq 1$ and is accurately 22 per cent at $\tau_0 = 1$, and 19 per cent at $\tau_0 = 3$. Therefore these values can serve as an indicator of the optical thickness of the medium.

It is somewhat more difficult to estimate similar parameters when studying the evolution of the emission line-spectrum formed due to reflection from the medium. In this case, as it follows from Figs 1-3, the maximum value of the reflection probability density function, as well as the duration of the whole process, are significantly dependent on the value of the scattering factor. Instead, these values for equal λ are weakly dependent on the optical thickness.of the medium, what gives an indication of the value of the scattering coefficient.

Figs. 4,5 illustrate the evolution of the spectral line profiles both before and after the PDF maximum is reached. The time variable grows with the increment $h = 0.1$. In the immediate vicinity of the maximum, profile changes slow down. After this, the decline begins, shown in Fig.4. The time variable z varies here from top to bottom in ten times greater steps $h = 1$. What draws attention is the fact that after a long time the 'information' of the initial illumination of one of the boundaries of the medium is gradually lost and it radiates at the expense of a small number of still diffusing quanta. In this case, as it follows from Fig. 4, double-peaked emission lines appear at both boundaries of the medium.

5. Illumination by radiation of the form of Heaviside's unit-jump H-function. Establishment of a stationary regime in the medium

Let us now proceed to next problem, important from the point of view of astrophysical applications. We assume now that the finite atmosphere is illuminated continuously after its onset until the steady state field of radiation is established. As above, we are interested both in evolution of emission lines of reflected spectrum and absorption lines arising from the passage of radiation through the medium. The time dependence of the line profiles are described by cumulative distribution functions (CDF) given by Eqs.(15) and (16).

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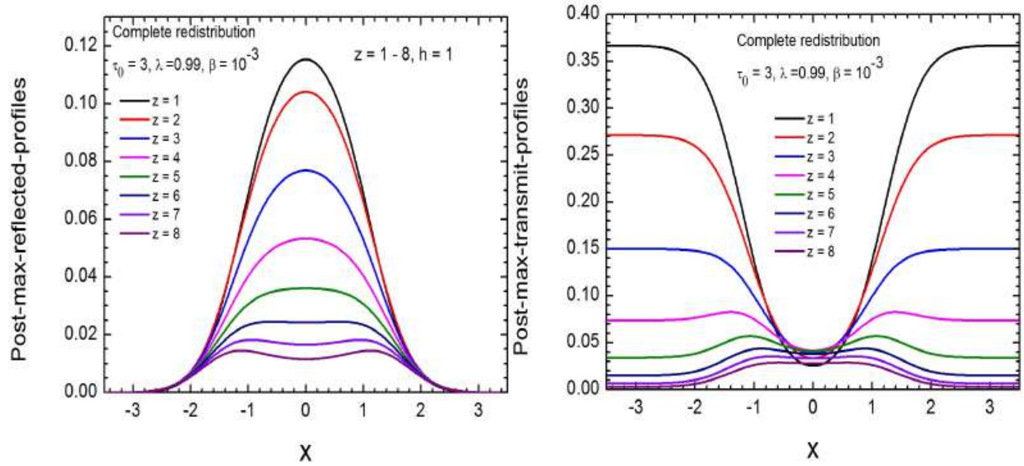

Figure 5. Evolution of the reflected (left) and transmitted (right) spectral lines profiles formed after PDF maximum. The value of the time variable z grows from the bottom to top in increment $h = 1$

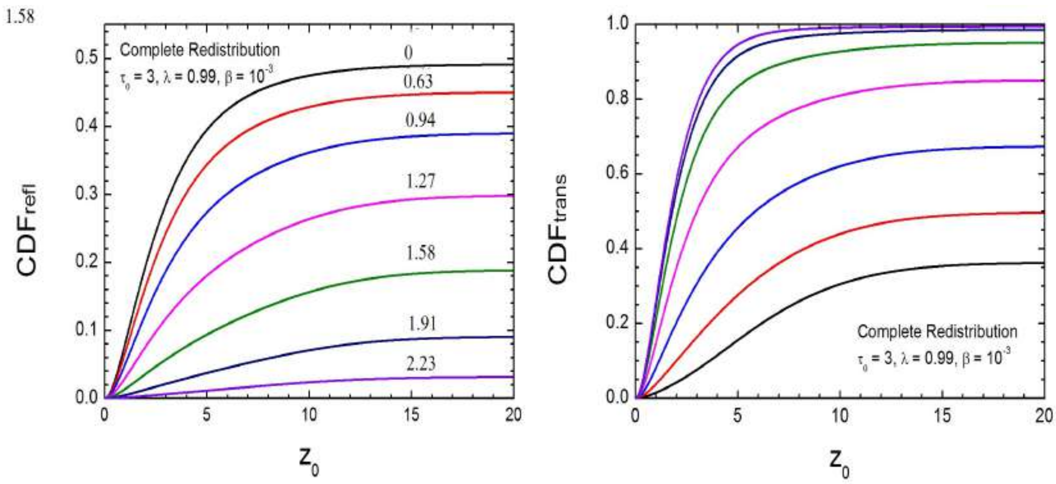

Figure 6. Evolution of the reflected (left) and transmitted (right) spectral lines profiles intending towards stationary regime for $\tau_0 = 3$ and $\lambda = 0.99$ and indicated values of frequencies within a line.

Here, in addition to the initial phase of the appearance of the line spectra, we will be interested in their evolution and the rate at which the steady state is established, which as above depends both on the thickness of the medium and on the value of the scattering coefficient. Typical curves showing the variation of the spectral line profiles before reaching the stationary regime are shown in Figs. 6-8. They are characterised by a plateau due to an asymptotic tendency of the line radiation towards their limiting values. As should be expected, the rate of the tendency towards stationary regime in the optically thin media we consider is sufficiently high. In fact, the curves achieve their limit values within the time interval $[5 \leq z_0 \leq 10]$. The illustrations above clearly show that the absorption lines formed as a result of transmission through the medium are established faster than the emission lines in the reflected spectrum. Another feature of the processes under study is their longer duration when the role of multiple scattering in the medium increases, i.e. at larger values of λ.

The absorption spectrum, as can also be seen in Figures 9-11, provides rich material for estimating both the optical thickness of the medium and the value of the scattering coefficient. The central residual line intensities together with equivalent widths provide important time-dependent information about these quantities, and, as a consequence, to study the physics of the observed phenomenon.

The spectrum formed by the reflection of the medium can, of course, also be used to obtain all sorts of information about its physical characteristics. For this purpose one can use the limiting values of the quantities under consideration known as solutions of the classical stationary transfer problem. In this case, the values of the optical thickness of the medium and the scattering coefficient are found

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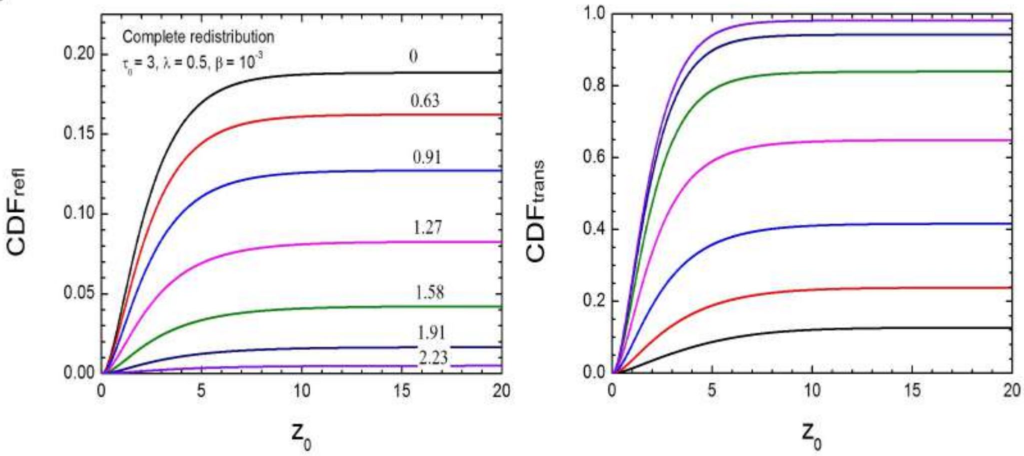

Figure 7. The same as in Fig.6 for $\tau_0 = 3$ and $\lambda = 0.5$.

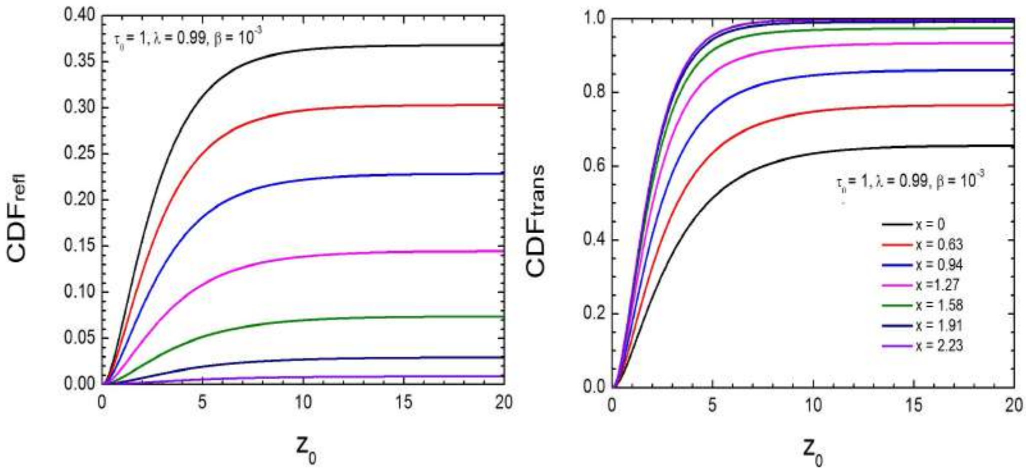

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Figure 8. The same as in Fig.6 for $\tau_0 = 1$ and $\lambda = 0.99$.

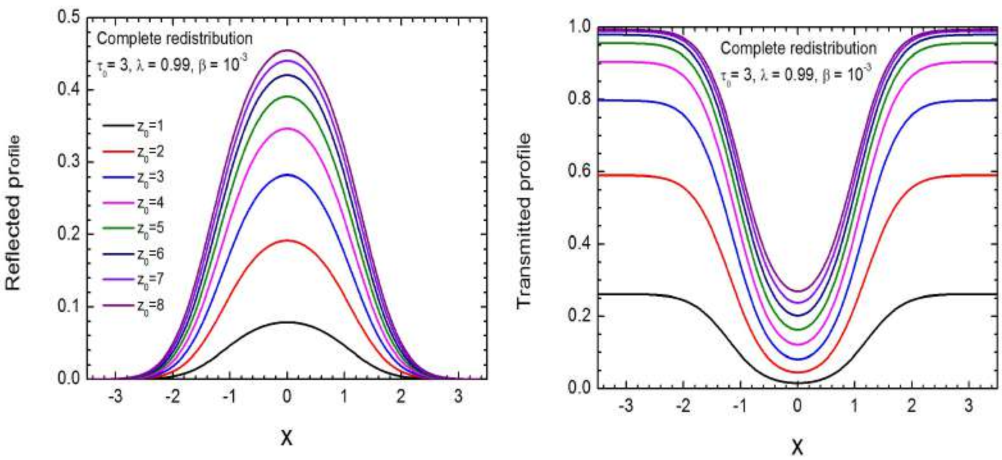

Figure 9. Evolution of the reflected (left) and transmitted (right) spectral lines profiles intending towards stationary regime for $\tau_0 = 3$ and $\lambda = 0.99$ during indicated time intervals.

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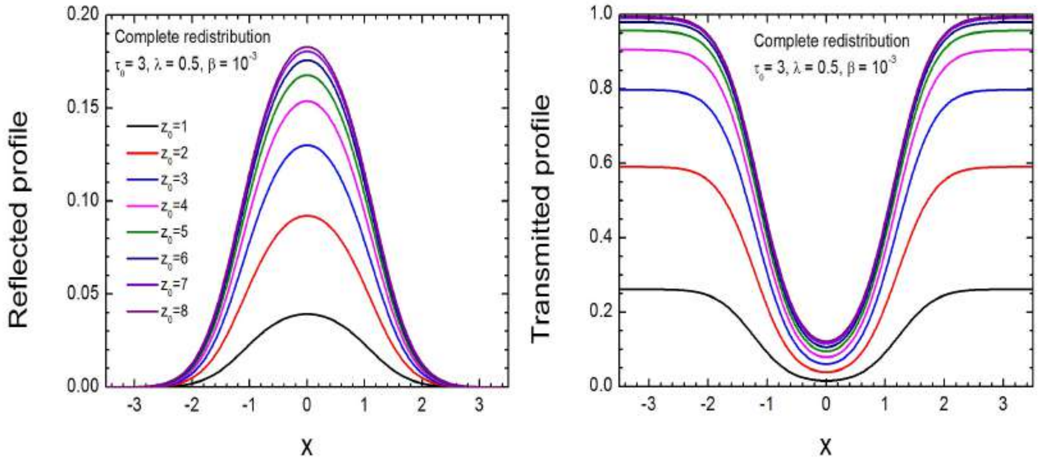

Figure 10. The same as in Fig.9 for $\tau_0 = 3$ and $\lambda = 0.5$.

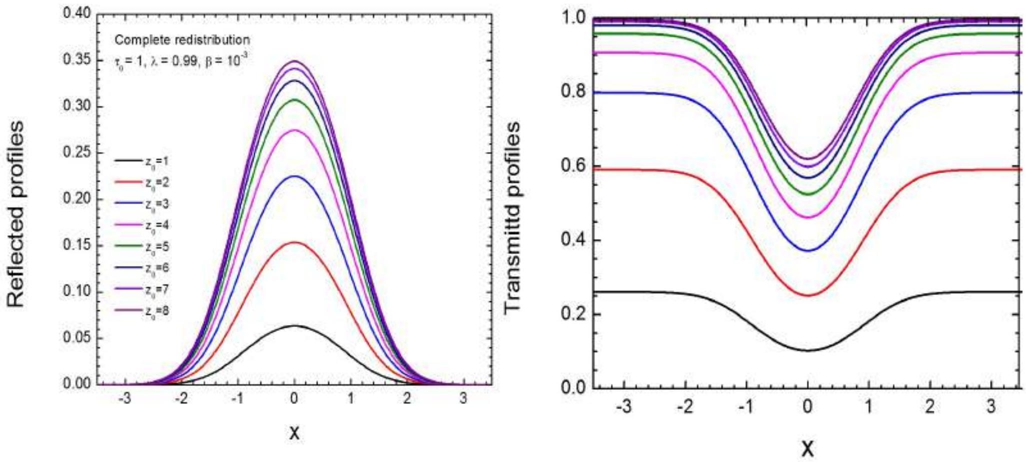

Figure 11. The same as in Fig.9 for $\tau_0 = 1$ and $\lambda = 0.99$.

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jointly from the known solution of the stationary problem. The identification of the spectral line and its assignment to a certain chemical element together with the dynamics of change and saturation rate, as in the case of the absorption spectrum, reveal the physical picture of the dynamic process under study.

6. Concluding remarks

The paper solves the problem of spectral line formation in a finite medium illuminated from one side by non-stationary radiation. Two possible types of non-stationarity of the sources are considered: $\delta(t)$-shaped energy radiation and that of the form of a unit-jump given by the Heaviside $H(t)$ function. This choice is due to the fact that knowledge of the solutions of the problem for these two cases makes it possible, if necessary, to refer to problems with a more general formulation involving more complex laws of variation of the incident radiation over time. Separately, we considered the problem of establishing a stationary radiation field under continuous illumination of the medium.

One of the main goals of this work was to identify the possibility of using data on the dynamics of changes in the observed spectra to obtain additional information about the properties of the radiating medium and the optical parameters of the lines contained in them. It has been shown that the specific type of probability distributions describing the temporal variations of spectral lines provides this possibility because of a number of its characteristics. One of them is the location of their maximum value taken at $z = 1$. The comparison of this time value with the real time for a given line makes it possible to estimate a number of characteristics of the emitting medium, such as the degree of ionisation of the atom or ion concerned. To determine the optical thickness of the medium and the scattering coefficient by comparing theoretical results with observational data, line-dependent values of various parameters characterising the evolution of spectral lines can be used. These include the knowledge of the maximum value of the temporal distribution, the amplification and subsequent attenuation rates of the spectral lines, the relative values of the energy emitted by reflection as well as the energy absorbed due to the passage through the medium both in the pre-maximal and post-maximal periods.

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