# A planetary resonant effect in Parker stellar dynamo

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**Abstract**

The effect of periodic pumping on dynamo generation in the simplest Parker model is studied in this work. Pumping is understood in the sense that the periodic parameters oscillations in the dynamo system leads to a change in the rate of the exponential growth of the mean magnetic field. And since the Parker model simultaneously describes its time oscillations as the field grows, this phenomenon is very similar to parametric resonance in the classical model of a harmonic oscillator. With the help of asymptotic analysis and numerical simulation, we demonstrate both pump regions similar to parametric resonance, as well as different amplification regions at high driving force frequencies, and suppression regions at low frequencies, find the gain maximum and investigate the behavior of the critical pump frequency separating the regions of generation and suppression.

***Keywords:*** *MHD dynamo, Parker dynamo model, parametric resonance.*

## 1. Introduction

Solar activity cycle as well as stellar activity cycles are believed to be driven by stellar dynamo action based on stellar differential rotation and mirror asymmetric convection. The point however is that, the length of solar activity cycle (about 11 year) is quite close to the Jupiter orbital period and many astronomers supposed that the physical nature of solar activity cycle is somehow associated with the Jupiter influence on solar magnetohydrodynamics. Obridko et al. (2022) recently demonstrated that solar activity cycle is the only known case among a dozen similar cases accessible for contemporary observations where an activity cycle is observable and its length is closed to the planets orbital period and we have to accept that we face just a coincidence in solar case.

Obridko et al. (2022) stress however that this result do not exclude that a planetary effect on stellar dynamo is possible in principle. The aim of this short paper is to show that a weak periodic modulation of stellar dynamo drivers indeed can affect the dynamo threshold and transform a slightly subcritical dynamo action in a supercritical one.

## 2. Dynamo model

Obviously, a gravitation of an exoplanet or a star in a binary system leads to a weak modulation of stellar dynamo drivers which can be in principle include in dynamo model (e.g. Moss et al., 2002). The problem is how to separate this weak influence from various nonstationary phenomena associated with dynamo action. Our aim here is to demonstrate a physical phenomena rather to suggest a realistic model of a particular explanatory system and we sole the above problem as follows. We consider the simplest stellar dynamo model originated by Parker (1955) and include a weak periodic modulation of differential rotation.

The model proposed by Parker (1955) to describe the solar dynamo cycle is a direct consequence of the averaged magnetic induction equation written for the poloidal and toroidal components of the magnetic field. In the approximation of azimuthal symmetry - independence of the angle $\varphi$, and of a thin spherical layer -

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independence of the radial distance $r$, the only remaining spatial variable $\theta$ can be taken into account through the expansion of the magnetic field components in the first few harmonics. The simplest approximation describing the physical structure of the dynamo cycle can be obtained for only two components and is called the low-mode approximation. This is the minimal set of the Fourier modes sufficient to obtain a growing oscillating solution with nonzero magnetic moment (Nefedov & Sokoloff, 2010). Thus, for a dimensionless average magnetic field expanded in terms of symmetric (for $A$) and antisymmetric (for $B$) harmonics with respect to the equator

$$\mathbf{B} = B\mathbf{e}_\varphi + [\nabla, A\mathbf{e}_\varphi] = (b_1 \sin(2\theta) + b_2 \sin(4\theta))\,\mathbf{e}_\varphi + [\nabla, (a_1 \sin(\theta) + a_2 \sin(3\theta))\,\mathbf{e}_\varphi], \tag{1}$$

This yields in the following simple dynamical system

$$\begin{aligned} \dot{a}_1 &= (R_\alpha/2)b_1 - \mu^2 a_1, & \dot{b}_1 &= R_\omega(a_1 - a_2) - \mu^2 b_1, \\ \dot{a}_2 &= (R_\alpha/2)(b_1 + b_2) - \mu^2 a_2, & \dot{b}_2 &= 2R_\omega a_2 - \mu^2 b_2. \end{aligned} \tag{2}$$

Two dimensionless control parameters of the system $R_\alpha$ and $R_\omega$ are responsible for the hydrodynamic helicity in the convective shell and for the differential rotation, while it is well known that if their product, the dynamo number $D$, is large enough, then the magnetic field in the system will increase exponentially. The third parameter $\mu$ is responsible for diffusion and, in general, must take into account all components of the Laplacian: radial, axial, and crosswise. Depending on the specific model, $\mu$ can be considered the same for all four equations (if the main part of diffusion is radial) or different (if the axial part of the Laplacian determines the main contribution, since the second derivatives with respect to $\theta$ will be different for different harmonics). Here we assume the equality of all diffusion coefficients, and then we make an assumption about what will change if these coefficients differ for each of the equations of the system.

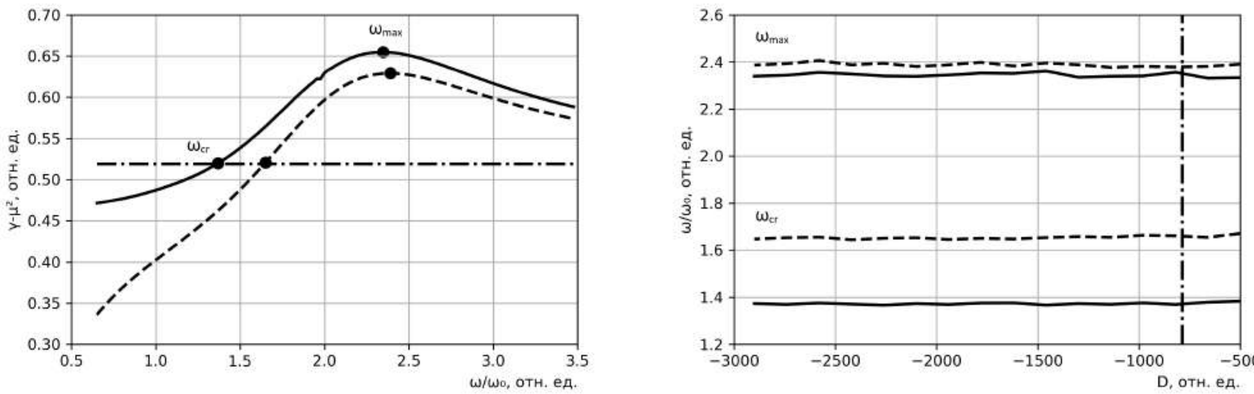


Figure 1. The left panel: dependence of the exponential growth rate $\gamma - \mu^2$ on the parametric excitation frequency $\omega$. The dash-dotted horizontal line is the growth rate without parametric pumping $\sigma = 0$. The right panel: dependence of the critical excitation frequency $\omega_{cr}$ and the frequency of maximal growth $\omega_{max}$ on the dynamo-number $D = R_\alpha R_\omega$. The dash-dotted vertical line shows the dynamo-number when the dynamo-growth stops. The black solid and dashed lines are the numerical and analytic results respectively. The frequencies are normalised on correspondingly eigenfrequencies $\omega_0$.

Having expressed two components: $b_1(t)$ and $b_2(t)$ from the first two equations, we substitute them into the second two. We obtain a system of two equations of the second order, which, after the replacement $a_{1,2}(t) = f_{1,2}(t)\exp(-\mu^2 t)$, can be reduced to a Mathieu-type system:

$$\begin{aligned} \ddot{f}_1 - (R_\alpha R_\omega/2)(1 + \sigma\sin(\omega t))(f_1 - f_2) = 0, \\ \ddot{f}_2 - (R_\alpha R_\omega/2)(1 + \sigma\sin(\omega t))(f_1 + f_2) = 0. \end{aligned} \tag{3}$$

For such a system, in the absence of a periodic force $\sigma = 0$, it is easy to calculate the eigenfrequencies $\omega_0$ and the generation rates $\gamma_0$ of the harmonic solution:

$$\lambda_0 = \gamma_0 \pm i\omega_0 = \pm\sqrt{-\frac{R_\alpha R_\omega}{\sqrt{2}}}\exp(\pm 3i\pi/8). \tag{4}$$


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For $\sigma \neq 0$, the solution to the system can be sought in the form of a harmonic with a shifted frequency and a changed generation rate, however, unlike the analysis of parametric resonance for a harmonic equation, this method does not give anything. Therefore, we are looking for a solution (3) as a sum of not two, but four complex conjugate exponents with exponents $\gamma \pm i\beta \pm i\omega/2$. Then, if the external periodic action has a double frequency $\omega$, then, neglecting the higher harmonics and collecting the terms from each of the four exponentials, we obtain the solvability of the system for

$$\gamma + i\beta = \pm\sqrt{\lambda_0^2 \pm i\gamma_0\omega\sqrt{1 \pm \frac{iR_\alpha^2 R_\omega^2\sigma^2}{8\lambda_0^2\omega^2}} - \frac{\omega^2}{4}}. \tag{5}$$

For small $\sigma$, this asymptotic expression for the exponent of the exponential solution can be approximately written as

$$\gamma + i\beta = \lambda_0 \pm \frac{i\omega}{2} \pm \frac{R_\alpha^2 R_\omega^2\sigma^2}{32\lambda_0\omega(\lambda_0 \pm i\omega/2)} + o(\sigma^2). \tag{6}$$

Thus, periodic pumping of the Parker model selects two harmonics with frequencies shifted by $2\beta$ relative to each other and $\gamma$ generation rates close to $\gamma_0$. The appearance of diffusion proportional to $\mu^2$, see the system (2), only leads to a decrease in the generation rate by $\mu^2$ – it transforms in $(\gamma - \mu^2)$ – while the very nature of the beats remains the same. Note that in the course of the analytical evaluation, we neglect the higher harmonics in $i\omega/2$, so in the formula (6), the signs should be chosen such that only the lower harmonics remain. It can be seen that for $\sigma = 0$ the solution completely coincides with $\lambda_0$ defined by Eq. (4), while for $\sigma \neq 0$ the real part of the solution (6) is greater than $\lambda_0$ for $\omega$ is greater than some critical frequency, and less than $\lambda_0$ for $\omega$ less than this critical frequency. In the region of the doubled frequency of the external force, the positive addition to the generation rate has a local maximum, and then, at $\omega \to \infty$, the generation rate tends to $\gamma_0$.

The described features of the dependence of external pumping on the frequency of the driving force are clearly visible in the figure 1, left panel: the analytical results are shown in the figure by dashed line, and the numerical results of calculating the generation rate are shown by a black solid line. The divergence of the solutions is due to the asymptotic nature of the results obtained, therefore, a decrease in $\sigma$ leads to the fact that the two curves tend to each other and simultaneously converge to the straight line $Re\,\gamma = \lambda_0$, dashed-dotted horizontal line. A distinctive feature of such a response to a parametric action is the absence of a clearly defined narrow resonance maximum at multiple frequencies, which, however, is explained by the degeneracy of the symmetric system and the absence of a pure harmonic solution for the system (3).

By the degeneracy of the system, we mean that the fourth-order equation for the eigenvalues of the system (2) has roots with real parts and frequencies that coincide in absolute value, respectively, among them there is no fastest growing harmonic with a selected frequency, since two equally growing harmonics have the same frequency. As a result, under parametric pumping, they do not have a solution in the form of a quasi-harmonic signal, as in the classical case of parametric resonance, but instead, beats with a specific resonance pattern are observed. If the diffusion $\mu$ for each equation of the system (2) differs, then a distinguished frequency will appear with the fastest growing harmonic, and the parametric resonance will acquire classical features with distinguished narrow peaks at doubled and multiple frequencies. Indeed, a numerical test showed that for different diffusion coefficients – the resonance pattern is a superposition of the pattern 1 and sharp peaks at double and multiple frequencies. At the same time, the gain maximum at $\omega_{max}$ corresponding to the figure 1 and the presence of the critical frequency $\omega_{cr}$ (below which the generation rate is suppressed by the periodic influence, and above which it is enhanced) remain.

Finally, let's pay attention to the resulting asymptotic formula for the exponential growth rate (6), the real part of which is shown in the figure 1. The graph has a wide maximum, in comparison with the classical resonant peak, near the frequency $\omega = 2.4\omega_0$ and a critical boundary $\omega = 1.6\omega_0$, which separates the region of amplification and suppression of generation. The position of these characteristic markers depends on the natural frequency $\omega_0$, and, accordingly, on the dynamo number $D = R_\alpha R_\omega$, but its minimum value is limited by the generation region – see the analytical and numerical estimate of the critical frequency in the figure 1, right panel. In other words, at sufficiently high frequencies of the excitation force, greater than this critical frequency, a periodic change in $R_\alpha$ will lead to an increase in generation at frequencies close to the natural frequency $\omega_0$. Of course, in the case of nonlinear suppression $D = R_\alpha R_\omega$ the natural frequency of the system will also change, but the generation will still be enhanced at sufficiently high frequencies. In this case, it is difficult to predict in advance what kind of amplification - at high frequencies near a wide maximum or


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at a doubled frequency near a resonant peak - will be the main one, since this will be determined by the diffusion part, but both can be present in the general formulation.

## 3. Conclusion and Discussion

We demonstrated that even a weak planetary effect on dynamo drivers can in principle lead to substantial modification of dynamo driven magnetic field, i.e. transform a decaying magnetic field in a growing one and *vice versa*. Indeed, playing with parameter $\mu$ responsible for turbulent losses in our dynamical system we can make the dynamo number $D$ for unperturbed system to be just a threshold one and dynamo driven magnetic field to be just marginally stable. Then if the frequency of parametric excitation is large enough we obtain excitation (right part of Fig. 1) and decay if the frequency is low enough (left part of Fig. 1). If the unperturbed dynamo system is slightly subcitical a moderate $\sigma$ can be still sufficient to get an excitation. Of course, if perturbation is weak the subcritical dynamo should be very close to the excitation level so the effect hardly can happen in many exoplanetary systems.

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