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Spectral lines evolution in the media illuminated by non-stationary energy sources

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Abstract

A brief review of the author's recent work on the development and application of the theory of time-dependent transfer of radiation in the spectral line is given. Various factors that influence the temporal characteristics of changes in line spectra are studied. This creates a fundamental opportunity to use them to gain insight into both external energy sources, the physical and geometrical properties of the medium itself as well as the various optical parameters of the observed spectral lines.

1. Introduction

We give a brief overview of our recent results in the theory of non-stationary radiative transfer concerning the evolution of observed line spectra that are formed under influence of time-dependent energy sources. The area is of considerable astrophysical interest, having in mind the great diversity of this kind of well-known cosmic phenomena, differing both in scale and in duration (bursts of Novas and Supernovas, flare and other activity manifestations in the Sun, Fuor-like changes, etc.).

The existing mathematical theory includes two main directions for solving time-dependent problems of radiative transfer in spectral line frequencies. They are based on the Laplace transform of derived equations (Chandrasekhar, 1948, 1950, Minin, 1964, 1965, 1967, Sobolev, 1950, 1963)and on the construction of Neumann series for desired physical quantities (Ganapol, 1979, Matsumoto, 1967, 1974, Uesugi & Irvine, 1970). Both approaches with their advantages encounter difficulties in addressing relatively more realistic model problems. Ways to overcome the difficulties encountered, both theoretical and computational, are shown in our recent papers (Nikoghossian, 2021a,b, 2022a,b).

The ultimate goal of this study is to describe the temporal variations of the observed line spectra and their dependence on various physical conditions and initial assumptions. This will make it possible to use the observed temporal characteristics of the evolution of spectral lines to interpret the observed phenomenon and to determine various optical parameters of both these lines and the medium itself.

2. Statement of the problem

In the transfer problem under consideration, the geometric and optical properties of the medium are assumed to be time-independent. Temporal changes are related only to the energy of the primary energy sources, which generally can be located both inside and outside the medium. We consider energy sources of two types of time distribution, namely, of the form of the Dirac $\delta(t)$ -function and the form of the Heaviside unit jump function $H(t)$. This choice allows us to judge the solutions of the considered problems also with a more complex temporal dependence of incident energy.

The medium is assumed to be one-dimensional and of finite optical thickness, one of boundaries of which is illuminated by non-stationary energy sources (see Fig. 1). Solution of the transfer problem for finite medium allows, in particular, to trace the asymptotical behavior of the studied time-dependent phenomenon in passing to the semi-infinite limit. Based on such considerations, the optical thickness

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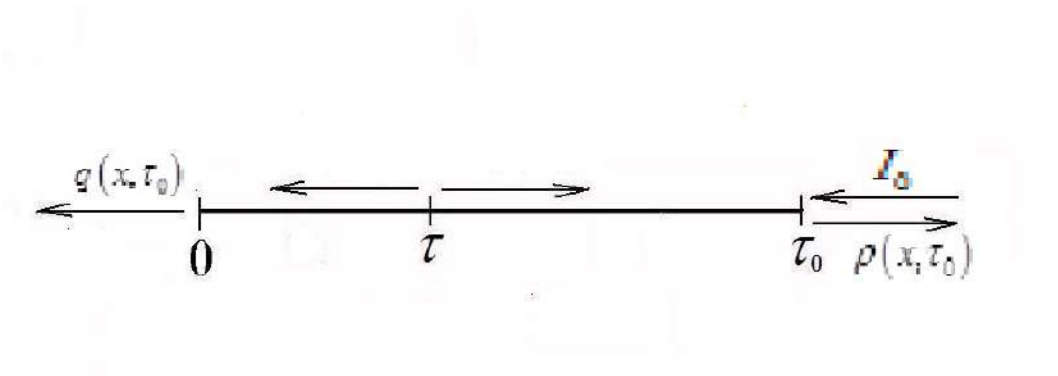

Figure 1. Schematic description of radiation transport in a one-dimensional medium of finite optical thickness

of the absorbing and scattering medium is everywhere assumed to be equal to the boundary value of $\tau_0 = 3$, above which the obtained solutions reach an asymptotic plateau. Choosing a smaller value for the thickness would make the role of multiple scattering in the medium less discernible. Some problems, we formulate under assumption that the scattering process in the medium is monochromatic, which is due to fact, that the scattering in the continuum, which influence is also of importance, is accounted for in a very simple way. The effects associated with the redistribution of radiation in spectral line frequencies we discuss separately.

3. Neumann series

The approach to the solution of time-dependent problems of the theory of radiation transfer, based on the construction of Neumann series, was developed by Matsumoto cited above). In the simplest model problems, these series are expansions of the required solutions for reflectance $\rho(x, \tau_0)$ and transmittance $q(x, \tau_0)$, in the degrees of probability of re-emission of quanta at an elementary event of scattering λ. The physical meaning of each term of the series is obviously the same as that of the sought quantity, referred, however, to a certain number of scattering events n. The latter satisfy the functional equations obtained by the well-known invariant imbedding technique (Bellman & Wing, 1973, Casti & Kalaba, 1976). In the monochromatic formulation of the problem when the medium is inhomogeneous medium, these series are of more complex nature:

$$\rho\left(x,\tau_0\right)=\sum_{n=1}^{\infty}\rho_n\left(x,\tau_0\right), \qquad q\left(x,\tau_0\right)=\sum_{n=0}^{\infty}q_n\left(x,\tau_0\right) \tag{1}$$

where x is the dimensionless frequency measured by displacement from the center of the line in Doppler units.

The essence of the method we suggest consists in constructing appropriate recurrence relations using the method of invariant imbedding, in which the first terms of the series not related to the double reflection are calculated beforehand. We have

$$\rho_1\left(x,\tau_0\right)=\frac{1}{2}\int_0^{\tau_0}\tilde{\lambda}\left(x,\tau\right)\mathrm{e}^{-2v(x)(\tau_0-\tau)}v\left(x\right)d\tau \tag{2}$$

$$\rho_2\left(x,\tau_0\right)=\int_0^{\tau_0}\tilde{\lambda}\left(x,\tau\right)\rho_1\left(x,\tau\right)\mathrm{e}^{-2v(x)(\tau_0-\tau)}v\left(x\right)d\tau, \tag{3}$$

$$q_0\left(x,\tau_0\right)=\mathrm{e}^{-v(x)\tau_0} \qquad q_1\left(x,\tau_0\right)=\frac{1}{2}\int_0^{\tau_0}\tilde{\lambda}\left(x,\tau\right)q_0\left(x,\tau\right)\mathrm{e}^{-v(x)(\tau_0-\tau)}v\left(x\right)d\tau \tag{4}$$

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$$q_2(x,\tau_0) = \frac{1}{2}\int\limits_0^{\tau_0} \tilde{\lambda}(x,\tau)\, q_1(x,\tau)\, e^{-v(x)(\tau_0-\tau)} v(x)\, d\tau \tag{5}$$

where the following notations are adopted: $\tilde{\lambda}(\tau_0) = [\lambda(\tau_0)\,\alpha(x) + \gamma]/v(x)$, $v(x) = \alpha(x) + \beta + \gamma$. We retain the commonly accepted designation $\alpha(x)$ for the profile of absorption coefficient and β and γ, correspondingly for the ratio of absorption and scattering coefficient in continuum to that in the center of the spectral line.

The above formulas have a transparent physical meaning and can be written directly. The remaining terms of the series, are expressed through auxiliary functions Φ_n and Ψ_n according to the formulas

$$\rho_n(x,\tau_0) = \int\limits_0^{\tau_0} \Phi_n(x,\tau) e^{-2v(x)(\tau_0-\tau)} v(x)\, d\tau, \tag{6}$$

$$q_n(x,\tau_0) = \int\limits_0^{\tau_0} \Psi_n(x,\tau)\, e^{-v(x)(\tau_0-\tau)} v(x)\, d\tau. \tag{7}$$

The last ones, in their turn, are expressed through the previous, already found, terms of corresponding series

$$\Phi_n(x,\tau_0) = \frac{1}{2}\tilde{\lambda}(x,\tau_0)\left[\rho_{n-1}(x,\tau_0) + \sum_{k=1}^{n-2} \rho_k(x,\tau_0)\, \rho_{n-k-1}(x,\tau_0)\right], \qquad n > 2 \tag{8}$$

$$\Psi_n(x,\tau_0) = \frac{1}{2}\tilde{\lambda}(x,\tau_0)\left[q_{n-1}(x,\tau_0) + \sum_{k=1}^{n-2} q_k(x,\tau_0)\, \rho_{n-k-1}(x,\tau_0)\right], \qquad n > 2 \tag{9}$$

As we see, each stage of the iterative process is accompanied by the calculation of integrals (6) and (7). The convergence of the Neumann series at a given frequency depends obviously on the optical thickness of the medium and the value of the scattering coefficient. The fastest convergence occurs in the wings of the line, where the medium is relatively transparent and the role of diffusion undergone by the quanta is small. In the problem we consider, the thickness of the medium is relatively small, so even in the central frequencies, the number of iterations, depending on the required accuracy, is about 6-8. Note that the above relations are quite general, since the possible dependence of the scattering coefficient on depth in the medium, discussed below, is taken into account. If the medium is considered homogeneous, the Neumann series turn into expansions of the corresponding physical quantities by degrees of the scattering coefficient.

4. Time-dependent problem

As is known (see, for example, Sobolev, 1963), the time spent by quanta during diffusion in the medium consists of two components: the time lost by quanta for staying in an excited state of the atom and - the time necessary for passing a path between the scattering acts undergone. In solving the time-dependent problems of radiation transfer in the line, one has most often limited ourselves to taking into account either the first of these causes or the second. The simpler first case essentially is reduced to determining the average number of scattering events. Relatively complicated is the second case, which has been considered by various authors using different methods (Minin, 1964, 1965, 1967, Sobolev, 1963), in particular, using the Laplace transform approach. Even here, however, the resulting equations, despite their resemblance to similar equations in the stationary problems, encounter considerable difficulties in their numerical solution.

Having at our disposal the solution of the stationary problem in the form of Neumann series it is not difficult to construct, as we showed in Nikoghossian (2021a,b, 2022a,b), the corresponding solutions of the non-stationary problems we consider:

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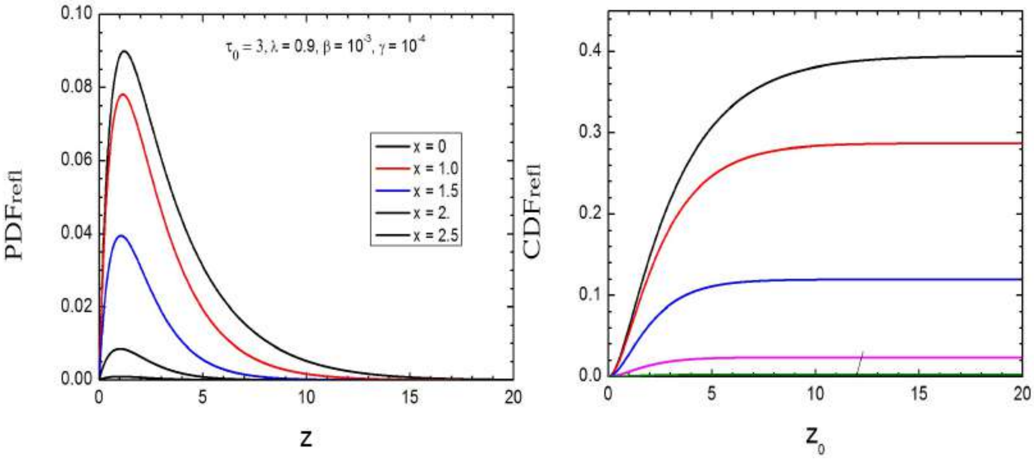

Figure 2. Probability density and cumulative distribution functions for reflected quanta for $\lambda = 0,99$.

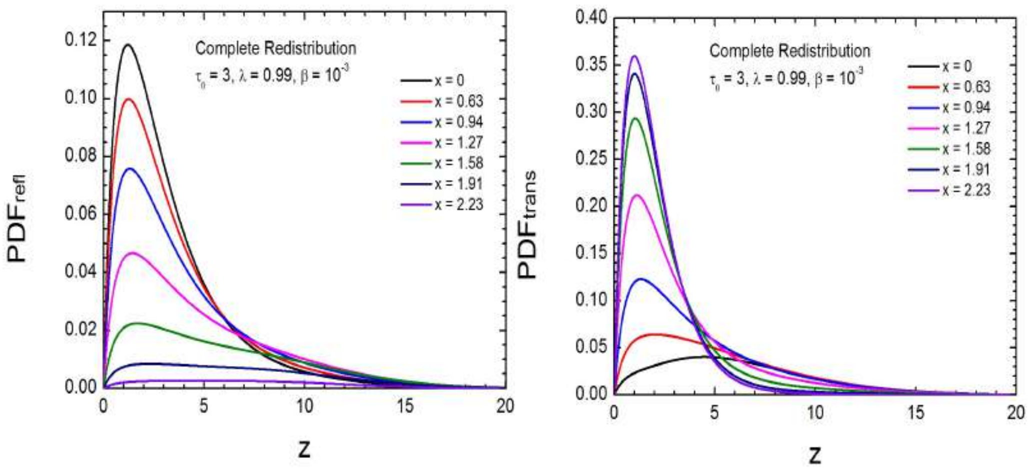

Figure 3. The same as in Fig.2. for transmitted quanta

$$\rho\,(x,\tau_0) = \sum_{n=1}^{\infty} \rho_n\,(x,\tau_0) F_n(z), \qquad q\,(x,\tau_0) = \sum_{n=1}^{\infty} q_n\,(x,\tau_0) F_n(z), \tag{10}$$

where

$$F_n(z) = \frac{z^{2n-1}}{(2n-1)!}\mathrm{e}^{-z} \qquad n = 1, 2, ... \tag{11}$$

is the so-called Erlang-n distribution. The dimensionless temporal variable $z = t/\bar{t}$ is measured in units of $\bar{t} = t_1 t_2/\,(t_1 + t_2)$, where t_1 is the average time an atom stays in an excited state and t_2 is the average time it takes a quantum to travel between two successive acts of scattering. The value of t_1 in most cases is on the order of 10^{-7} -10^{-8}sec which usually is much smaller than the values of t_2 commonly found in astrophysical applications. The latter is determined by the density of absorbing atoms/ions in a given line and physical conditions in the medium.

The physical meaning of terms in the series Eq. 10 is obvious: they describe the probability density function of the quantum's escape through one of the boundaries as a result of a certain number of n scattering acts. Typical examples of probability density functions (PDF) are shown in the left panels of Fig. 2 and Fig. 3.

These functions describe the evolution of the spectral line profiles formed at the boundaries of the medium when it is illuminated by a δ-function shaped pulse. We see that all components of the Neumann series, and hence the profiles themselves, take their maximal values, as it was said, at $z = 1$ to which some real time $t = \bar{t}$ corresponds. Knowledge of $\bar{t}$ allows to estimate the value of t_2 and

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making then an idea about density of a given sort of atoms and on some physical characteristics of the medium such as the level of ionization. Thus, it is possible in principle to use the real-time values of when certain lines reach their maxima to estimate the relative abundance of various chemical elements

In addition, we consider also the establishment of the equilibrium state regime under prolonged illumination of the medium by a source of the unit jump form given by the Heaviside H-function. The evolution of the line profiles in this case is described by the cumulative distribution function (CDF) given by

$$C_\rho(x,\tau_0,z_0) = \mathrm{e}^{-z_0} \sum_{n=1}^{\infty} \rho_n\left(x,\tau_0\right) \sum_{k=0}^{\infty} \frac{z^{2n+k}}{(2n+k)!}, \tag{12}$$

$$C_q(x,\tau_0,z_0) = \mathrm{e}^{-z_0} \sum_{n=1}^{\infty} q_n\left(x,\tau_0\right) \sum_{k=0}^{\infty} \frac{z^{2n+k}}{(2n+k)!} \tag{13}$$

A typical example of cumulative distributions for reflected and transmitted radiation is shown in the right panels of Fig. 2 and Fig. 3. In both processes we considered it is assumed that the incident radiation has a unit intensity of continuum. The CDF curves are characterised by a plateau due to an asymptotic tendency of the line radiation towards their limiting values which are found by treating on the base of classical theory of stationary radiation transfer in spectral lines. Compared with the PDF distribution, the growth here is much slower (Δz_0 is of the order of 1), however, the steady state regime can be usually considered established in about the time interval $5 \leq z_0 \leq 10$. The illustrations above clearly show that the absorption lines formed as a result of transmission through the medium are established faster than the emission lines in the reflected spectrum. Another feature of these processes is their duration which depends for a given medium on the level of multiple scattering in the medium and increases at larger values of λ. We will return to the dynamics of changes in the spectral line profiles when the stationary mode is established in the next section.

The identification of spectral lines alongside with the dynamics and saturation rate are the power tool in interpreting the physical picture of the dynamic process under study.

5. The evolution of the line profiles

Fig. 2 and 3 show the difference between the speeds of the processes associated with the reflection of radiation from the medium and the transmission through it. The latter have a faster time course, which is due to the possibility of transmittance without scattering.

Obviously, the picture of the processes in both reflection and transmission cases depends on both the optical thickness of the medium and the value of the scattering coefficient. In particular, the duration of changes in the observed spectral lines essentially depends for a fixed value of the optical thickness on the value of the scattering coefficient. As one would expect, the greater the role of multiple scattering in the medium, the longer duration of the line evolution. For the same reason the process is comparatively short in a medium of smaller optical thickness.

A characteristic feature of the maximum value of the PDF for transmitted radiation is its weak dependence on both the optical thickness and the value of the scattering coefficient, which is associated with the asymptotic approximation of the distribution to unity in the lines wings. As a result, the maximum value of the PDF becomes approximately $\mathrm{e}^{-1} = 0.35$. This value depends only on the intensity of the primary radiation incident on the medium in the continuum, which is taken as one in the paper. Therefore, a comparison of the theoretical and observed maximum values allows us to estimate the intensity of radiation falling on the medium. This is important when comparing two spectral lines distant from each other, when the incident energy depends on the frequency.

Further, by following the evolution of the absorption line profiles arising as the result of transmittance through the medium, it is possible to gain insight into both the optical thickness of the medium and the role of diffusion in the medium in the observed spectra. This can be done by comparing the different parameters of the theoretically derived distributions and the profiles of the observed lines. For example, the relative fraction of the total energy absorbed in the line during the period up to the maximum is calculated to be weakly dependent on the scattering coefficient over the whole interval of

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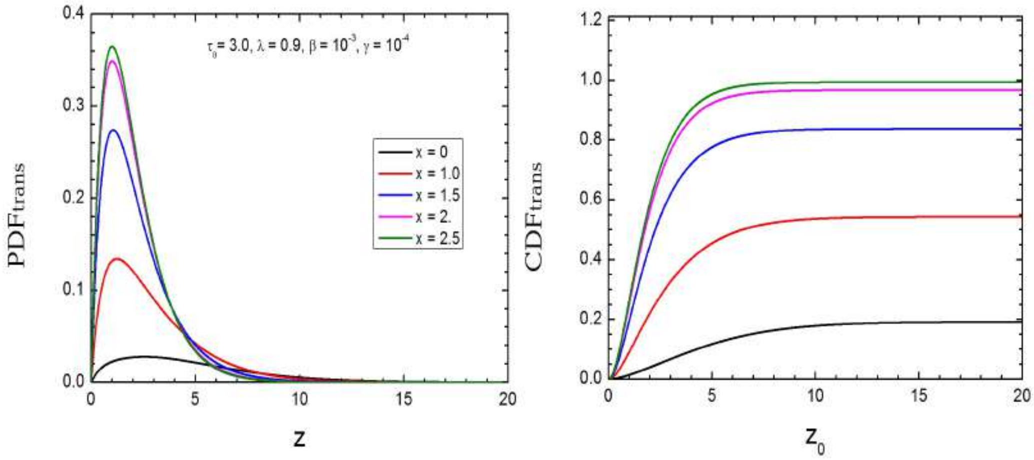

Figure 4. Probability distribution functions for reflected and transmitted quanta for $\lambda = 0,99$ in the case of completely non-coherent scattering

$0 < \lambda \leq 1$ and is accurately 22 per cent at $\tau_0 = 1$, and 19 per cent at $\tau_0 = 3$. Therefore these values can serve as an indicator of the optical thickness of the medium.

It is somewhat more difficult to estimate similar parameters when studying the evolution of the reflected emission line-spectrum. In this case, as it follows from Fig. 2 and 3, the maximum value of the reflection probability density function, as well as the duration of the whole process, are significantly dependent on the value of the scattering factor. Instead, these values for equal λ are weakly dependent on the optical thickness of the medium, what gives an indication of the value of the scattering coefficient.

For a confident interpretation of the temporal variations of the observed line-spectra, it is necessary to have data on the influence of various factors on the recorded evolution of the spectrum. Below we will consider some of these factors, the contribution of which should be considered important.

5.1. The effect of completely non-coherent scattering

In the above examples we assumed that the scattering in the spectral line is monochromatic. However, as we know, in most cases with strong lines we have to take into account the effect of redistribution of radiation over frequencies within the line. Here we consider the resulting effect for the case of completely incoherent scattering frequently used in various astrophysical problems. The corresponding PDF distributions for the reflected and transmitted radiation are depicted in Fig.4. Comparing them with the results shown in Fig. 2 and 3, we conclude that in the case of the frequency redistribution the lines evolve longer than in monochromatic scattering. It is also important the fact that, at a certain stage of the decline after the maximum, emission lines appear with double-peaked profiles at both boundaries of the medium. It may seem especially strange that an emission line can appear in the absorption spectrum after a long time (see also Fig. 6 below). Such phenomenon finds a simple physical explanation, consisting in that at a final stage of evolution, even at insignificant illumination of medium, still there is several number of diffusing quanta and the difference between radiation regimes at boundaries is reduced to a minimum.

A clearer picture of the evolution of spectral lines can be obtained from changes in their profiles over time. Fig. 5 and 6 demonstrate this evolution both before and after the PDF maximum is reached. The time variable grows with the increment $h = \Delta z$ =0.1. In the immediate vicinity of the maximum, profile changes slow down. After this, the decline begins, shown in Fig. 6. The time variable z varies here from top to bottom in ten times greater steps $h = \Delta z$=1.

Finally, let us also consider the case of prolonged illumination of the medium, as a result of which the radiation field in the medium tends to a stationary regime. Fig. 7 shows the evolution of spectral line profiles formed at the boundaries of the medium. One should pay attention to the essential differences between the evolutions of the observed profiles under the two illumination laws considered

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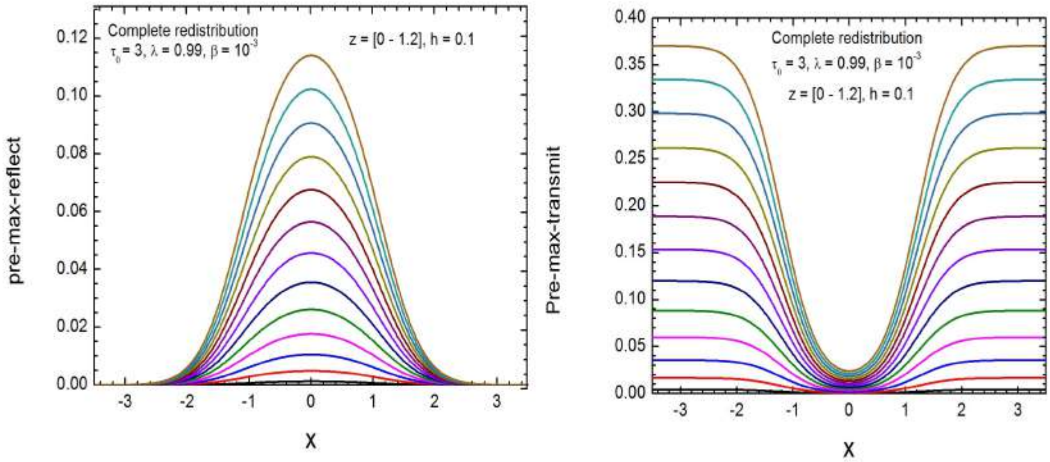

Figure 5. Evolution of the spectral line profile formed as a result of reflection (left) and transmission (right) before reaching the PDF maximum. The value of the time variable z grows from the bottom to top in increment $h = 0.1$.

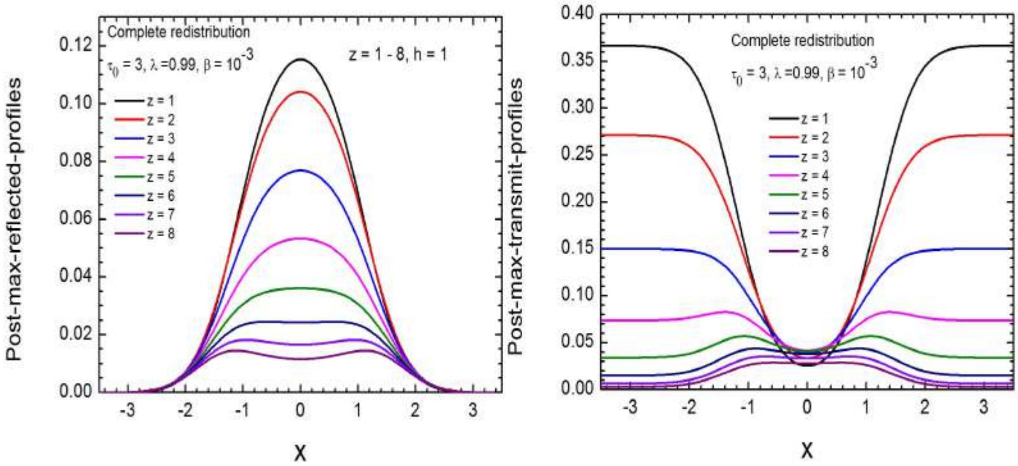

Figure 6. Evolution of the spectral lines profiles formed as a result of reflection (left) and transmission (right) after reaching the PDF maximum. The value of the time variable z grows from the top to bottom in increment $h = 1$.

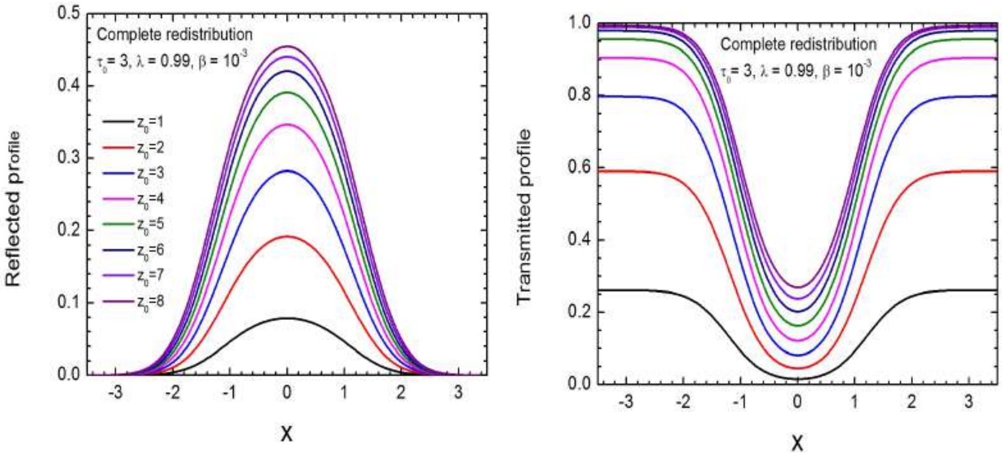

Figure 7. Evolution of the reflected (left) and transmitted (right) spectral lines profiles intending towards stationary regime during indicated time intervals.

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in the paper (see Fig. 5 and 7). It is of interest to compare the processes of spectral line amplification in the two types of energy sources of illumination considered in the paper. The main difference is that the specified growth in this case occurs about ten times slower. In addition, the speed of formation of the line profiles themselves are different in the line core and its wings. At a delta-shaped source, the line formed at reflection has its wings set earlier and only then the core, and the line formed after radiation passing through the medium has the opposite picture: the wings are set later than the core (Fig. 5). At the same time, as it is evident from Fig. 7, the scenario of formation of the spectral line profile is directly opposite.

5.2. The effect of inhomogeneity of the medium. The influence of the continuum scattering

Here we reveal the influence of medium inhomogeneity on the evolution of spectral lines. Obviously, in reality, each of the local optical properties of the medium can change from point to point in it. The crucial role in establishing the field of radiation in the medium plays the photon re-radiation probability in an elementary act of scattering λ, so we find it expedient to treat the effect of its probable variation with depth. Accounting for this type of medium inhomogeneity, while important, is also relatively complicated from the point of view of its theoretical implementation. For a clearer presentation of the results obtained, the characteristics of the continuous spectrum, such as the absorption and scattering coefficients, we consider here to be independent of the optical depth, despite the fact that their consideration does not lead to any fundamental difficulties.

As a simple example of the medium inhomogeneity effect, we consider a one-dimensional problem of monochromatic radiation transport, where the scattering coefficient in the line is given by the exponential law

$$\tilde{\lambda}\,(,\tau_0) = \left[1 + a(x)\mathrm{e}^{\mp v(x)\tau_0}\right]^{-1}, \tag{14}$$

where

$$a(x) = \frac{1-\tilde{\lambda}(x,0)}{\tilde{\lambda}(x,0)} = \frac{(1-\lambda\,(0))\,\alpha\,(x)+\beta}{\lambda\,(0)\,\alpha\,(x)+\gamma} \tag{15}$$

This dependence is encountered in the problems of spectral line formation within the framework of the two-level model, when, in addition to radiative transitions, collision transitions are taken into account. This takes place at relatively low temperatures and rather high concentrations of free electrons. We consider two opposite cases of exponential changes in the scattering coefficient with depth, which are either increasing or decreasing in approaching the illuminated boundary. They depend on choosing the boundary value of $\lambda(0)$ which we take, for better expository reasons, equal to $\lambda(0) = 0,5$ or 0,9 correspondingly for the negative and positive exponentials. Fig. 8 demonstrates the functions $\tilde{\lambda}(x,\tau)$ for these particular cases

We should pay attention to the dependence of the scattering coefficient in each of the cases under consideration on the frequency.

A visual representation of the differences between the two cases under consideration is given in Figs. 9, 10 which present changes in the spectral line profiles, arising from the reflection of radiation from the medium. For illustration of their evolution in the amplifying pre-maximum period, both in the absence of scattering in the continuum (left panels), and in its presence (right panels) are presented. Also for clarity, the value of the gamma parameter is assumed to be large enough, far from being always realized in astrophysical applications. The given figures allow us to make a number of important conclusions. As one might expect, accounting for scattering in the continuous spectrum leads to additional continuous emission, which weakens the lines themselves. The effect of inhomogeneity is demonstrated by Fig.10, which consists in the appearance of two-peaked profiles in the case when the level of diffusion of radiation in the medium decreases as it approaches the externally illuminated boundary. It can be also seen that as a result of additional continuous emission in this case one can observe two emission lines separated by the weak absorbtion component.

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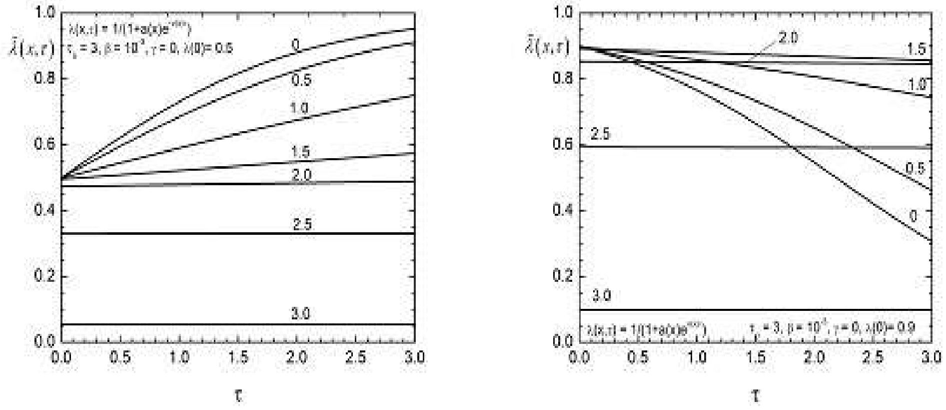

Figure 8. Functions $\tilde{\lambda}(x,\tau)$ in different frequencies for indicated values optical parameters.

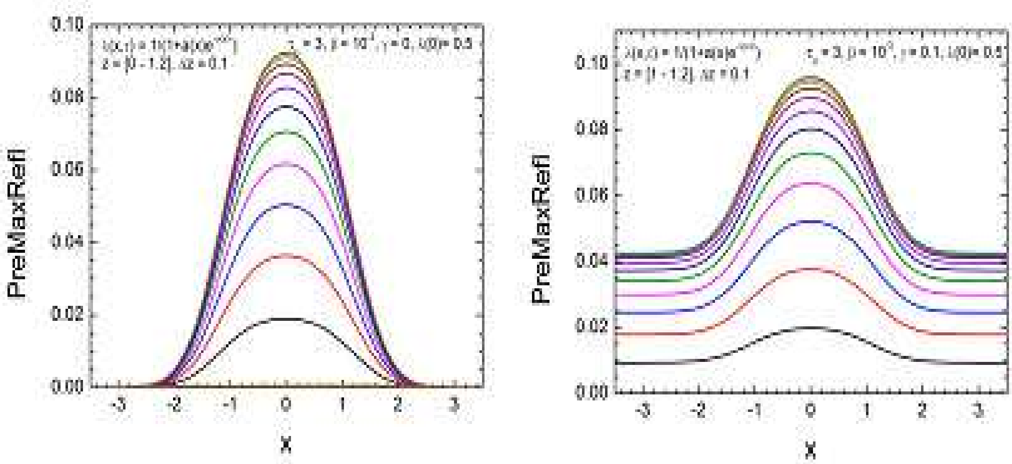

Figure 9. The evolution of the reflected pre-maximum lines profiles for $\lambda(0) = 0.5$.

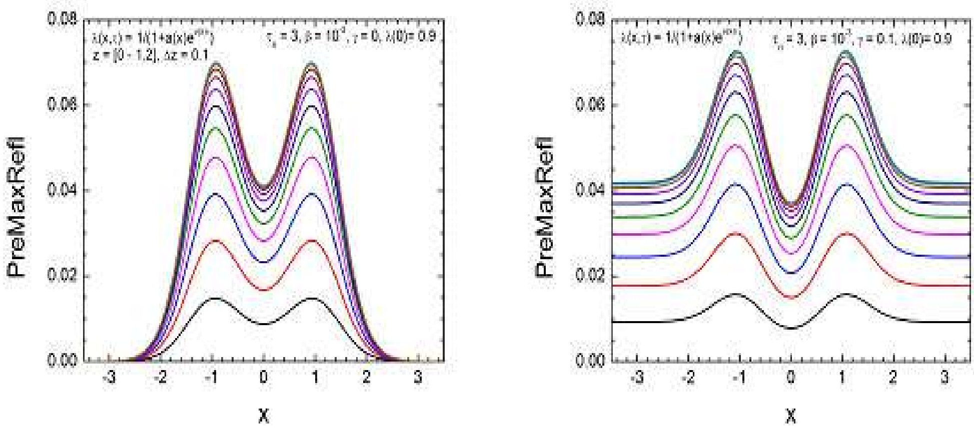

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Figure 10. The evolution of the reflected pre-maximum lines profiles for $\lambda(0) = 0.9$.

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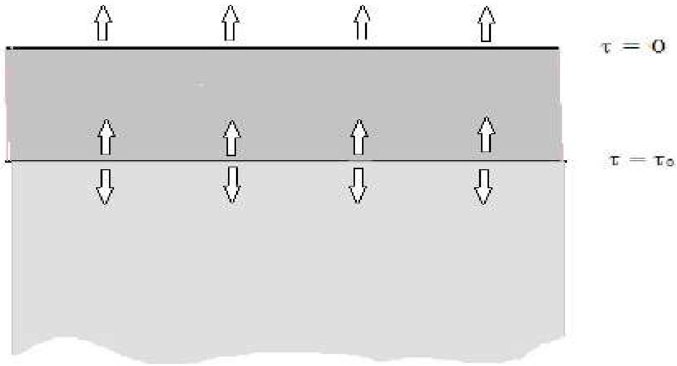

Figure 11. Schematic picture of the radiation transport in the presence of internal energy sources.

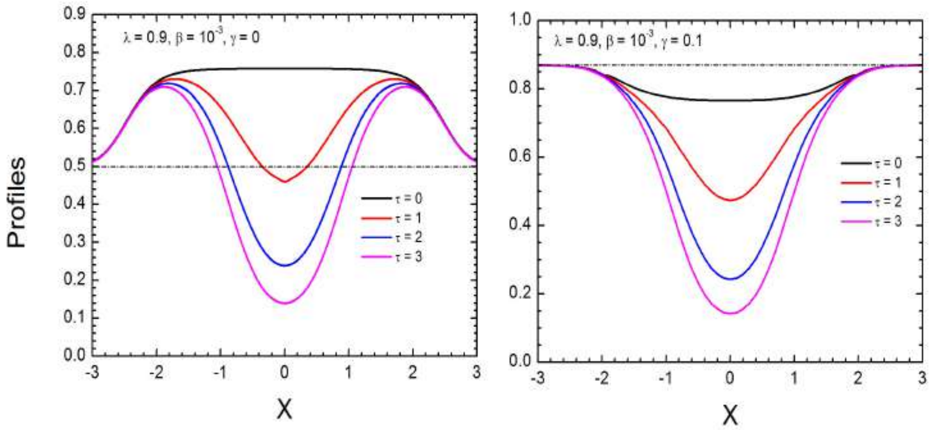

Figure 12. Profiles of spectral lines formed in the stationary regime in the semi-infinite absorbing and scattering atmosphere without (left) and with (right) continuum scattering.

5.3. Location of primary energy sources

In conclusion, let us apply the method we developed to reveal the effect of the location of the primary energy sources on the observed temporal characteristics of changes in the spectral lines. Consider this question in the simplest problem of spectral line formation in a one-dimensional semi-infinite atmosphere in the presence of an energy source at some optical depth τ_0, which radiates with the same probability of 0.5 in both directions by emission in a continuous spectrum with an intensity summarily equal to unity (Fig. 11). Fig. 12 shows the observed profiles of the lines formed by sources at different depths without considering the scattering in the continuum (left panel) and with this scattering taken into account (right panel). In both cases we obtain absorbtion profiles in spite of difference in the levels of the continuum spectrum. However in the absence of continuum scattering the lines have a specific form with emission components in the wings and absorbtion component in their core. There exists a clear correlation between the depth of the energy source location and the equivalent width and central residual intensity of lines (Nikoghossian, 2022b).

Fig. 13 demonstrates the PDF and CDF of at the center of observed spectral lines as a function of the depth of the energy source. Note that the time reading $z = z_0$ in the figure coincides with the moment of the beginning of the quanta' exit from the semi-infinite atmosphere. The strong relation between the depth of the energy source and the observed characteristics of the line allows to make an idea on the location and the some other characteristics of this source.

6. Concluding remarks

The goal we pursue is twofold: on the one hand, it is the further development of the non-stationary theory of the formation of a linear spectrum with the development of computational methods. On the

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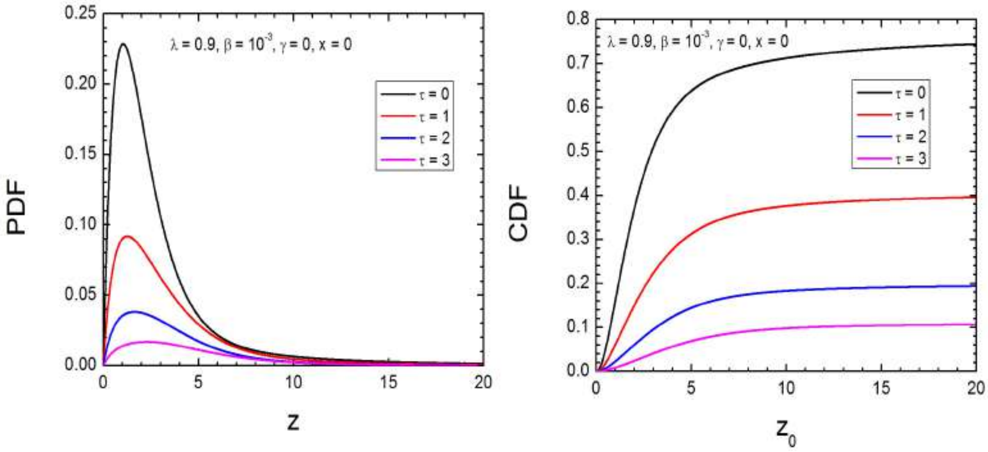

Figure 13. Typical graphs of PDF and CDF for the photons in the center of the line in the absence of continuum scattering when $\lambda = 0.9$.

other hand, it is their application in describing the evolution of spectral lines under certain physical conditions. Ultimately, the goal is how to use the information contained in the dynamic pattern of observed changes in spectra to find out the physical and optical properties of the medium in question and the chemical composition, spectrum of external and internal energy sources.

Two possible realizations of the observations are considered: spectra formed by reflection from the medium and those formed as a result of transmittance through it. Two types of primary energy sources are considered: those of the Dirac delta function and Heaviside H-functions forms. The results of numerical calculations concerning the evolution of spectral line profiles for each of the above cases are presented. The influence of various physical conditions, such as redistribution of radiation over frequencies, inhomogeneity of the medium, scattering in the continuous spectrum, location of energy sources, and various parameters of spectral lines on the shape, duration, and other characteristics of the evolution of the line spectra are described. All this provides ample opportunities for studying and interpreting, determining various physical characteristics of a large range of non-stationary processes of astrophysical interest.

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