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# Inertia I: The global $\mathrm{MS}_p$-SUSY induced uniform motion

G.Ter-Kazarian*

Byurakan Astrophysical Observatory, Byurakan, Aragatsotn Province, Armenia

**Abstract**

In this communication our main emphasis is on the review of the foundations of standard Lorentz code (SLC) of a particle motion. To this aim, we develop the theory of global, so-called, `double space´- or *master space* ($\mathrm{MS}_p$)-supersymmetry, subject to certain rules, wherein the superspace is a 14D-extension of a direct sum of background spaces $M_4\oplus$ $\mathrm{MS}_p$ by the inclusion of additional 8D fermionic coordinates. The latter is induced by the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to $\mathrm{MS}_p$. While all the particles are living on $M_4$, their superpartners can be viewed as living on $\mathrm{MS}_p$. This is a main ground for introducing $\mathrm{MS}_p$, which is *unmanifested* individual companion to the particle of interest. Supersymmetry transformation is defined as a translation in superspace, specified by the group element with corresponding anticommuting parameters. The multiplication of two successive transformations induce the motion. As a corollary, we derive SLC in a new perspective of global double $\mathrm{MS}_p$-SUSY transformations in terms of Lorentz spinors $(\underline{\theta},\bar{\underline{\theta}})$. This calls for a complete reconsideration of our ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property. In $\mathrm{MS}_p$-SUSY theory, obviously as in standard unbroken SUSY theory, the vacuum zero point energy problem, standing before any quantum field theory in $M_4$, is solved. The particles in $M_4$ themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4\oplus$ $\mathrm{MS}_p$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2\equiv\nu_b=\nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy. On these premises, we derive the two postulates on which the Special Relativity (SR) is based.

***Keywords:*** *Special relativity–Lorentz and Poincaré invariance–Supersymmetry–Supersymmetric models*

## 1. Introduction

In the present article we study the first part of the phenomenon of inertia dedicated to the *inertial uniform motion*. This article is a more detailed exposition of the first part of work (Ter-Kazarian, 2024). Governing the motions of planets, both fundamental phenomena of nature the gravity and inertia reside at the very beginning of physics. Despite the advocated success of general relativity (GR) in explaining the gravity, which was a significant landmark in the development of the field, the problem of inertia stood open and it is still the most important incomprehensive problem that needs to be solved. Today there is no known feasible way to account for credible explanation of this problem, consisting of two parts: the *inertial uniform motion* of a body, and how this is affected by applied forces (the *accelerated motion and inertia effects*).

The beginning of the study of phenomenon of inertia can be attributed to the works of Galileo (Drake, 1978) and Newton (Newton, 1687). Certainly, more than four centuries passed since the famous far-reaching discovery of Galileo (in 1602-1604) that all bodies fall at the same rate (Drake, 1978), which led to an early empirical version of the suggestion that the gravity and inertia may somehow result from a single mechanism. Besides describing these early gravitational experiments, Newton in *Principia Mathematica* (Newton, 1687) has proposed a comprehensive approach to studying the relation between the gravitational and inertial masses of a body. Ever since, there is an ongoing quest to understand the reason for the universality of the gravity and inertia, attributing to the *weak* principle of equivalence (WPE), which establishes the independence of free-fall trajectories of the internal composition and structure of bodies. The variety of consequences of the precision experiments from astrophysical observations makes it possible to probe this fundamental issue more deeply by imposing the constraints of various analyzes. Currently, the observations

*gago_50@yahoo.com


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performed in the Earth-Moon-Sun system, or at galactic and cosmological scales, probe more deeply the WPE.

The inertia effects cannot be in full generality identified with gravity within GR as it was proposed by Einstein in 1907, because there are many experimental controversies to question the validity of such a description, for details see (Ter-Kazarian, 2012) and references therein. The universality of the gravity and inertia effects attribute to the geometry but as having a different natures. Unlike gravity, here a curvature has arisen entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a "fictitious gravity" which can be globally removed by appropriate coordinate transformations, refers to this coordinate system itself, without relation to other systems or matter fields. The key to our construction procedure of the toy model (Ter-Kazarian, 2012) is an assignment to each and every particle individually a new fundamental constituent of hypothetical 2D, so-called, *master-space* ($MS_p$), subject to certain rules. The $MS_p$, embedded in the background 4D-space, is an *unmanifested* indispensable individual companion to the particle of interest. This together with the idea that the inertia effects arise as a deformation/(distortion of local internal properties) of $MS_p$, are the highlights of the alternative *relativistic theory of inertia* (RTI). The crucial point is to observe that, in spite of totally different and independent physical sources of gravity and inertia, the RTI furnishes justification for the introduction of the WPE.

However, the RTI obviously is incomplete theory unless it has conceptual problems for further motivation and justification of introducing the fundamental concept of $MS_p$. The way we assigned such a property to the $MS_p$ is completely *ad hoc* and there are some obscure aspects of this hypothesis. Moreover, this theory should certainly be incomplete without revealing the physical processes that underly the *inertial uniform motion* of a particle in flat space. Therefore, the present paper purports to develop a consistent solution of this problem, which is probably the most fascinating challenge for physical research.

From its historical development the principles of classical mechanics are constructed on the premises of our experience about relative locations and relative motions. In reducing the decision to that of whether a body is at rest or in *uniform motion* is disjunctive and, therefore, it is inherently indeterminate. This aspect of mechanics which deserves further investigation, unfortunately, has attracted little attention in subsequent developments. There is nothing in the basic postulates of physics to decide on the issue. The view that the problem of motion can be completely discussed in terms of observables implies that a kinematical description of all the relative motions in the universe completely specifies the system, so that kinematically equivalent motions must be dynamically equivalent. The notion of *uniform motion* of a particle, which is the mill-stone put into physics by hand, at first glance seemed to be classified as an empirical term rather than as a notion of pure reason and, thus, this is not a subject of perception. Although this question seems to be a purely philosophical problem, nevertheless, it has an outstanding physical significance for fundamentals of physics, and the SR in particular. Certainly, in recent years the violation of CPT and Lorentz invariance at huge energies has become a major preoccupation of physicists. This idea gathers support from a breakthrough made in recent observational and theoretical efforts in this field.

One of the achievements of present experimental high energy astrophysics is the testing of violation of SR for ultra-high energy cosmic rays (UHECRs) and TeV-$\gamma$ photons observed (for a comprehensive review see (Batista & et al., 2019)). The Lorentz invariance violation (LIV) phenomenology for UHECRs has been intensely studied in the last few decades, even though the progress has been dramatic. A propagation in intergalactic space through the cosmic microwave background (CMB) radiation necessarily should reduce energy of UHECRs below the Greisen-Zatsepin-Kuzmin (GZK) limit, $5 \times 10^{19}$ eV. The existence of this effect is uncertain owing to conflicting observational data and small number statistics. This is commonly referred as the GZK anomaly. Observed air showers experiment show an excess of muons compared to predictions of standard hadronic interaction models above $\simeq 10^{16}$ eV, which indicates a longer-than-expected muon attenuation length. If the muon excess cannot be explained by improved hadronic event generators with in the Standard Model, new physics could qualitatively play a role. Comparatively large increases of muon number over a small primary energy range would likely be a hint for Lorentz symmetry violation at very high Lorentz factors.

The TeV-$\gamma$ paradox is another one related to the transparency of the CMB. The HEGRA has detected high-energy photons with a spectrum ranging up to 24 TeV (Aharonian & et al., 1999) from Mk 501, a BL Lac object at a redshift of 0.034 ($\sim$ 157 Mpc). The recent confirmation that at least some $\gamma$-ray bursts originate at cosmological distances (Metzger & et al., 1997, van Paradis & et al., 1997) suggests that the radiation from these sources could be used to probe some of the fundamental laws of physics.

Most ideas in phenomenology reflect the expectation that the characteristic scale of quantum spacetime effects, which will strongly affect the nature of spacetime, should be within a two or three orders of magnitude


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of the Planck scale, $E_P = M_P c^2$ ($\sim 1.22 \times 10^{19}$ GeV). Enormous efforts have been made over the past decade to test LIV as quantum spacetime effects in terms of expansions in powers of Planck energy scale in various scenarious of quantum gravity, see e.g. (Alfaro & Palma, 2003, Amelino-Camelia & Piran, 2001, Mattingly, 2005). But since the theory of quantum gravity based on noncommutative geometry is at earlier stage of development, and the possibility to preserve a Lorentz-invariant interpretation of noncommutative spacetime is still not excluded (Balachandran et al., 2008, Chaichian et al., 2004, Fiore & Wess, 2007), it is natural to tie the LIV to various alternatives for the yet nonexistent theory, see e.g. (Alfaro & Palma, 2002, 2003, Alfaro et al., 2002a,b). However, even thanks to the fruitful interplay between phenomenological analysis and high energy astronomical experiments, the scientific situation remains, in fact, more inconsistent to day. Moreover, a systematic analysis of these properties happens to be surprisingly difficult by conventional theoretical methods.

In the present paper we develop on so-called \`double space´- or \`$\mathrm{MS}_p$-SUSY´ theory, subject to certain rules. The fundamental notion of the particle *motion* in full generality relies on the concept of locality which refers to the original continuous spacetime. The important reason to question the validity of such a description is the fact that we do not understand the nature of phenomenon of *motion*. It must suffice to expect some objections against the idealization of an arbitrarily precise localization in terms of points in spacetime. In the first step it proves necessary to introduce a constitutive ansatz of the simple, yet tentative, *unmanifested* intermediate *motion* state, as shown in Section 2. We derive SLC in a new perspective of global double $\mathrm{MS}_p$-SUSY transformations in terms of Lorentz spinors $(\underline{\theta}, \bar{\underline{\theta}})$ referred to $\mathrm{MS}_p$. This allows to introduce the physical finite *relative time* interval between two events as integer number of the *own atomic duration time* of double transition of a particle from $M_4$ to $\mathrm{MS}_p$ and back. This is a main ground for introducing $\mathrm{MS}_p$, which is *unmanifested* individual companion to the particle of interest. We place the emphasis on the fundamental difference between the standard SUSY theories and some rather unusual properties of $\mathrm{MS}_p$-SUSY theory. While the standard SUSY theory can be realized only as a spontaneously broken symmetry since the experiments do not show elementary particles to be accompanied by superpartners with different spin but identical mass, the $\mathrm{MS}_p$-SUSY, in contrary, is realized as an *exact* SUSY, where all particles are living on 4D Minkowski space, but their superpartners can be viewed as living on $\mathrm{MS}_p$.

With this perspective in sight, we will proceed according to the following structure. To start with, Section 2 is devoted to probing SLC behind the \`double space´- or $\mathrm{MS}_p$-SUSY. Giving a first glance at $\mathrm{MS}_p$ in Subsection 2.1, we motivate and justify its introduction, and outline the objectives of the proposed symmetry. In Section 3 we give a hard look at $\mathrm{MS}_p$. The $\mathrm{MS}_p$-SUSY is worked out as a guiding principle in Section 4. In Section 5, we turn to non-trivial linear representation of $\mathrm{MS}_p$-SUSY algebra. In Section 6, we discuss the general superfields. On these premises, in Section 7, we derive the two postulates on which the theory of SR is based. In light of the absence of compelling experimental evidence for LIV, there does not appear to be any immediate motivation for the proposed theory. Therefore, as a physical outlook and concluding remarks, we list in Section 8 of what we think is the most important that distinguish this theory from phenomenological approaches of differing MAVs in the published literature. For brevity, whenever possible undotted and dotted spinor indices often can be ruthlessly suppressed without ambiguity. Unless indicated otherwise, we take natural units, $h = c = 1$.

## 2. Probing SLC behind the \`double space´- or $\mathrm{MS}_p$-SUSY

With regard to our original question as to the understanding of the physical processes underlying the *motion*, we tackle the problem in the framework of quantum field theory. In what follows, we should compare and contrast the particle quantum states defined on the two background spaces $M_4$ and $\mathrm{MS}_p$, forming a basis in the Hilbert space. Let us consider functional integrals for a quantum-mechanical system with one degree of freedom. Denote by $x(t)$ the position operator in the Heisenberg picture, and by $|x, t>$ its eigenstates. The Schwinger transformation function, $F(x't'; xt)$, (Milton, 2000, 2015, Schwinger, 1960, 2000), is the probability amplitude that a particle which was at $x$ at time $t$, on the path with no coinciding points, will be at point $x'$ at time $t'$. To express the function $F(x't'; xt)$ as a path integral in $M_4$, we usually choose $n$ intermediate points $(x''_i, t''_i)$ on the path and divide the finite time interval into $n+1$ small intervals: $t = t_0, t_1, \ldots, t_{n+1} = t'$;  $t_i = t_0 + i\varepsilon$, where $\varepsilon$ can be made arbitrarily small by increasing $n$. According to Trotter product formula (Hall, 2015), the noncommutativity of the kinetic and potential energy operators


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can be ignored. Then $F(x't';xt)$ can be computed as a product of ordinary integrals

$$F(x't';xt) = \int dx'' < x't' \, |x''t'' >< x''t''|xt > . \tag{1}$$

In the limit $n \to \infty$, the function $F(x't';xt)$ becomes an operational definition of the path integral.

### 2.1. Motivation of MS$_p$: In search of symmetry

We assume that a flat MS$_p$ is the 2D composite space MS$_p$ $\equiv \underline{M}_2$ (see (3)). The elementary act of particle motion at each time step $(t_i)$ through the infinitely small spatial interval $\triangle x_i = (x_{i+1} - x_i)$ during the time interval $\triangle t_i = (t_{i+1} - t_i) = \varepsilon$ is probably the most fascinating challenge for physical research. Since this is beyond our perception, it appears legitimate to consider extension to the infinitesimal Schwinger transformation function, $F_{ext}(x_{i+1}, t_{i+1}; x_i, t_i)$, in fundamentally different aspect.

We hypothesize that

*in the limit $n \to \infty (\varepsilon \to 0)$, the elementary act of motion consists of an `annihilation´ of a particle at point $(x_i, t_i) \in M_4$, which can be thought of as the transition from initial state $|x_i, t_i >$ into unmanifested intermediate state, so-called, `motion´ state, $|\underline{x}_i, \underline{t}_i >$, and of subsequent `creation´ of a particle at infinitely close final point $(x_{i+1}, t_{i+1}) \in M_4$, which means the transition from `motion´ state, $|\underline{x}_i, \underline{t}_i >$, into final state, $|x_{i+1}, t_{i+1} >$. The motion state, $|\underline{x}_i, \underline{t}_i) >$, should be defined on unmanifested `master´ space, $\underline{M}_2$, which includes the points of all the atomic elements, $(\underline{x}_i, \underline{t}_i) \in \underline{M}_2$ $(i = 1, 2, ...)$.*

This furnishes justification for an introduction of *unmanifested master space*, $\underline{M}_2$. A fundamental composition property of transformation functions (Schwinger, 2000), written in this limit for the extended Schwinger transformation function,

$$\lim_{n\to\infty} \left( F_{ext}(x_{i+1}, t_{i+1}; x_i, t_i) = \sum_{\underline{x}_i} < x_{i+1}, t_{i+1} | \underline{x}_i, \underline{t}_i >< \underline{x}_i, \underline{t}_i | x_i, t_i > \right), \tag{2}$$

would evidently imply that the number of intermediate points $(\underline{x}_i)$ should be set to one. The possibility of contemplating such a mechanism of motion is facilitated by the fact that although at first glance this is the only, not necessarily the most perfect, but rather clear way of setting the problem. To clarify the setup of the extended Schwinger transformation function, it should help a few noteworthy points of Fig. 1. The net result of each atomic double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ (from $M_4$ to $\underline{M}_2$ and back) is as if we had operated with a space-time *translation* on the original space $M_4$. So, the symmetry we are looking for must mix the particle quantum states during its propagation in order to reproduce the central relationship between the two successive transformations of this symmetry and the generators of space-time translations. Namely, the subsequent operation of two finite transformations will induce a translation in space and time of the states on which they operate. Such successive transformations will induce in $M_4$ the inhomogeneous Lorentz group, or Poincaré group, and that the unitary linear transformation $|x, t > \to U(\Lambda, a)|x, t >$ on vectors in the physical Hilbert space. Thus, the underlying algebraic structure of this symmetry generators closes with the algebra of *translations* on the original space $M_4$ in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra, including the generators of translations. Essentially the only truly appealing possibility of known symmetry possessing such manageable properties is the supersymmetry (SUSY), see e.g. (Aitchison, 2007, Baer & Tata, 2006, Dreiner et al., 2004, Fayet & Ferrara, 1977, Ferrara et al., 1974, Sohnius, 1985, Wess & Bagger, 1993, West, 1987). SUSY is accepted as a legitimate feature of nature, although the presence of specific `sparticle´ modes may be of some concern, since they have not (yet) been observed, except a few examples (Aharonov & Casher, 1979, Jackiw, 1984, Landau, 1930, Ravndal, 1980). SUSY-multiplets contain different spins but are always degenerate in mass and SUSY must be broken in nature where elementary particles do not come in mass-degenerate multiplets.

The successive atomic double transitions of a particle $M_4 \rightleftharpoons \underline{M}_2$ can necessarily be investigated coherently, at least conceptually, with SUSY-theory techniques, but certainly, we need to drastically change the scope of the standard SUSY to build up the `double space´- or `MS$_p$´-SUSY theory. As we will see, this is firstly related to the `superspace´ which is a direct sum extension of background double spaces $M_4 \oplus \underline{M}_2$, with an inclusion of additional fermionic coordinates induced by the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, which refer to $\underline{M}_2$. Secondly, thanks to the embedding map (10), the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, in turn, induce the spinors $\theta(\underline{\theta}, \bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta}, \bar{\underline{\theta}})$ (see (93)), as to $M_4$. Consequently, the `symmetric´ superspace can be parameterized by


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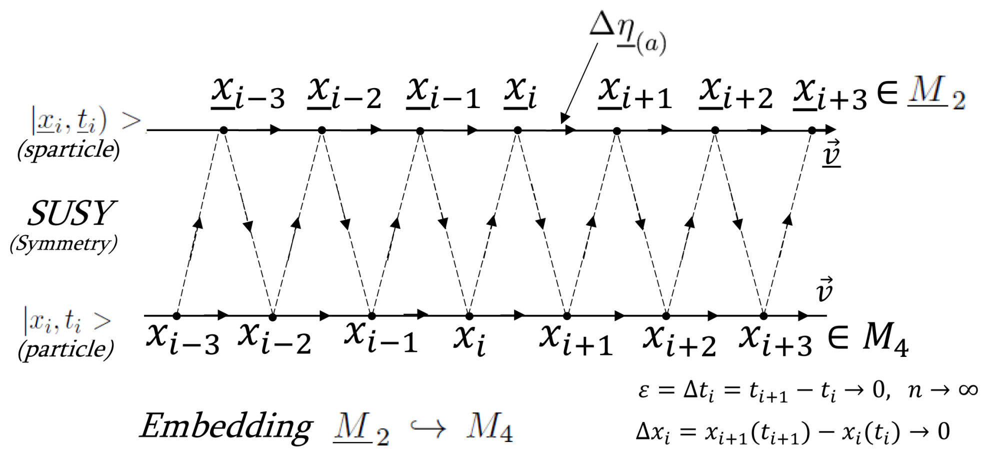


Figure 1. The extended Schwinger transformation function geometry. The `master´ space, MS$_p \equiv \underline{M}_2$, embedded in the background 4D-space $M_4$, is an unmanifested indispensable individual companion to the particle of interest, devoid of any external influence. A *creation* of a particle in $\underline{M}_2$ means its transition from initial state defined on $M_4$ into intermediate state defined on $\underline{M}_2$, while an *annihilation* of a particle in $\underline{M}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in $M_4$. The net result of each atomic double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ (from $M_4$ to $\underline{M}_2$ and back) is as if we had operated with a space-time *translation* on the original space $M_4$. The *atomic displacement*, $\Delta\underline{\eta}_{(a)}$, is caused in $\underline{M}_2$ by double transition of a particle, $\underline{M}_2 \rightleftharpoons M_4$. All the particles are living on $M_4$, while their superpartners can be viewed as living on $\underline{M}_2$.

$\Omega = \Omega_q(x^m,\, \theta(\underline{\theta},\bar{\underline{\theta}}),\, \bar{\theta}(\underline{\theta},\bar{\underline{\theta}})) \times \Omega_{\underline{q}}(\underline{\eta}^{\underline{m}},\, \underline{\theta},\, \bar{\underline{\theta}})$(*see* (66)). This is just enough to achieve the desired goal of deriving SLC in terms of spinors related to $\underline{M}_2$. This allows to introduce the physical finite *relative time* interval between two events, as integer number of the *own atomic duration time* of double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ (97). It is to be stressed that the ground state of MS$_p$-SUSY model has a vanishing energy value and is nondegenerate (SUSY unbroken). All the particles are living on $M_4$, and their superpartners can be viewed as living on $\underline{M}_2$. The particles in $M_4$ themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4 \oplus \underline{M}_2$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2 \equiv \nu_b = \nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy.

## 3. A hard look at MS$p$: Embedding $\underline{M}_2 \hookrightarrow M_4$

A notable conceptual element of our approach is the concept of, so-called, `master´ space (MS$_p$), which is indispensably tied to propagating particle of interest without relation to the other particles. The geometry of MS$_p$ is a new physical entity, with degrees of freedom and a dynamics of its own. We assume that a flat MS$_p$ is the 2D composite space,

$$\underline{M}_2 = \underline{R}^1_{(+)} \oplus \underline{R}^1_{(-)}. \tag{3}$$

The ingredient 1D-space $\underline{R}^1_{\underline{m}}$ is spanned by the coordinates $\underline{\eta}^{\underline{m}}$. The following notational conventions are used throughout this paper: all quantities related to the space $\underline{M}_2$ will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, ... = (\pm)$ denote the world indices related to $\underline{M}_2$. The Lorentz metric in $\underline{M}_2$ is

$$\underline{g} = \underline{g}(\underline{e}_{\underline{m}},\, \underline{e}_{\underline{n}})\, \underline{\vartheta}^{\underline{m}} \otimes \underline{\vartheta}^{\underline{n}}, \tag{4}$$

where $\underline{\vartheta}^{\underline{m}} = d\underline{\eta}^{\underline{m}}$ is the infinitesimal displacement. The basis $\underline{e}_{\underline{m}}$ at the point of interest in $\underline{M}_2$ is consisted of two real *null vectors*:

$$\underline{g}(\underline{e}_{\underline{m}},\, \underline{e}_{\underline{n}}) \equiv < \underline{e}_{\underline{m}},\, \underline{e}_{\underline{n}} >= {}^*o_{\underline{m}\,\underline{n}},\; ({}^*o_{\underline{m}\,\underline{n}}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

The norm, $i\underline{d} \equiv d\hat{\underline{\eta}}$, reads $i\underline{d} = \underline{e}\,\underline{\vartheta} = \underline{e}_{\underline{m}} \otimes \underline{\vartheta}^{\underline{m}}$, where $i\underline{d}$ is the tautological tensor field of type (1,1), $\underline{e}$ is a shorthand for the collection of the 2-tuplet $(\underline{e}_{(+)},\, \underline{e}_{(-)})$, and $\underline{\vartheta} = \begin{pmatrix} \underline{\vartheta}^{(+)} \\ \underline{\vartheta}^{(-)} \end{pmatrix}$. We may equivalently use a


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temporal $\underline{x}^{\underline{0}} \in \underline{T}^{\underline{1}}$ and a spatial $\underline{x}^{\underline{1}} \in \underline{R}^{\underline{1}}$ variables $\underline{x}^{\underline{r}}(\underline{x}^{\underline{0}},\, \underline{x}^{\underline{1}})(\underline{r} = \underline{0},\, \underline{1})$, such that

$$\underline{M}_2 = \underline{R}^{\underline{1}} \oplus \underline{T}^{\underline{1}}. \tag{6}$$

The norm, $i\underline{d}$, now can be rewritten in terms of displacement, $d\underline{x}^{\underline{r}}$, as

$$i\underline{d} = d\hat{\underline{x}} = \underline{e}_{\underline{0}} \otimes d\underline{x}^{\underline{0}} + \underline{e}_{\underline{1}} \otimes d\underline{x}^{\underline{1}}, \tag{7}$$

where $\underline{e}_{\underline{0}}$ and $\underline{e}_{\underline{1}}$ are, respectively, the temporal and spatial basis vectors:

$$\begin{array}{l} \underline{e}_{\underline{0}} = \frac{1}{\sqrt{2}}\left(\underline{e}_{(+)} + \underline{e}_{(-)}\right), \quad \underline{e}_{\underline{1}} = \frac{1}{\sqrt{2}}\left(\underline{e}_{(+)} - \underline{e}_{(-)}\right), \\ \underline{g}(e_{\underline{r}},\, e_{\underline{s}}) \equiv < e_{\underline{r}},\, e_{\underline{s}} >= diag(1, -1), \end{array} \tag{8}$$

and the corresponding coordinates are

$$\underline{x}^{\underline{0}} = \frac{1}{\sqrt{2}}\left(\underline{\eta}^{(+)} + \underline{\eta}^{(-)}\right), \quad \underline{x}^{\underline{1}} = \frac{1}{\sqrt{2}}\left(\underline{\eta}^{(+)} - \underline{\eta}^{(-)}\right). \tag{9}$$

Suppose the position of the particle is specified by the coordinates $x^m(s)$ $(x^0 = t)$ in the basis $e_m$ (m=0,1,2,3) at given point in the background $M_4$ space. Consider a smooth (injective and continuous) embedding $\underline{M}_2 \hookrightarrow M_4$. That is, a smooth map $f:\ \underline{M}_2 \longrightarrow M_4$ is defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{e}_{\underline{0}} = e_0, \quad \underline{x}^{\underline{0}} = x^0, \quad \underline{e}_{\underline{1}} = \vec{n}, \quad \underline{x}^{\underline{1}} = |\vec{x}|, \tag{10}$$

where $\vec{x} = e_i x^i = \vec{n}|\vec{x}|$ $(i = 1, 2, 3)$. Given the inertial frames $S_{(4)}, S'_{(4)}, S''_{(4)}, \ldots$ in unaccelerated uniform motion in $M_4$, we may define the corresponding inertial frames $\underline{S}_{(2)},\, \underline{S'}_{(2)},\, \underline{S''}_{(2)}, \ldots$ in $\underline{M}_2$, which are used by the non-accelerated observers for the positions $\underline{x}^{\underline{r}}, \underline{x}'^{\underline{r}}, \underline{x}''^{\underline{r}}, \ldots$ of a free particle in flat $\underline{M}_2$. According to (10), the time axes of the two systems $\underline{S}_{(2)}$ and $S_{(4)}$ coincide in direction, and the time coordinates are taken the same. For the case at hand,

$$\underline{v}^{(\pm)} = \frac{d\underline{\eta}^{(\pm)}}{d\underline{x}^{\underline{0}}} = \frac{1}{\sqrt{2}}(\underline{v}^{\underline{0}} \pm \underline{v}^{\underline{1}}), \quad \underline{v}^{\underline{1}} = \frac{d\underline{x}^{\underline{1}}}{d\underline{x}^{\underline{0}}} = |\vec{v}| = |\frac{d\vec{x}}{dx^0}|, \tag{11}$$

and that

$$\underline{u} = \underline{e}_{\underline{m}}\underline{v}^{\underline{m}} = (\underline{\vec{v}}_{\underline{0}}, \underline{\vec{v}}_{\underline{1}}), \quad \underline{\vec{v}}_{\underline{0}} = \underline{e}_{\underline{0}}\underline{v}^{\underline{0}}, \quad \underline{\vec{v}}_{\underline{1}} = \underline{e}_{\underline{1}}\underline{v}^{\underline{1}} = \vec{n}|\vec{v}| = \vec{v}, \tag{12}$$

therefore, $\underline{u} = u = (e_0, \vec{v})$. To explain why $MS_p$ is two dimensional, we note that only 2D real *null vectors* (8) are allowed as the basis at given point in $MS_p$, which is embedded in $M_4$. Literally speaking, the $\underline{M}_2$ can be viewed as 2D space living on the 4D world sheet.

Suppose the elements of the Hilbert space can be generated by the action of field-valued operators $\phi(x)\,(\chi(x),\, A(x))$ $(x \in M_4)$, where $\chi(x)$ is the Weyl fermion and $A(x)$ is the complex scalar bosonic field defined on $M_4$, and accordingly, of field-valued operators $\underline{\phi}(\underline{\eta})\,(\underline{\chi}(\underline{\eta})),\, \underline{A}(\underline{\eta}))$ $\underline{\eta} \in \underline{M}_2)$, where $\underline{\chi}(\eta)$ is the Weyl fermion and $\underline{A}(\eta)$ is the complex scalar bosonic field defined on $\underline{M}_2$, on the translationally invariant vacuum:

$$\begin{array}{lll} |x >= \phi(x)|0 >, & |x_1, x_2 >= \phi(x_1)\phi(x_2)|0 > & \text{(referring to } M_4), \\ |\underline{\eta} >= \underline{\phi}(\underline{\eta})|0 >, & |\underline{\eta}_1, \underline{\eta}_2 >= \underline{\phi}(\underline{\eta}_1)\underline{\phi}(\underline{\eta}_2)|0 > & \text{(referring to } \underline{M}_2). \end{array} \tag{13}$$

The displacement of the field takes the form

$$\phi(x_1 + x_2) = e^{ix_2^m p_m}\,\phi(x_1)\,e^{-ix_2^m p_m}, \quad \underline{\phi}(\underline{\eta}_1 + \underline{\eta}_2) = e^{i\underline{\eta}_2^{\underline{m}} \underline{p}_{\underline{m}}}\,\underline{\phi}(\underline{\eta}_1)\,e^{-i\underline{\eta}_2^{\underline{m}} \underline{p}_{\underline{m}}}, \tag{14}$$

where $p_m = i\partial_m$ is the generator of translations on quantum fields $\phi(x)$, and $\underline{p}_{\underline{m}} = i\underline{\partial}_{\underline{m}}$ is the generator of translations on quantum fields $\underline{\phi}(\underline{\eta}) \equiv \underline{\phi}(\underline{x}^{\underline{0}}, \underline{x}^{\underline{1}})$:

$$[\phi, p_m] = i\partial_m\,\phi, \quad [\underline{\phi}, \underline{p}_{\underline{m}}] = i\underline{\partial}_{\underline{m}}\,\underline{\phi}, \tag{15}$$

where, according to embedding map (10), $(\underline{m} = (\pm)$ or $\underline{0}, \underline{1})$. The relation between the fields $\phi(x)$ and $\underline{\phi}(\underline{\eta})$ can be given by the proper orthochronous Lorentz transformation. For a field of spin-$\vec{S}$, the general transformation law reads

$$\phi'_\alpha(x') = M_\alpha{}^\beta\,\phi_\beta(x) = \exp\left(-\tfrac{1}{2}\vartheta^{mn}S_{mn}\right)_\alpha{}^\beta\,\phi_\beta(x) = \exp\left(-i\vec{\vartheta}\cdot\vec{S} - i\vec{\zeta}\cdot\vec{K}\right)_\alpha{}^\beta\,\phi_\beta(x), \tag{16}$$


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where the two-by-two matrix $M$ ($M \in SL(2,C)$) of determinant one represents the action of the Lorentz group on two-component Weyl spinors, $\vec{\vartheta}$ is the rotation angle about an axis $\vec{k}$ ($\vec{\vartheta} \equiv \vartheta\vec{k}$), and $\vec{\zeta}$ is the boost vector $\vec{\zeta} \equiv \vec{e}_v \cdot \tan h^{-1}|\vec{v}|$, provided $\vec{e}_v \equiv \vec{v}/|\vec{v}|$, $\vartheta^i \equiv (1/2)\varepsilon^{ijk}\vartheta_k$ $(i,j,k = 1,2,3)$, and $\zeta^i \equiv \vartheta^{i0} = -\vartheta^{0i}$. The antisymmetric tensor $S_{mn} = -S_{nm}$, satisfying the commutation relations of the $SL(2.C)$, is the (finite-dimensional) irreducible matrix representations of the Lie algebra of the Lorentz group, and $\alpha$ and $\beta$ label the components of the matrix representation space, the dimension of which is related to the spin $S^i \equiv (1/2)\varepsilon^{ijk}S_k$ of the particle. The spin $\vec{S}$ generates three-dimensional rotations in space and the $K^i \equiv S^{0i}$ generate the Lorentz-boosts. The fields of spin-zero ($\vec{S} = \vec{K} = 0$) scalar field $A(x)$ and spin-one $A^n(x)$, corresponding to the $(1/2,1/2)$ representation, transform under a general Lorentz transformation as folloqws:

$$\begin{array}{ll} \underline{A}(\underline{\eta}) \equiv A(x), & (\text{spin} \quad 0); \\ \underline{A}^{\underline{m}}(\underline{\eta}) = \Lambda^{\underline{m}}_{\ n} A^n(x), & (\text{spin} \quad 1). \end{array} \tag{17}$$

The map from $SL(2,C)$ to the Lorentz group is established through the $\vec{\sigma}$-Pauli spin matrices, $\sigma^m = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \equiv (I_2, \vec{\sigma})$, $\bar{\sigma}^m \equiv (I_2, -\vec{\sigma})$, where $I_2$ is the identity two-by-two matrix. Both hermitian matrices $P$ and $P'$ or $\underline{P}$ and $\underline{P'}$ have expansions, respectively, in $\sigma$ or $\underline{\sigma}$:

$$(\sigma^m p'_m) = M(\sigma^m p_m)M^\dagger, \quad (\sigma^{\underline{m}} p'_{\underline{m}}) = M(\sigma^{\underline{m}} p_{\underline{m}})M^\dagger, \tag{18}$$

where $M(M \in SL(2,C))$ is unimodular two-by-two matrix. According to embedding map (10), the $\underline{\sigma}$-matrices are

$$\sigma^{\underline{m}} = \sigma^{(\pm)} = \tfrac{1}{\sqrt{2}}(\sigma^{\underline{0}} \pm \sigma^{\underline{1}}) = \tfrac{1}{\sqrt{2}}(\sigma^0 \pm \sigma^3). \tag{19}$$

The matrices $\sigma^{\underline{m}}$ form a basis for two-by-two complex matrices $\underline{P}$:

$$\underline{P} = (p_{\underline{m}}\sigma^{\underline{m}}) = (p_{(\pm)}\sigma^{(\pm)}) = (p_{\underline{0}}\sigma^{\underline{0}} + p_{\underline{1}}\sigma^{\underline{1}}), \tag{20}$$

provided $p_{(\pm)} = i\partial_{\underline{\eta}^{(\pm)}}$, $p_{\underline{0}} = i\partial_{\underline{x}^{\underline{0}}}$ and $p_{\underline{1}} = i\partial_{\underline{x}^{\underline{1}}}$. The real coefficients $p'_{\underline{m}}$ and $p_{\underline{m}}$, like $p'_m$ and $p_m$, are related by a Lorentz transformation $p'_{\underline{m}} = \Lambda^{\underline{n}}_{\underline{m}} p_{\underline{n}}$, because the relations $det(\sigma^{\underline{m}} p_{\underline{m}}) = p_{\underline{0}}^2 - p_{\underline{1}}^2$ and $det\, M = 1$ yield ${p'_{\underline{0}}}^2 - {p'_{\underline{1}}}^2 = p_{\underline{0}}^2 - p_{\underline{1}}^2$. Correspondence of $p_{\underline{m}}$ and $\underline{P}$ is uniquely: $p_{\underline{m}} = \frac{1}{2}Tr(\sigma^{\underline{m}}\underline{P})$, which combined with (18) yields

$$\Lambda^{\underline{m}}_{\underline{n}}(M) = \tfrac{1}{2}\, Tr\left(\sigma^{\underline{m}} M \sigma^{\underline{n}} M^\dagger\right). \tag{21}$$

Meanwhile $(\underline{\chi}\,\sigma^{\underline{m}}\bar{\underline{\zeta}})\,\underline{A}_{\underline{m}}$ is a Lorentz scalar if the following condition is satisfied:

$$\Lambda^{\underline{m}}_{\underline{n}}(M)\sigma^{\underline{n}}_{\alpha\dot{\alpha}} = (M^{-1})_\alpha{}^\beta \sigma^{\underline{m}}_{\beta\dot{\beta}} (M^{-1})^{\dagger\dot{\beta}}{}_{\dot{\alpha}}. \tag{22}$$

A two-component $(1/2,0)$ Weyl fermion, $\chi_\beta(x)$, therefore, transforms under Lorentz transformation to yield $\underline{\chi}_\alpha(\underline{\eta})$:

$$\chi_\beta(x) \longrightarrow \underline{\chi}_\alpha(\underline{\eta}) = (M_R)_\alpha{}^\beta\, \chi_\beta(x), \quad \alpha, \beta = 1, 2, \tag{23}$$

where the orthochronous Lorentz transformation, corresponding to a rotation by the angles $\vartheta_3$ and $\vartheta_2$ about, respectively, the axes $n_3$ and $n_2$, is given by rotation matrix

$$M_R = e^{i\frac{1}{2}\sigma_2\vartheta_2} e^{i\frac{1}{2}\sigma_3\vartheta_3}. \tag{24}$$

There with the rotation of an hermitian matrix $P$ is

$$p_{\underline{m}}\sigma^{\underline{m}} = M_R\, p_m \sigma^m\, M_R^\dagger, \tag{25}$$

where $p_m$ and $p_{\underline{m}}$ denote the momenta $p_m \equiv m(ch\beta,\ sh\beta \sin\vartheta_2 \cos\vartheta_3,\ sh\beta \sin\vartheta_2 \sin\vartheta_3,$ $sh\beta \cos\vartheta_2)$, and $p_{\underline{m}} \equiv m(ch\underline{\beta},\ 0,\ 0,\ sh\underline{\beta})$.

A two-component $(0,1/2)$ Weyl spinor field is denoted by $\bar{\chi}^{\dot{\beta}}(x)$, and transforms as

$$\bar{\chi}^{\dot{\beta}}(x) \longrightarrow \underline{\bar{\chi}}^{\dot{\alpha}}(\underline{\eta}) = (M_R^{-1})^{\dagger\dot{\alpha}}{}_{\dot{\beta}}\, \bar{\chi}^{\dot{\beta}}(x), \quad \dot{\alpha}, \dot{\beta} = 1, 2. \tag{26}$$

The so-called `dotted´ indices have been introduced to distinguish the $(0,1/2)$ representation from the $(1/2,0)$ representation. The `bar´ over the spinor is a convention that this is the $(0,1/2)$-representation.


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We used the Van der Waerden notations for the Weyl two-component formalism: $(\bar{\underline{\chi}}_{\dot{\alpha}})^* = \underline{\chi}_\alpha$ and $\bar{\underline{\chi}}_{\dot{\alpha}} = (\underline{\chi}_\alpha)^*$. The infinitesimal Lorentz transformation matrices for the $(1/2, 0)$ and $(0, 1/2)$ representations,

$$\begin{array}{ll} M \simeq I_2 - \frac{i}{2}\vec{\vartheta}\cdot\vec{\sigma} - \frac{1}{2}\vec{\zeta}\cdot\vec{\sigma}, & \text{for } (\frac{1}{2}, 0); \\ (M^{-1})^\dagger \simeq I_2 - \frac{i}{2}\vec{\vartheta}\cdot\vec{\sigma} + \frac{1}{2}\vec{\zeta}\cdot\vec{\sigma}, & \text{for } \quad (0, \frac{1}{2}) \end{array} \tag{27}$$

give $S^{mn} = \sigma^{mn}$ for the $(1/2, 0)$ representation, and $S^{mn} = \bar{\sigma}^{mn}$ for the $(0, 1/2)$ representation, where the bilinear covariants that transform as a Lorentz second-rank tensor read

$$(\sigma^{mn})_\alpha^{\ \beta} \equiv \frac{i}{4}(\sigma^m_{\alpha\dot{\alpha}}\,\bar{\sigma}^{n\dot{\alpha}\beta} - \sigma^n_{\alpha\dot{\alpha}}\,\bar{\sigma}^{m\dot{\alpha}\beta}), \quad (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\ \dot{\beta}} \equiv \frac{i}{4}(\bar{\sigma}^{m\dot{\alpha}\alpha}\,\sigma^n_{\alpha\dot{\beta}} - \bar{\sigma}^{n\dot{\alpha}\alpha}\,\sigma^m_{\alpha\dot{\beta}}), \tag{28}$$

provided $\bar{\sigma}^m \equiv (I_2;\, -\vec{\sigma})$, $(\sigma^{m\,*})_{\alpha\dot{\beta}} = \sigma^m_{\beta\dot{\alpha}}$ and $(\bar{\sigma}^{m\,*})^{\dot{\alpha}\beta} = \bar{\sigma}^{m\dot{\beta}\alpha}$.

## 4. The MS$_p$-SUSY

The theoretical significance resides in constructing the MS$_p$-SUSY as a guiding principle. If that is the case as above, a *creation* of a particle in $\underline{M}_2$ means its transition from initial state defined on $M_4$ into intermediate state defined on $\underline{M}_2$, while an *annihilation* of a particle in $\underline{M}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in $M_4$. All the fermionic and bosonic states taken together form a basis in the Hilbert space. The basis vectors in the Hilbert space composed of $H_B \otimes H_F$ is given by

$$\{|\underline{n}_b > \otimes |0>_f,\ |\underline{n}_b > \otimes f^\dagger\,|0>_f\},$$

or

$$\{|n_b > \otimes |\underline{0}>_f,\ |n_b > \otimes \underline{f}^\dagger\,|\underline{0}>_f\},$$

where we consider two pairs of creation and annihilation operators $(b^\dagger,\, b)$ and $(f^\dagger,\, f)$ for bosons and fermions, respectively, referred to the background space $M_4$, as well as $(\underline{b}^\dagger,\, \underline{b})$ and $(\underline{f}^\dagger,\, \underline{f})$ for bosons and fermions, respectively, as to background master space $\underline{M}_2$. The boson and fermion number operators are $N_b = b^\dagger b$ or $\underline{N}_b = \underline{b}^\dagger\underline{b}$, where $N_b|n_b >= n_b|n_b >$ and $\underline{N}_b|\underline{n}_b >= \underline{n}_b|\underline{n}_b >$ $(= 0, 1, ..., \infty)$, and $N_f = f^\dagger f$ or $\underline{N}_f = \underline{f}^\dagger\underline{f}$, provided $N_f|n_f >= n_f|n_f >$ and $\underline{N}_f|\underline{n}_f >= \underline{n}_f|\underline{n}_f >$ $(= 0, 1)$. Taking into account the action of $(b, b^\dagger)$ or $(\underline{b}, \underline{b}^\dagger)$ upon the eigenstates $|n_b >$ or $|\underline{n}_b >$, respectively:

$$\begin{array}{ll} b|n_b >= \sqrt{n_b}|n_b - 1>, & b^\dagger|n_b >= \sqrt{n_{b+1}}|n_b + 1>, \\ \underline{b}|\underline{n}_b >= \sqrt{\underline{n}_b}|\underline{n}_b - 1>, & \underline{b}^\dagger|\underline{n}_b >= \sqrt{\underline{n}_b + 1}|\underline{n}_b + 1>, \end{array} \tag{29}$$

we may construct the quantum operators, $(q^\dagger,\, \underline{q}^\dagger)$ and $(q,\, \underline{q})$, which replace bosons by fermions and vice versa:

$$\begin{array}{l} q\,|\underline{n}_b,\, n_f >= q_0\sqrt{\underline{n}_b}\,|\underline{n}_b - 1,\, n_f + 1>, \\ q^\dagger\,|\underline{n}_b,\, n_f >= q_0\sqrt{\underline{n}_b + 1}\,|\underline{n}_b + 1,\, n_f - 1>, \end{array} \tag{30}$$

and that

$$\begin{array}{l} \underline{q}\,|n_b,\, \underline{n}_f >= q_0\sqrt{n_b}\,|n_b - 1,\, \underline{n}_f + 1>, \\ \underline{q}^\dagger\,|n_b,\, \underline{n}_f > q_0\sqrt{n_b + 1}\,|n_b + 1,\, \underline{n}_f - 1>. \end{array} \tag{31}$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators $q$ and $\underline{q}$. Putting two operators in one $B = (\underline{b}$ or $b)$ and $F = (f$ or $\underline{f})$, the canonical quantization rules can be written most elegantly as

$$\begin{array}{l} [B,\, B^\dagger] = 1; \quad \{F,\, F^\dagger\} = 1; \quad [B,\, B] = [B^\dagger,\, B^\dagger] = \{F,\, F\} = \{F^\dagger,\, F^\dagger\} \\ = [B,\, F] = [B,\, F^\dagger] = [B^\dagger,\, F] = [B^\dagger,\, F^\dagger] = 0, \end{array} \tag{32}$$

where we note that $\delta_{ij}\delta^3(\vec{p} - \vec{p}')$ and $\delta_{ij}\delta^3(\vec{\underline{p}} - \vec{\underline{p}}')$ are the unit element 1 of the convolution product $*$, while according to embedding map (10), we have $\vec{\underline{p}} = \vec{n}|\vec{p}| = \vec{p}$ and $\vec{\underline{p}}' = \vec{p}'$. The operators $q$ and $\underline{q}$ can be constructed as

$$q = q_0,\underline{b}\,f^\dagger, \quad q^\dagger = q_0\,\underline{b}^\dagger\,f, \quad \underline{q} = q_0\,b\underline{f}^\dagger, \quad \underline{q}^\dagger = q_0\,b^\dagger\,\underline{f}. \tag{33}$$

So, we may refer the action of the supercharge operators $q$ and $q^\dagger$ to the background space $M_4$, having applied in the chain transformations of fermion $\chi$ (accompanied with the auxiliary field $F$ as it will be seen later on) to boson $\underline{A}$, defined on $\underline{M}_2$:

$$\longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow. \tag{34}$$


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Respectively, we may refer the action of the supercharge operators $\underline{q}$ and $\underline{q}^\dagger$ to the $\underline{M}_2$, having applied in the chain transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field $\underline{F}$) to boson $A$, defined on the background space $M_4$:

$$\longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow . \tag{35}$$

Written in one notation, $Q = (q \text{ or } \underline{q})$, the operators (33) become

$$Q = q_0\, B^\dagger F = (q \text{ or } \underline{q}), \quad Q^\dagger = q_0\, BF^\dagger = (q^\dagger \text{ or } \underline{q}^\dagger). \tag{36}$$

Due to nilpotent fermionic operators $F^2 = (F^\dagger)^2 = 0$, the operators $Q$ and $Q\dagger$ also are nilpotent: $Q^2 = (Q^\dagger)^2 = 0$. Hence, the quantum system can be described in one notation by the selfadjoint Hamiltonian $\mathcal{H} \equiv \{Q^\dagger, Q\} = (H_q \equiv \{q^\dagger, q\} \text{ or } H_{\underline{q}} \equiv \{\underline{q}^\dagger, \underline{q}\})$, and the generators $Q$ and $Q^\dagger$, where the commutators as well as anticommutators appear in the algebra of symmetry generators. Such an algebra involving commutators and anticommutators is called a Lie algebra (GLA):

$$\mathcal{H} = \{Q^\dagger, Q\} \geq 0; \quad [\mathcal{H}, Q] = [\mathcal{H}, Q^\dagger] = 0. \tag{37}$$

This is a sum of Hamiltonian of bosonic and fermionic noninteracting oscillators, which decouples, for $Q = q$, into

$$H_q = q_0^2\, (\underline{b}^\dagger \underline{b} + f^\dagger f) = q_0^2\, (\underline{b}^\dagger \underline{b} + \tfrac{1}{2}) + q_0^2\, (f^\dagger f - \tfrac{1}{2}) \equiv H_{\underline{b}} + H_f, \tag{38}$$

or, for $Q = \underline{q}$, into

$$H_{\underline{q}} = q_0^2\, (b^\dagger b + \underline{f}^\dagger \underline{f}) = q_0^2\, (b^\dagger b + \tfrac{1}{2}) + q_0^2\, (\underline{f}^\dagger \underline{f} - \tfrac{1}{2}) \equiv H_b + H_{\underline{f}}, \tag{39}$$

with the corresponding energies:

$$E_q = q_0^2\, (\underline{n}_b + \tfrac{1}{2}) + q_0^2\, (n_f - \tfrac{1}{2}), \quad E_{\underline{q}} = q_0^2\, (n_b + \tfrac{1}{2}) + q_0^2\, (\underline{n}_f - \tfrac{1}{2}). \tag{40}$$

The proposed algebra (37) becomes more clear in a normalization $q_0 = \sqrt{m}$:

$$\{Q^\dagger, Q\} = 2m; \quad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0. \tag{41}$$

The latter has underlying algebraic structure of the superalgebra for massive *one-particle* states in the rest frame of $N = 1$ SUSY theory without central charges. This is rather technical topic, and it requires care to do correctly. In what follows we only give a brief sketch. The extension of the MS$_p$-SUSY superalgebra (41) in general case when $\vec{p} = i\vec{\partial} \neq 0$ in $M_4$ or $\underline{\vec{p}_1} = i\underline{\vec{\partial}_1} \neq 0$ in $\underline{M}_2$, and assuming that the resulting motion of a particle in $M_4$ is governed by the Lorentz symmetries, the MS$_p$-SUSY algebra can then be summarized as a non-trivial extension of the Poincaré group algebra those of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to $M_4$. Moreover, if there are several spinor generators $Q_\alpha{}^i$ with $i = 1, ..., N$ - theory with $N-$extended supersymmetry, can be written as a GLA of SUSY field theories, with commuting and anticommuting generators:

$$\begin{array}{l} \{Q_\alpha{}^i, \bar{Q}^j{}_{\dot{\alpha}}\} = 2\delta^{ij}\, \sigma^{\hat{m}}_{\alpha\dot{\alpha}}\, p_{\hat{m}}; \\ \{Q_\alpha{}^i, Q_\beta{}^j\} = \{\bar{Q}^i{}_{\dot{\alpha}}, \bar{Q}^j{}_{\dot{\beta}}\} = 0; \\ [p_{\hat{m}}, Q_\alpha{}^i] = [p_{\hat{m}}, \bar{Q}^j{}_{\dot{\alpha}}] = 0, \quad [p_{\hat{m}}, p_{\hat{n}}] = 0. \end{array} \tag{42}$$

The odd part of the supersymmetry algebra is composed entirely of the spin-1/2 operators $Q_\alpha{}^i$, $Q_\beta{}^j$. In order to trace a maximal resemblance in outward appearance to the standard SUSY theories, here we set one notation $\hat{m} = (m \quad \text{if} \quad Q = q, \text{ or } \underline{m} \quad \text{if} \quad Q = \underline{q})$, and as before the indices $\alpha$ and $\dot{\alpha}$ run over 1 and 2.

The supersymmetry charges, $Q_\alpha{}^i$, can be obtained as usual Noether charges associated with a conserved fermionic Noether current, $J_\alpha{}^{i\,\hat{m}} = (j_\alpha{}^{i\,m} \text{ or } \underline{j}_\alpha{}^{i\,\underline{m}})$,

$$q_\alpha{}^i = \int d^3x\, j_\alpha{}^{i\,0}, \quad \text{or} \quad \underline{q}_\alpha{}^i = \int d\eta\, \underline{j}_\alpha{}^{i\,\underline{0}}. \tag{43}$$

The supercurrent $J_\alpha{}^{i\,\hat{m}}$ is an anticommuting six-vector, which also carries a spinor index, as befits the current associated with a symmetry with fermionic generators, where the supercurrent and its hermitian


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conjugate are separately conserved ($\partial_m j_\alpha{}^{im} = 0$ and $\partial_m \bar{j}_\alpha{}^{im} = 0$), and ($\partial_{\underline{m}} \underline{j}_\alpha{}^{i\underline{m}} = 0$ and $\partial_{\underline{m}} \bar{\underline{j}}_\alpha{}^{i\underline{m}} = 0$). So for both supercharges, $q$ and $\underline{q}$, we get a supersymmetric models, respectively:

$$\begin{aligned}
&\{q_\alpha{}^i,\, \bar{q}^j{}_{\dot\alpha}\} = 2\delta^{ij}\,\sigma^m_{\alpha\dot\alpha}\, p_m;\\
&\{q_\alpha{}^i,\, q_\beta{}^j\} = \{\bar{q}^i{}_{\dot\alpha},\, \bar{q}^j{}_{\dot\beta}\} = 0;\\
&[p_m,\, q_\alpha{}^i] = [p_m,\, \bar{q}^j{}_{\dot\alpha}] = 0, \quad [p_m,\, p_n] = 0.
\end{aligned} \tag{44}$$

and

$$\begin{aligned}
&\{\underline{q}_\alpha{}^i,\, \bar{\underline{q}}^j{}_{\dot\alpha}\} = 2\delta^{ij}\,\sigma^{\underline{m}}_{\alpha\dot\alpha}\, p_{\underline{m}};\\
&\{\underline{q}_\alpha{}^i,\, \underline{q}_\beta{}^j\} = \{\bar{\underline{q}}^i{}_{\dot\alpha},\, \bar{\underline{q}}^j{}_{\dot\beta}\} = 0;\\
&[p_{\underline{m}},\, \underline{q}_\alpha{}^i] = [p_{\underline{m}},\, \bar{\underline{q}}^j{}_{\dot\alpha}] = 0, \quad [p_{\underline{m}},\, p_{\underline{n}}] = 0.
\end{aligned} \tag{45}$$

**Remark:** In the standard theory, the Q's operate with fields defined in the single $M_4$ space. It is why the result of a Lorentz transformation in $M_4$ followed by a supersymmetry transformation is different from that when the order of the transformations is reversed (Sohnius, 1985). But, in the MS$_p$-SUSY theory, the Q's ((33), (36)) operate with fields defined on both $M_4$ and $\underline{M}_2$ spaces, fulfilling a transition of a particle between these spaces ($M_4 \rightleftharpoons \underline{M}_2$). The particle motion arises as a complex process of the global MS$_p$-SUSY double transformations, therefore we will obtain the same result if we reverse the order of the Lorentz and supersymmetry transformations.

We cut short further description of the unitary supersymmetry representations that give rise to the concept of supermultiplets, since they are so well known.

## 5. Non-trivial linear representation of the MS$_p$-SUSY algebra

With these guidelines to follow, we start by considering the simplest example of a supersymmetric theory in six dimensional background space $M_4 \oplus \underline{M}_2$ as the MS$_p$-generalization of free Wess-Zumino toy model (Wess & Zumino, 1974) of standard theory. To obtain a feeling for this model we may consider first example of non-trivial linear representation $(\psi, \mathcal{A}, \mathcal{F})$, of the MS$_p$-SUSY algebra. This has $N = 1$ and $s_0 = 0$, and contains two Weyl spinor states of a massive Majorana spinor $\psi(\chi,\, \underline{\chi})$, two complex scalar fields $\mathcal{A}(A,\, \underline{A})$, and two more real scalar degrees of freedom in the complex auxiliary fields $\mathcal{F}(F,\, \underline{F})$, which provide in supersymmetry theory the fermionic and bosonic degrees of freedom to be equal off-shell as well as on-shell, and are eliminated when one goes on-shell. The component multiplets, $(\psi, \mathcal{A}, \mathcal{F})$, are called the chiral or scalar multiplets. This model is instructive because it contains the essential elements of the MS$_p$-induced SUSY and, therefore, intended to be rather complementary to the superfield derivations given in next section.

The anticommuting (Grassmann) parameters $\epsilon^\alpha(\xi^\alpha,\, \underline{\xi}^\alpha)$ and $\bar\epsilon^\alpha(\bar\xi^\alpha,\, \bar{\underline{\xi}}^\alpha)$:

$$\begin{aligned}
&\{\epsilon^\alpha,\, \epsilon^\beta\} = \{\bar\epsilon^\alpha,\, \bar\epsilon^\beta\} = \{\epsilon^\alpha,\, \bar\epsilon^\beta\} = 0,\\
&\{\epsilon^\alpha,\, Q_\beta\} = \cdots = [p_{\hat m},\, \epsilon^\alpha] = 0,
\end{aligned} \tag{46}$$

allow us to write the algebra (42) for ($N = 1$) entirely in terms of commutators:

$$\begin{aligned}
&[\epsilon Q,\, \bar{Q}\bar\epsilon] = 2\epsilon\sigma^{\hat m}\bar\epsilon p_{\hat m},\\
&[\epsilon Q,\, \epsilon Q] = [\bar{Q}\bar\epsilon,\, \bar{Q}\bar\epsilon] = [p^{\hat m},\, \epsilon Q] = [p^{\hat m},\, \bar{Q}\bar\epsilon] = 0.
\end{aligned} \tag{47}$$

For brevity, here the indices $\epsilon Q = \epsilon^\alpha Q_\alpha$ and $\bar\epsilon\bar{Q} = \bar\epsilon_{\dot\alpha}\bar{Q}^{\dot\alpha}$ will be suppressed unless indicated otherwise. This supersymmetry transformation maps tensor fields $\mathcal{A}(A,\, \underline{A})$ into spinor fields $\psi(\chi,\, \underline{\chi})$ and vice versa. From the algebra (47) we see that $Q$ has mass dimension $1/2$. Therefore, as usual, fields of dimension $\ell$ transform into fields of dimension $\ell + 1/2$ or into derivatives of fields of lower dimension.

Starting with the scalar field $\underline{A}(\underline{\eta}) = A(x)$ (17), in the view of chain transformations (34), we should define the spinor $\chi$ as the field into which $\underline{A}(\underline{\eta})$ transforms:

$$\delta_\xi \underline{A} = (\xi q + \bar\xi\bar q) \times \underline{A} = \sqrt{2}\xi\chi. \tag{48}$$

The field $\chi$ transforms into a tensor field of higher dimension and into the derivative of $\underline{A}$ itself:

$$\delta_\xi \chi = (\xi q + \bar\xi\bar q) \times \chi = i\sqrt{2}\sigma^m\bar\xi(\delta^0_m\,\underline{\partial}_0 + \tfrac{1}{|\vec{x}|}x^i\delta_{im}\,\underline{\partial}_1)\underline{A} + \sqrt{2}\xi F, \tag{49}$$


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where $i = 1, 2, 3$. The coefficient of $(\delta^0_m\,\underline{\partial_0} + \frac{1}{|\vec{x}|}x^i\delta_{im}\,\underline{\partial_1})\underline{A}$ is chosen to guarantee that the commutator of

$$\delta_{\xi_1}\delta_{\xi_2}\underline{A} = i2\xi_1\sigma^m\bar{\xi}_2(\delta^0_m\,\underline{\partial_0} + \tfrac{1}{|\vec{x}|}x^i\delta_{im}\,\underline{\partial_1})\underline{A} + 2\xi_1\bar{\xi}_2F \tag{50}$$

closes in the sense of

$$(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})\underline{A} = -2i(\xi_1\sigma^m\bar{\xi}_2 - \xi_2\sigma^m\bar{\xi}_1) \times (\delta^0_m\underline{\partial_0} + \tfrac{1}{|\vec{x}|}x^i\delta_{im}\underline{\partial_1})\underline{A}. \tag{51}$$

The same commutator acting on the field $\chi$: $(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})\chi$, closes if

$$\delta_\xi F = (\xi q + \bar{\xi}\bar{q}) \times F = i\sqrt{2}\bar{\xi}\bar{\sigma}^m\partial_m\chi. \tag{52}$$

In the same way, we should define the spinor $\underline{\chi}$ as the field into which $A(x)$ transforms. In this case, the infinitesimal supersymmetry transformations for $Q = \underline{q}$, by virtue of (35), read

$$\begin{aligned}
&\delta_{\underline{\xi}}A = (\underline{\xi}\,\underline{q} + \bar{\underline{\xi}}\,\bar{\underline{q}}) \times A = \sqrt{2}\,\underline{\xi}\,\underline{\chi},\\
&\delta_{\underline{\xi}}\,\underline{\chi} = (\underline{\xi}\,\underline{q} + \bar{\underline{\xi}}\,\bar{\underline{q}}) \times \underline{\chi} = i\sqrt{2}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}}\,(\delta^0_{\underline{m}}\,\partial_0 + \tfrac{|\vec{x}|}{x^i}\,\delta^1_{\underline{m}}\,\partial_i)A + \sqrt{2}\underline{\xi}\,\underline{F},\\
&\delta_{\underline{\xi}}\,\underline{F} = (\underline{\xi}\,\underline{q} + \bar{\underline{\xi}}\,\bar{\underline{q}}) \times \underline{F} = i\sqrt{2}\,\bar{\underline{\xi}}\,\bar{\sigma}^{\underline{m}}\,\partial_{\underline{m}}\,\underline{\chi}.
\end{aligned} \tag{53}$$

If $\mathcal{A}$ has dimension 1, then $\psi$ has dimension 3/2, while $\mathcal{F}$ has dimension 2 and must assume the role of auxiliary field. We see that the latter transforms into a space derivative under $\delta_\xi$ or $\delta_{\underline{\xi}}$. This will always be the case for the component of highest dimension in any given multiplet. The first relation in (47) means that there should be a particular way of going from one subspace (bosonic/fermionic) to the other and back, such that the net result is as if we had operator of translation $p_{\hat{m}}$ on the original subspace. Actually, in general, the supersymmetry transformations close supersymmetry algebra for $\mathcal{A}$, $\psi$ and $\mathcal{F}$.

To construct an invariant action it is sufficient to find combinations of fields which transform into space derivatives. The supersymmetric kinetic energy defined in terms of superfields, $\underline{\phi}(\underline{z}^{(M_2)})$, constructed in the superspace $\underline{z}^{(M_2)} = (\underline{\eta}^{\underline{m}},\, \underline{\theta},\, \bar{\underline{\theta}})$, is

$$\int d^2\underline{\eta}\, d^4\underline{\theta}\, \underline{\phi}^\dagger\, \underline{\phi}, \tag{54}$$

where the superspace Lagrangian is written

$$\begin{aligned}
&\underline{\phi}^\dagger\,\underline{\phi} = A^*A + \cdots + \underline{\theta}\,\underline{\theta}\,\bar{\underline{\theta}}\,\bar{\underline{\theta}}\left[\tfrac{1}{4}A^*\Box A + \tfrac{1}{4}\Box A^*A - \tfrac{1}{2}\partial_m A^*\partial^m A\right.\\
&\left.+\underline{F}^*\,\underline{F} + \tfrac{i}{2}\partial_{\underline{m}}\,\bar{\underline{\chi}}\,\bar{\sigma}^{\underline{m}}\,\underline{\chi} - \tfrac{i}{2}\,\bar{\underline{\chi}}\bar{\sigma}^{\underline{m}}\,\partial_{\underline{m}}\,\underline{\chi}\right].
\end{aligned} \tag{55}$$

In (54), as usual, we imply the measures in terms of four Grassmannian variables $\underline{\theta}_1, \underline{\theta}_2$ and $\bar{\underline{\theta}}^1, \bar{\underline{\theta}}^2$ as follows:

$$\begin{aligned}
&d^4\underline{\theta} = d^2\underline{\theta}\, d^2\bar{\underline{\theta}}, \quad d^2\underline{\theta} = -\tfrac{1}{4}\varepsilon_{\alpha\beta}\,\underline{\theta}^\alpha\,\underline{\theta}^\beta,\\
&d^2\bar{\underline{\theta}} = -\tfrac{1}{4}\varepsilon^{\dot{\alpha}\dot{\beta}}\,\bar{\underline{\theta}}_{\dot{\alpha}}\,\bar{\underline{\theta}}_{\dot{\beta}},
\end{aligned} \tag{56}$$

where involving Berezin's integration in the superspace, we have

$$\int \underline{\theta}^2\, d^2\underline{\theta} = \int \bar{\underline{\theta}}^2\, d^2\bar{\underline{\theta}} = 1, \quad \int \underline{\theta}^2\bar{\underline{\theta}}^2\, d^4\underline{\theta} = 1. \tag{57}$$

All other integrations give zero. Similarly, consider the superspace $z^{(M_4)} = (x^m,\, \theta(\underline{\theta}, \bar{\underline{\theta}})),\, \bar{\theta}(\underline{\theta}, \bar{\underline{\theta}}))$, which is an extension of $M_4$ by the inclusion of additional spinors, induced by the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ (see (93)). The supervolume integrals of products of superfields, $\phi(z^{(M_4)})$, constructed in the superspace $z^{(M_4)}$ will lead to the supersymmetric kinetic energy for the $MS_p$-Wess-Zumino model

$$\int d^4x\, d^4\theta\, \phi^\dagger\phi, \tag{58}$$

where the superspace Lagrangian reads

$$\begin{aligned}
&\phi^\dagger\phi = \underline{A}^*\,\underline{A} + \cdots + \theta\theta\bar{\theta}\bar{\theta}\,\left[\tfrac{1}{4}\underline{A}^*\,\underline{\Box}\,\underline{A} + \tfrac{1}{4}\underline{\Box}\,\underline{A}^*\,\underline{A} - \tfrac{1}{2}\partial_{\underline{m}}\underline{A}^*\,\partial^{\underline{m}}\underline{A}\right.\\
&\left.+F^*F + \tfrac{i}{2}\partial_m\bar{\chi}\bar{\sigma}^m\chi - \tfrac{i}{2}\bar{\chi}\bar{\sigma}^m\partial_m\chi\right],
\end{aligned} \tag{59}$$

and $\Box A = \underline{\Box}\,\underline{A}$, $\partial_{\underline{m}}\underline{A}^*\,\partial^{\underline{m}}\underline{A} = \partial_m A^*\,\partial^m A$.

Thus, the equations of motion for non-trivial linear representation model can be derived from the following Lagrangians:

$$\mathcal{L}_{Q=q} = \mathcal{L}_0 + m\mathcal{L}_m, \quad \mathcal{L}_{Q=\underline{q}} = \underline{\mathcal{L}}_0 + m\underline{\mathcal{L}}_m, \tag{60}$$


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provided,

$$\begin{array}{ll}\mathcal{L}_0 = i\partial_m\bar{\chi}\bar{\sigma}^m\chi + \underline{A}^*\underline{\Box}\,\underline{A} + F^*F, & \mathcal{L}_m = \underline{A}\,F + \underline{A}^*\,F^* - \frac{1}{2}\chi\chi - \frac{1}{2}\bar{\chi}\bar{\chi}, \\ \underline{\mathcal{L}}_0 = i\partial_{\underline{m}}\,\underline{\bar{\chi}}\bar{\sigma}^m\underline{\chi} + A^*\Box A + \underline{F}^*\underline{F}, & \underline{\mathcal{L}}_{\underline{m}} = A\,\underline{F} + A^*\,\underline{F}^* - \frac{1}{2}\underline{\chi}\,\underline{\chi} - \frac{1}{2}\underline{\bar{\chi}}\,\underline{\bar{\chi}},\end{array} \tag{61}$$

and $\Box = \underline{\Box}$, $A \equiv \underline{A}$. Whereupon, the equations for the Weyl spinor $\psi$ and complex scalar $\mathcal{A}$ of the same mass $m$, are

$$\begin{array}{lcl} i\bar{\sigma}^m\,\partial_m\,\chi + m\bar{\chi} = 0, & & i\bar{\sigma}^{\underline{m}}\,\partial_{\underline{m}}\,\underline{\chi} + m\underline{\bar{\chi}} = 0, \\ F + m\underline{A}^* = 0, & \text{and} & \underline{F} + mA^* = 0, \\ \underline{\Box}\,\underline{A} + mF^* = 0, & & \Box A + m\underline{F}^* = 0. \\ (a) & & (b) \end{array} \tag{62}$$

In accord to (34) and (35), respectively, (a) stands for $Q = q$ (referring to the motion of a fermion, $\chi$, in $M_4$) and (b) stands for $Q = \underline{q}$ (so, of a boson, $A$, in $M_4$). Finally, the algebraic auxiliary field $\mathcal{F}$ can be eliminated to find

$$\begin{array}{l}\mathcal{L}_{Q=q} = i\partial_m\bar{\chi}\bar{\sigma}^m\chi - \frac{1}{2}(\chi\chi + \bar{\chi}\bar{\chi}) + \underline{A}^*\underline{\Box}\,\underline{A} - m^2\underline{A}^*\underline{A}, \\ \underline{\mathcal{L}}_{Q=\underline{q}} = i\partial_{\underline{m}}\,\underline{\bar{\chi}}\bar{\sigma}^m\underline{\chi} - \frac{1}{2}(\underline{\chi}\,\underline{\chi} + \underline{\bar{\chi}}\,\underline{\bar{\chi}}) + A^*\Box A - m^2A^*\,A.\end{array} \tag{63}$$

To complete the model, we also need superspace expressions for the masses and couplings, which can be easily found in analogy of the standard theory, namely: 1) fermion masses and Yukawa couplings, $(\partial^2\mathcal{P}/\partial\mathcal{A}^2)\psi\psi$; and 2) the scalar potential, $\mathcal{V}(\mathcal{A},\,\mathcal{A}^*) = |\partial\mathcal{P}/\partial\mathcal{A}|^2$; where $\mathcal{P} = (1/2)\,m\,\Phi^2 + (1/3)\,\lambda\,\Phi^3$ is the most general renormalizable interaction for a single chiral superfield. Thereby, the auxiliary field equation of motion reads $\mathcal{F}^* + (\partial\mathcal{P}/\partial\mathcal{A}) = 0$. Similarly, we can treat the vector superfields, etc. Here we shall forbear to write them out as the standard theory is so well known. Divergences in SUSY field theories are greatly reduced. Indeed all the quadratic divergences disappear in the renormalized supersymmetric Lagrangian and the number of independent renormalization constants is kept to a minimum.

## 6. Rigid superspace geometry, superfields

In the framework of standard generalization of the coset construction (Callan et al., 1969, Coleman et al., 1969, Salam & Strathdee, 1974, Weinberg, 1968), we will take $G = G_q \times G_{\underline{q}}$ to be the supergroup generated by the MS$_p$-SUSY algebra (42). Let the stability group $H = H_q \times H_{\underline{q}}$ be the Lorentz group (referred to $M_4$ and $\underline{M}_2$), and we choose to keep all of $G$ unbroken. Given $G$ and $H$, we can construct the coset, $G/H$, by an equivalence relation on the elements of $G$: $\Omega \sim \Omega h$, where $\Omega = \Omega_q \times \Omega_{\underline{q}} \in G$ and $h = h_q \times h_{\underline{q}} \in H$, so that the coset can be pictured as a section of a fiber bundle with total space, $G$, and fiber, $H$. So, the Maurer-Cartan form, $\Omega^{-1}d\Omega$, is valued in the Lie algebra of $G$, and transforms as follows under a global G transformation,

$$\begin{array}{l}\Omega \longrightarrow g\Omega h^{-1}, \\ \Omega^{-1}d\Omega \longrightarrow h(\Omega^{-1}d\Omega)h^{-1} - dh\,h^{-1},\end{array} \tag{64}$$

with $g \in G$. We also consider a superspace which is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ (spanned by the 6D-coordinates $X^{\hat{m}} = (x^m,\,\underline{\eta}^{\underline{m}})$ by the inclusion of additional 8D-fermionic coordinates $\Theta^\alpha = (\theta^\alpha,\,\underline{\theta}^\alpha)$ and $\bar{\Theta}_{\dot{\alpha}} = (\bar{\theta}_{\dot{\alpha}},\,\underline{\bar{\theta}}_{\dot{\alpha}})$, as to $(q,\,\underline{q})$, respectively. These spinors satisfy the following relations:

$$\begin{array}{l}\{\Theta^\alpha,\,\Theta^\beta\} = \{\bar{\Theta}_{\dot{\alpha}},\,\bar{\Theta}_{\dot{\beta}}\} = \{\Theta^\alpha,\,\bar{\Theta}_{\dot{\beta}}\} = 0, \\ [x^m,\,\theta^\alpha] = [x^m,\,\bar{\theta}_{\dot{\alpha}}] = 0, \\ [\underline{\eta}^{\underline{m}},\,\underline{\theta}^\alpha] = [\underline{\eta}^{\underline{m}},\,\underline{\bar{\theta}}_{\dot{\alpha}}] = 0.\end{array} \tag{65}$$

and $\Theta^{\alpha *} = \bar{\Theta}^{\dot{\alpha}}$. Points in superspace are identified by the generalized coordinates

$$z^{(M)} = (X^{\hat{m}},\,\Theta^\alpha,\,\bar{\Theta}_{\dot{\alpha}}) = (x^m,\,\theta^\alpha,\,\bar{\theta}_{\dot{\alpha}}) \oplus (\underline{\eta}^{\underline{m}},\underline{\theta}^\alpha,\,\underline{\bar{\theta}}_{\dot{\alpha}}).$$

We have then the one most commonly used `real´ or `symmetric´ superspace parametrized by

$$\Omega(X,\,\Theta,\,\bar{\Theta}) = e^{i(-X^{\hat{m}}p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})} = \Omega_q(x,\,\theta,\,\bar{\theta}) \times \Omega_{\underline{q}}(\underline{\eta},\,\underline{\theta},\,\underline{\bar{\theta}}), \tag{66}$$

where we now imply a summation over $\hat{m}$, etc., such that

$$\Omega_q(x,\,\theta,\,\bar{\theta}) = e^{i(-x^m p_m + \theta^\alpha q_\alpha + \bar{\theta}_{\dot{\alpha}}\bar{q}^{\dot{\alpha}})}, \quad \Omega_{\underline{q}}(\underline{\eta},\,\underline{\theta},\,\underline{\bar{\theta}}) = e^{i(-\underline{\eta}^{\underline{m}}p_{\underline{m}} + \underline{\theta}^\alpha \underline{q}_\alpha + \underline{\bar{\theta}}_{\dot{\alpha}}\underline{\bar{q}}^{\dot{\alpha}})}. \tag{67}$$

Supersymmetry transformation will be defined as a translation in superspace, specified by the group element

$$g(0,\,\epsilon,\,\bar{\epsilon}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})} = g_q(0,\,\xi,\,\bar{\xi}) \times g_{\underline{q}}(0,\,\underline{\xi},\,\underline{\bar{\xi}}) = e^{i(\xi^\alpha q_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{q}^{\dot{\alpha}})} \times e^{i(\underline{\xi}^\alpha \underline{q}_\alpha + \underline{\bar{\xi}}_{\dot{\alpha}}\underline{\bar{q}}^{\dot{\alpha}})}, \tag{68}$$


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with corresponding anticommuting parameters $\epsilon = (\xi \text{ or } \underline{\xi})$. To study the effect of supersymmetry transformations (64) and $h=1$, we consider

$$g(0,\,\epsilon,\,\bar\epsilon)\,\Omega(X,\,\Theta,\,\bar\Theta) = e^{i(\epsilon^\alpha Q_\alpha+\bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha})}\,e^{i(-X^{\hat m}p_{\hat m}+\Theta^\alpha Q_\alpha+\bar\Theta_{\dot\alpha}\bar Q^{\dot\alpha})}. \tag{69}$$

The multiplication of two successive transformations can be computed with the help of the Baker-Campbell-Hausdorf formula $e^Ae^B = e^{A+B+(1/2)[A,\,B]+\cdots}$. Hence the transformation (69) induces the motion as is evident from the first relation of (47):

$$g(0,\,\epsilon,\,\bar\epsilon)\,\Omega(X^{\hat m},\,\Theta,\,\bar\Theta)\;\to(X^{\hat m}+i\,\Theta\,\sigma^{\hat m}\,\bar\epsilon-i\,\epsilon\,\sigma^{\hat m}\,\bar\Theta,\;\Theta+\epsilon,\,\bar\Theta+\bar\epsilon), \tag{70}$$

namely,

$$\begin{array}{l} g_q(0,\,\xi,\,\bar\xi)\,\Omega_q(x,\,\theta,\,\bar\theta)\;\to(x^m+i\,\theta\,\sigma^m\,\bar\xi-i\,\xi\,\sigma^m\,\bar\theta,\;\theta+\xi,\,\bar\theta+\bar\xi),\\ g_{\underline{q}}(0,\,\underline{\xi},\,\bar{\underline{\xi}})\,\Omega_{\underline{q}}(\underline{\eta},\,\underline{\theta},\,\bar{\underline{\theta}})\;\to(\underline{\eta}^{\underline{m}}+i\,\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}}-i\,\underline{\xi}\,\sigma^{\underline{m}}\,\bar{\underline{\theta}},\;\underline{\theta}+\underline{\xi},\,\bar{\underline{\theta}}+\bar{\underline{\xi}}). \end{array} \tag{71}$$

The spinors $\theta(\underline{\theta},\bar{\underline{\theta}})$ and $\bar\theta(\underline{\theta},\bar{\underline{\theta}})$ satisfy the embedding map (10), namely $\Delta\underline{x}^0=\Delta x^0$ and $\Delta\underline{x}^2=(\Delta\vec x)^2$, so from (71) we obtain

$$\begin{array}{l} \underline{\theta}\,\sigma^0\,\bar{\underline{\xi}}-\underline{\xi}\,\sigma^0\,\bar{\underline{\theta}}=\theta\,\sigma^0\,\bar\xi-\xi\,\sigma^0\,\bar\theta,\\ (\underline{\theta}\,\sigma^3\,\bar{\underline{\xi}}-\underline{\xi}\,\sigma^3\,\bar{\underline{\theta}})^2=(\theta\,\vec\sigma\,\bar\xi-\xi\,\vec\sigma\,\bar\theta)^2. \end{array} \tag{72}$$

The superfield $\Phi(z^{(M)})=\phi(z^{(M_4)})$ or $\underline{\phi}(\underline{z}^{(M_2)})$, which has a finite number of terms in its expansion in terms of $\Theta$ and $\bar\Theta$ owing to their anticommuting property, can be considered as the generator of the various components of the supermultiplets. We will consider only a scalar superfield $\Phi'(z'^{(M)})=\Phi(z^{(M)})$, an infinitesimal supersymmetry transformation of which is given as

$$\delta_\epsilon\,\Phi(z^{(M)})=(\epsilon^\alpha Q_\alpha+\bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha})\times\Phi(z^{(M)}). \tag{73}$$

Acting on this space of functions, the $Q$ and $\bar Q$ can be represented as differential operators:

$$Q_\alpha=\tfrac{\partial}{\partial\Theta^\alpha}-i\sigma^{\hat m}{}_{\alpha\dot\alpha}\bar\Theta^{\dot\alpha}\partial_{\hat m},\quad \bar Q^{\dot\alpha}=\tfrac{\partial}{\partial\bar\Theta_{\dot\alpha}}-i\Theta^\alpha\sigma^{\hat m}{}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_{\hat m}, \tag{74}$$

where, as usual, the undotted/dotted spinor indices can be raised and lowered with a two dimensional undotted/dotted $\varepsilon-$tensors, and the anticommuting derivatives obey the relations

$$\tfrac{\partial}{\partial\Theta^\alpha}\,\Theta^\beta=\delta^\beta_\alpha,\quad \tfrac{\partial}{\partial\Theta^\alpha}\,\Theta^\beta\Theta^\gamma=\delta^\beta_\alpha\Theta^\gamma-\delta^\gamma_\alpha\Theta^\beta, \tag{75}$$

and similarly for $\bar\Theta$. In order to write the exterior product in terms of differential operators, one induces a new basis in rigid superspace of supervielbein 1-form

$$e^A(z)=dz^M\,e^A_M(z), \tag{76}$$

and that

$$D_A=e_A{}^N(z)\tfrac{\partial}{\partial z^N}, \tag{77}$$

where to be brief we left implicit the symbol $\wedge$ in writing of exterior product. The inverse vielbein $E_A{}^M(z)$ is defined by the relations $E^A_M(z)E_A{}^N(z)=\delta_M{}^N$, and $E_A{}^M(z)E^B_M(z)=\delta_A{}^B$. The covariant derivative operators

$$\begin{array}{l} D_{\hat m}=\partial_{\hat m},\quad D_\alpha=\tfrac{\partial}{\partial\Theta^\alpha}+i\sigma^{\hat m}{}_{\alpha\dot\alpha}\bar\Theta^{\dot\alpha}\partial_{\hat m},\\ \bar D^{\dot\alpha}=\tfrac{\partial}{\partial\bar\Theta_{\dot\alpha}}+i\Theta^\alpha\sigma^{\hat m}{}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_{\hat m}, \end{array} \tag{78}$$

anticommute with the $Q$ and $\bar Q$

$$\{Q_\alpha,\,D_\beta\}=\{\bar Q_{\dot\alpha},\,\bar D_{\dot\beta}\}=\{Q_\alpha,\,\bar D_{\dot\beta}\}=\{\bar Q_{\dot\alpha},\,D_\beta\}=0, \tag{79}$$

and satisfy the following structure relations:

$$\begin{array}{l} \{D_\alpha,\,D_{\dot\alpha}\}=-2i\sigma^{\hat m}{}_{\alpha\dot\alpha}\partial_{\hat m},\\ \{D_\alpha,\,D_\beta\}=\{\bar D_{\dot\alpha},\,\bar D_{\dot\beta}\}=0. \end{array} \tag{80}$$


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From (78), we obtain

$$e_A{}^M = \begin{pmatrix} e_{\hat a}{}^{\hat m} = \delta_{\hat a}^{\hat m} & e_{\hat a}{}^{\mu} = 0 & e_{\hat a\,\dot\mu} = 0 \\ e_{\alpha}{}^{\hat m} = i\sigma^{\hat m}_{\alpha\dot\alpha}\bar\Theta^{\dot\alpha} & e_{\alpha}{}^{\mu} = \delta_\alpha^\mu & e_{\alpha\,\dot\mu} = 0 \\ e^{\dot\alpha\,\hat m} = i\Theta^\alpha\sigma^{\hat m}_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha} & e^{\dot\alpha\,\mu} = 0 & e^{\dot\alpha}{}_{\dot\mu} = \delta^{\dot\alpha}_{\dot\mu} \end{pmatrix}, \tag{81}$$

where $\hat a = (a \text{ or } \underline{a})$, $\quad a = 0, 1, 2, 3$; $\quad \underline{a} = (+), (-)$. The supersymmetry transformations of the component fields can be found using the differential operators (78).

The covariant constraint

$$\bar D_{\dot\alpha}\Phi(z^{(M)}) = 0, \tag{82}$$

which does not impose equations of motion on the component fields, defines the chiral superfield, $\Phi$. Under the supersymmetry transformation (70) the chiral field transforms as follows:

$$\delta_\xi\Phi = \begin{cases} (\xi q + \bar\xi\bar q)\times\phi = \delta_\xi\,\underline{A}(\eta) + \sqrt{2}\theta\delta_\xi\chi(x) + \theta\theta\delta_\xi F(x) + \cdots, & \text{at } Q = q, \\ (\underline{\xi}\,\underline{q} + \bar{\underline{\xi}}\,\bar{\underline{q}})\times\underline{\phi} = \delta_{\underline{\xi}}\,A(x) + \sqrt{2}\underline{\theta}\delta_{\underline{\xi}}\,\underline{\chi}(\underline{\eta}) + \underline{\theta}\,\underline{\theta}\delta_{\underline{\xi}}\,\underline{F}(\underline{\eta}) + \cdots, & \text{at } Q = \underline{q}. \end{cases} \tag{83}$$

Equation (83) show that the chiral superfield contains the same component fields as the $\mathrm{MS}_p$-Wess-Zumino toy model.

## 7. Two postulates of SR

In this section we derive two postulates on which the theory of SR is based. Let us focus on the simple case of a peculiar anticommuting spinors $(\underline{\xi}, \bar{\underline{\xi}})$ and $(\xi, \bar\xi)$ defined as

$$\underline{\xi}^\alpha = i\,\tfrac{\tau}{2}\,\underline{\theta}^\alpha, \quad \bar{\underline{\xi}}_{\dot\alpha} = -i\,\tfrac{\tau^*}{2}\,\bar{\underline{\theta}}_{\dot\alpha}, \quad \xi^\alpha = i\tfrac{\tau}{2}\theta^\alpha, \quad \bar\xi_{\dot\alpha} = -i\tfrac{\tau^*}{2}\bar\theta_{\dot\alpha}. \tag{84}$$

The *atomic displacement* caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$, according to (12), reads

$$\Delta\underline{\eta}_{(a)} = \underline{e}_{\underline{m}}\Delta\underline{\eta}^{\underline{m}}_{(a)} = \underline{u}\tau, \tag{85}$$

where the components $\Delta\underline{\eta}^{\underline{m}}_{(a)}$, according to (71), are written

$$\Delta\underline{\eta}^{\underline{m}}_{(a)} = \underline{v}^{\underline{m}}\,\tau = i\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}} - i\underline{\xi}\,\sigma^{\underline{m}}\,\bar{\underline{\theta}}. \tag{86}$$

Here the real parameter $\tau\,(=\tau^*)$ can physically be interpreted as the *atomic duration time* of double transition. By virtue of (84), the (86) is reduced to

$$\Delta\underline{\eta}^{\underline{m}}_{(a)} = (\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\theta}})\tau. \tag{87}$$

In Van der Warden notations for the Weyl two-component formalism $\bar{\underline{\theta}}_{\dot\alpha} = (\underline{\theta}_\alpha)^*$ (App.A), the (85), gives

$$\Delta\underline{\eta}^2_{(a)} = \underline{u}^2\tau^2 = \underline{v}^{(+)}\underline{v}^{(-)}\tau^2 = 2(\underline{\theta}_1\,\bar{\underline{\theta}}_1\underline{\theta}_2\,\bar{\underline{\theta}}_2)\tau^2 = 2(\underline{\theta}_1^2\underline{\theta}_2^2)\tau^2 \geq 0, \tag{88}$$

provided, $\underline{v}^{(+)} = \sqrt{2}\,\underline{\theta}_1\,\bar{\underline{\theta}}_1 = \sqrt{2}\,\underline{\theta}_1^2$ and $\underline{v}^{(-)} = \sqrt{2}\,\underline{\theta}_2\,\bar{\underline{\theta}}_2 = \sqrt{2}\,\underline{\theta}_2^2$. The (88), combined with (10), (11) and (19), can be recast into the form

$$\Delta\underline{\eta}^2_{(a)} = \tfrac{1}{2}\left[(\Delta\underline{x}^{\underline{0}}_{(a)})^2 - (\Delta\underline{x}^{\underline{1}}_{(a)})^2\right], \tag{89}$$

where $\Delta\underline{x}^{\underline{0}}_{(a)} = \underline{v}^{\underline{0}}\tau$, $\Delta\underline{x}^{\underline{1}}_{(a)} = \underline{v}^{\underline{1}}\,\tau$, and $\underline{v}^{(\pm)} = \frac{1}{\sqrt{2}}(\underline{v}^{\underline{0}} \pm \underline{v}^{\underline{1}})$. Hence the velocities of light in vacuum, $\underline{v}^{\underline{0}} = c$, and of a particle ,$\underline{\vec v}_{\underline{1}} = \underline{e}_{\underline{1}}\underline{v}^{\underline{1}} = \vec n|\vec v| = \vec v\ (|\vec v| \leq c)$, are

$$\begin{aligned} \underline{v}^{\underline{0}} &= \underline{\theta}\,\sigma^{\underline{0}}\,\bar{\underline{\theta}} = (\underline{\theta}_1\,\bar{\underline{\theta}}_1 + \underline{\theta}_2\,\bar{\underline{\theta}}_2) = \underline{\theta}\,\bar{\underline{\theta}}, \\ \underline{v}^{\underline{1}} &= \underline{\theta}\,\sigma^{\underline{1}}\,\bar{\underline{\theta}} = (\underline{\theta}_1\,\bar{\underline{\theta}}_1 - \underline{\theta}_2\,\bar{\underline{\theta}}_2). \end{aligned} \tag{90}$$

Both map relationship in (72) are reduced by (84) to

$$\theta\bar\theta = \underline{v}^{\underline{0}}, \quad \theta\theta\bar\theta\bar\theta = \tfrac{2}{3}(\underline{v}^{\underline{1}})^2. \tag{91}$$


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Here we have used the following spinor algebra relations:

$$(\theta\sigma^m\bar{\theta})(\theta\sigma^n\bar{\theta}) = \frac{1}{2}\,\theta\theta\bar{\theta}\bar{\theta}\,g^{mn}. \tag{92}$$

By virtue of relations $\theta_\alpha\theta_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\theta\theta$ and $\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}$, where the antisymmetric tensors $\varepsilon_{\alpha\beta}$ and $\varepsilon^{\alpha\beta}$ $(\varepsilon_{21} = \varepsilon^{12} = 1, \varepsilon_{12} = \varepsilon^{21} = -1, \varepsilon_{11} = \varepsilon_{22} = 0)$, and that of bilinear combinations $\theta\theta$ and $\bar{\theta}\bar{\theta}$, are invariant under Lorentz transformations, we obtain from (91): $\theta_1^2 + \theta_2^2 = \underline{v}^0$, and $\theta_1\theta_2 = \frac{1}{\sqrt{6}}\underline{v}^1$, which yield

$$\begin{aligned}\theta_1(\underline{\theta},\,\bar{\underline{\theta}}) &= \frac{1}{2}\left[\left(\underline{v}^0 + \sqrt{\frac{2}{3}}\underline{v}^1\right)^{1/2} + \left(\underline{v}^0 - \sqrt{\frac{2}{3}}\underline{v}^1\right)^{1/2}\right],\\ \theta_2(\underline{\theta},\,\bar{\underline{\theta}}) &= \frac{1}{2}\left[\left(\underline{v}^0 + \sqrt{\frac{2}{3}}\underline{v}^1\right)^{1/2} - \left(\underline{v}^0 - \sqrt{\frac{2}{3}}\underline{v}^1\right)^{1/2}\right].\end{aligned} \tag{93}$$

We conclude that the *motion of a particle in $M_4$ is encoded in the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to the master space $\underline{M}_2$, which is indispensably tied to the propagating particle of interest.* The Lorentz invariance is a fundamental symmetry and refers to measurements of ideal inertial observers that move uniformly forever on rectilinear timelike worldlines. In view of relativity of all kinds of motion, we are of course not limited to any particular constant spinor $\underline{\theta}$ which yields the velocity $\vec{v}(\underline{\theta})$, but can choose at will any other constant spinors $\underline{\theta}', \underline{\theta}'', \ldots$ yielding respectively the velocities $\vec{v'}(\underline{\theta}'), \vec{v}''(\underline{\theta}''), \ldots$, whose transformational law on the original spinor $\underline{\theta}$ is known (16):

$$\underline{\theta}'_\alpha = M_\alpha{}^\beta\underline{\theta}_\beta, \quad \bar{\underline{\theta}}_{\dot{\alpha}} = (M^*)_{\dot{\alpha}}{}^{\dot{\beta}}\,\bar{\underline{\theta}}_{\dot{\beta}}, \quad \dot{\alpha},\dot{\beta} = 1,2, \tag{94}$$

where $M \in SL(2,C)$, the hermitian matrix $M^*$ is related by a similarity transformation to $(M^{-1})^\dagger$, i.e. $(M^\dagger)^\beta{}_\alpha = (M^*)_\alpha{}^\beta$.

Aforesaid ensures to obtain some feeling about the origin of the two postulates of SR. Certainly, by virtue of (94) and (22), we derive the first founding property (i) that *the atomic displacement* $\Delta\underline{\eta}_{(a)}$, caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$, is an invariant:

$$\text{(i)} \quad \Delta\underline{\eta}_{(a)} = \Delta\underline{\eta}'_{(a)} = \cdots = inv. \tag{95}$$

The (94) also gives the second (ii) founding property that the bilinear combination $\underline{\theta}\,\bar{\underline{\theta}}$ is a constant:

$$\text{(ii)} \quad c = \underline{\theta}\,\bar{\underline{\theta}} = \underline{\theta}'\,\bar{\underline{\theta}}' = \cdots = const. \tag{96}$$

The latter yields a second postulate of SR (Einstein's postulate) - *the velocity of light, c, in free space appears the same to all observers regardless the relative motion of the source of light and the observer.* The $c$ is the maximum attainable velocity (90) for uniform motion of a particle in Minkowski background space, $M_4$. Equally noteworthy is the fact that (95) and (96) combined yield *invariance of the element of interval between two events $\Delta x = k\Delta\underline{\eta}_{(a)}$ (for given integer number k) with respect to the Lorentz transformation*:

$$\begin{aligned}&k^2\Delta\underline{\eta}^2_{(a)} = (c^2 - \underline{v}_1^2)\Delta t^2 = (c^2 - \vec{v}^{\,2})\Delta t^2 = (\Delta x^0)^2 - (\Delta\vec{x})^2 \equiv (\Delta s)^2 =\\ &(\Delta x'^0)^2 - (\Delta\vec{x})'^2 \equiv (\Delta s')^2 = \cdots = inv.,\end{aligned} \tag{97}$$

where $x^0 = ct, \quad x^{0'} = ct', \ldots$. We have here introduced a notion of physical relative finite *time intervals* between two events $\Delta t = k\tau/\sqrt{2},\, \Delta t' = k\tau'/\sqrt{2}, \ldots$.

The motion, say, of spin-0 particle in $\underline{M}_2$ can be described by the chiral superfield $\underline{\Phi}(\underline{\eta}^m,\, \underline{\theta},\, \bar{\underline{\theta}})$, while a similar motion of spin-1/2 particle in $M_4$ can be described by the chiral superfield $\Phi(x^m,\, \theta,\, \bar{\theta})$, etc. Therefore, what has been said above will require a complete revision of our ideas about the Lorentz code of motion, to be now referred to as the *individual code of a particle*, defined as its intrinsic property.

Having SLC to be equipped with the MS$_p$-SUSY mechanism, the spinors $\underline{\theta}$ encode all of the information necessary for the two founding properties (95) and (96) of SR. In subsequent paper (**?**), we will address the deformation of these spinors: $\underline{\theta} \to \tilde{\underline{\theta}} = \lambda^{1/2}\,\underline{\theta}$, etc., where $\lambda$ appears as a deformation function of the Lorentz invariance (LIDF). This yields a consistent microscopic theory of Lorentz invariance violation (LIV) caused by the deformation of both the line element, $\tilde{ds} = \lambda ds$, and maximum attainable velocity, $\tilde{c} = \lambda c$, of a particle. We will discuss the corresponding deformed geometry at LIV, and complement this conceptual investigation with testing of various LIDFs in the UHECR- and TeV-$\gamma$ threshold anomalies with in several instructive scenarios: the Coleman and Glashow-type perturbative extension of SLC, the LID extension of standard model, the LID in quantum gravity motivated space-time models and the LID in loop quantum gravity models. Consequently, in the third paper of this series, we will construct the framework of local MS$p$-SUSY (**?**), which will address the *accelerated motion* and inertia effects.


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# 8. The physical outlook and concluding remarks

Resuming the whole physical picture, in this Section 8 we expose the assertions made through a brief physical outlook of the key points of proposed theory. As concluding remarks, we also present what we think is the most important that distinguish this theory from phenomenological approaches of differing MAVs in the published literature.

**Underlying physical processes.** We identify the physical processes underlying the inertial uniform motion, which is probably the most fascinating challenge for physical research, and review the foundations of SLC.

**$MS_p$.** We motivate the subject by conceiving of double background spaces - 4D Minkowski space, $M_4$, and $MS_p$ for each particle. The geometry of $MS_p$ is a new physical entity, with degrees of freedom and a dynamics of its own. We assume that a flat $MS_p$ is the 2D composite space $\underline{M}_2$. Then, a smooth embedding map $f: \underline{M}_2 \longrightarrow M_4$ is defined to be an immersion - an embedding which is a function that is a homeomorphism onto its image. So that the $MS_p \equiv \underline{M}_2$ is viewed as 2D space living on the 4D world sheet. Given the inertial frames $S_{(4)}, S'_{(4)}, S''_{(4)}, ...$ in $M_4$, we define the corresponding inertial frames $\underline{S}_{(2)}$, $\underline{S'}_{(2)}$, $\underline{S''}_{(2)}$,... in $\underline{M}_2$, which used by the non-accelerated observers for the positions $\underline{x}^r, \underline{x}'^r, \underline{x}''^r, ...$ of a free particle in flat $\underline{M}_2$. Thereby the time axes of the two systems $\underline{S}_{(2)}$ and $S_{(4)}$ coincide in direction and that the time coordinates are taken the same.

We hypothesize that the elementary act of motion consists of an `annihilation´ of a particle at point $(x,t) \in M_4$, which can be understood as the transition from initial state $|x,t>$ to *unmanifested* intermediate state (`motion´ state), $|\underline{x},\underline{t}>$, and of subsequent `creation´ of a particle at infinitely close final point $(x',t') \in M_4$, which means the transition from `motion´ state, $|\underline{x},\underline{t}>$, to final state, $|x',t'>$. The motion state, $|\underline{x},\underline{t})>$, is defined on *unmanifested* `master´ space, $\underline{M}_2$, which includes the points of all the atomic elements, $(\underline{x},\underline{t}) \in \underline{M}_2$. This furnishes justification for an introduction of *master space*, $\underline{M}_2$. The $\underline{M}_2$ is indispensably tied to propagating particle, without relation to the other particles.

**`Double space´- or `$MS_p$´-SUSY.** The net result of each atomic double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ is as if we had operated with a space-time *translation* on the original space $M_4$. Such successive transformations will induce in $M_4$ the inhomogeneous Lorentz group, or Poincaré group, and that the unitary linear transformation $|x,t> \rightarrow U(\Lambda,a)|x,t>$ on vectors in the physical Hilbert space. We build up the `double space´- or `$MS_p$´-SUSY theory, wherein the superspace is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ by the inclusion of additional 8D fermionic coordinates. The latter is induced by the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to $\underline{M}_2$. By virtue of embedding map (10), the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$, in turn, induce the respective spinors $\theta(\underline{\theta},\bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta},\bar{\underline{\theta}})$ (see (93)), as to $M_4$. Consequently, the `symmetric´ superspace is parameterized (see (66)) by $\Omega = \Omega_q(x^m,\, \theta(\underline{\theta},\bar{\underline{\theta}}),\, \bar{\theta}(\underline{\theta},\bar{\underline{\theta}})) \times \Omega_{\underline{q}}(\underline{\eta}^m,\, \underline{\theta},\, \bar{\underline{\theta}})$. Supersymmetry transformation is defined as a translation in superspace, specified by the group element with corresponding anticommuting parameters. The multiplication of two successive transformations induce the motion. Obviously the ground state of $MS_p$-SUSY has a vanishing energy value and is nondegenerate (SUSY unbroken). All the particles are living on $M_4$, their superpartners can be viewed as living on $\underline{M}_2$. We emphasize that in $MS_p$-SUSY theory, alike in standard exact SUSY theory, the vacuum zero point energy problem, standing before any quantum field theory in $M_4$, will be solved. The particles in $M_4$ themselves can be considered as excited states above the underlying quantum vacuum of background double space $M_4 \oplus \underline{M}_2$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2 \equiv \nu_b = \nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy.

**Two founding properties.** On the premises of previous paragraph, we derive the two founding properties that (i) the atomic displacement $\Delta\underline{\eta}_{(a)}$, caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$, is an invariant; and (ii) the bilinear combination $\underline{\theta}\,\bar{\underline{\theta}}$ is a constant. These properties underly the two postulates on which the theory of SR is based. Thus, we show that the motion of a particle in $M_4$ is encoded in the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to the master space $\underline{M}_2$. This calls for a complete reconsideration of our ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property.


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This framework allows to introduce the physical finite *relative time* interval between two events as integer number of the *own atomic duration time* of double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$.

**Intrinsic code.** The predictions of proposed theory require a complete revision of our ideas about the Lorentz code, to be now referred to as the individual code of a particle, defined as its intrinsic property. The $MS_p$ , embedded in the background 4D-space, is an unmanifested indispensable individual companion to the particle of interest, devoid of any external influence.

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