Role of galactic disc thickness in magnetic field generation

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Abstract

A large variety of galaxies have magnetic structures of lengthscales comparable with their radius. Theoretically, their existence is based on the dynamo mechanism. It is based on alpha-effect characterizing helicity of the turbulent motions and differential rotation which is connected with changing angular velocity. The field can be destroyed by turbulent diffusion, so the mechanism is threshold and can be realized only for the case when first and second effect are more intensive than the third one. Equations of magnetohydrodynamics that are used to describe the dynamo mechanism, are too difficult to be solved. So, usually different approximation taking into account details of astrophysical objects are used. As for galaxies, a thin disc approximation has been developed. It works properly for galactic objects with small half-thickness. However, as for thick discs we should take a model which uses more complicated structure of the field. Here we find the threshold for the field generation for thick astrophysical discs.

Keywords: *magnetic fields, galaxies, eigenvalue problem*

1. Introduction

Lots of galaxies have regular magnetic fields (Arshakian et al., 2009). First arguments for it were obtained while studying cosmic rays. Simple models predicted that the rays should concentrate in the equatorial plane. However, their spatial distribution is quite uniform, so something should change their trajectories. This is caused by the magnetic field, and Enrico Fermi in his pioneer work estimated its value which is quite close to the modern observations (about 1 microgauus) (Fermi, 1949). A large number of observational proofs were connected with synchrotron emission studies (Ginzburg, 1959). Today most of the observational measurements are based on Farady rotation (Andreasyan & Makarov, 1989, Beck et al., 1996, Manchester, 1972). As for Milky Way, usually pulsars are taken (now there are 10^3 ones (Andreasyan et al., 2020). As for another galaxies, the extragalactic sources are taken to find the magnetic fields (Oppermann et al., 2011).

From the theoretical point of view, the process of the magnetic field generation seems to contain three stages. First of all, initial fields are connected with Biermann battery mechanism (Biermann & Schlüter, 1951). It is connected with fluxes of protons and electrons. They have principally different masses, and interact with the moving particles according to different laws. So there are circular currents which produce vertical magnetic fields (Mikhailov & Andreasyan, 2021).

After that, the field is enlarged by the turbulent dynamo. It is connected with properties of small-scale motions, and it can produce magnetic fields which have the strength of order of microgauss. The problem is that such field is associated with small turbulent cells, and it has random directions (Arshakian et al., 2009).

Then the large-scale dynamo begins its action. The average magnetic field is non-zero because the number of turbulent cells is finite. So, the field can be enlarged by the large-scale dynamo (Arshakian et al., 2009). It is based on two basic effects. Firstly, it is the differential rotation which characterizes non-solid rotation of the galactic disc. This process it connected with transition of the radial component of the field to the angular one. Secondly, the dynamo contains alpha-effect which is connected with helicity of the turbulent motions (curl of the turbulent velocity has a projection on the velocity itself, and if we average it, the corresponding coefficient should be included to the equations). Alpha-effect helps to make radial

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magnetic field from the angular one. These two effect work jointly and enlarge the solution. However, they should compete with the turbulent diffusivity, which tries to destroy structures of the magnetic field. So the magnetic field generation is connected with the treshold effect, and it is possible only if first two processes are more effective than the turbulent dissipation. Usually it is described by the so-called dynamo number, which includes vertical scaleheight of the disc, angular velocity and typical velocity of the turbulent motions (Beck et al., 1996). If it is larger than some critical value, we can say that the magnetic field in the galaxy can grow, else it decays.

The field growth can be studied using Steenbeck – Krause – Raedler equation (also called mean field dynamo equation) which is three-dimensional (Krause & Raedler, 1980). It is quite difficult both for the analytical and numerical solution, so different approximations are used. The approximations usually take into account symmetry properties of the object, and usually the equations are reduced to a pair of formulaes.

As for the galaxies, usually the thin disc approximation is used. It was developed by Moss (1995), Subramanian & Mestel (1993), and it considered half-thickness of the disc as a small parameter. The field component which is perpendicular to the disc can be neglected, and its partial derivative can be taken from the non-divergence condition. The z-structure of the field is assumed according to the cosine law, so the z-derivatives can be calculated algebraically. Also the coefficients which characterize averaged characteristics of the motion of the medium.

The evolution equations describe the magnetic field growth according to an exponential law (Mikhailov & Pashentseva (Frolova), 2022b). If we assume such law, we can obtain an eigenvalue problem for the magnetic field. As for the thin disc model in the axisymmetric case, the eigenvalues and the eigenfunctions can be found exactly (Mikhailov, 2020). The eigenvalue characterizes the possibility of the magnetic field growth. Also, positive values characterize if the field can principally grow. It is important that the magnetic field growth rate includes the dynamo number.

However, this model is not applicable for discs, where the half-thickness can be comparable with the radius of the disc. The vertical structure can differ from the simple cosine model, and the vertical flows can occur and change the magnetic field. So we should take the models which describe z-dependence in more detail (Mikhailov & Pashentseva (Frolova), 2022a).

As for this case, the eigenvalue problem is much more complicated. It can be solved only approximately using perturbation theory. As for the non-perturbed case we can take the results which have been obtained for thin disc model.

Here we give a review of results connected with the study of the magnetic field in thin and thick galactic discs (Mikhailov & Pashentseva (Frolova), 2022a,b). After that we show the numerical tests of the magnetic field growth for different parameters.

2. Thin disc approximation

In the case of thin disc, we can describe the magnetic field in the equatorial plane using two components depending on distance from the center r: the angular one B_φ and the radial one B_r. We will assume that components of the field are proportional to $\cos\left(\frac{\pi z}{2h}\right)$, where z is the distance from the equatorial plane, and h is the half-thickness of the disc. Using the dimensional variables and measuring the lengths in units of the radius of the galaxy, and times in the units of $\frac{h^2}{\eta}$, we will obtain the following system of equations (Moss 1995; Mikhailov 2020):

$$\frac{\partial B_r}{\partial t} = -R_\alpha B_\varphi - \frac{\pi^2 B_r}{4} + \lambda^2\left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2}\right);$$

$$\frac{\partial B_\varphi}{\partial t} = -R_\omega B_r - \frac{\pi^2 B_\varphi}{4} + \lambda^2\left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2}\right);$$

with boundary conditions:

$$B_r|_{r=0} = B_r|_{r=1} = B_\varphi|_{r=0} = B_\varphi|_{r=1} = 0.$$

Here R_α characterizes the alpha-effect, R_ω shows differential rotation and $\lambda = \frac{h}{R}$ is the dissipation parameter (Moss 1995).

Taking into account that the magnetic field is proportional to $\exp \gamma t$, we can rewrite the problem in eigenvalue form (Mikhailov 2020):

$$\gamma B_r = -R_\alpha B_\varphi - \frac{\pi^2 B_r}{4} + \lambda^2\left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2}\right);$$

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$$\gamma B_\varphi = -R_\omega B_r - \frac{\pi^2 B_\varphi}{4} + \lambda^2 \left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2} \right).$$

The eigenvalue will be:

$$\gamma_n^\pm = -\frac{\pi^2}{4} \pm \sqrt{D} - \lambda^2 \mu_{1,n}^2.$$

Corresponding eigenfunctions are (Mikhailov 2020):

$$B_r^+ = \sqrt{R_\alpha} J_1(\mu_{1,n} r);$$

$$B_\varphi^+ = -\sqrt{R_\omega} J_1(\mu_{1,n} r);$$

$$B_r^- = \sqrt{R_\alpha} J_1(\mu_{1,n} r);$$

$$B_\varphi^- = \sqrt{R_\omega} J_1(\mu_{1,n} r).$$

Here J_1 is the Bessel function and $\mu_{1,n}$ is the zero of this function ($J_1(\mu_{1,n}) = 0$). $D = R_\alpha R_\omega$ is the dynamo number. It can be obtained as $D = 9\frac{h^2\Omega^2}{u^2}$ where u is the turbulent velocity.

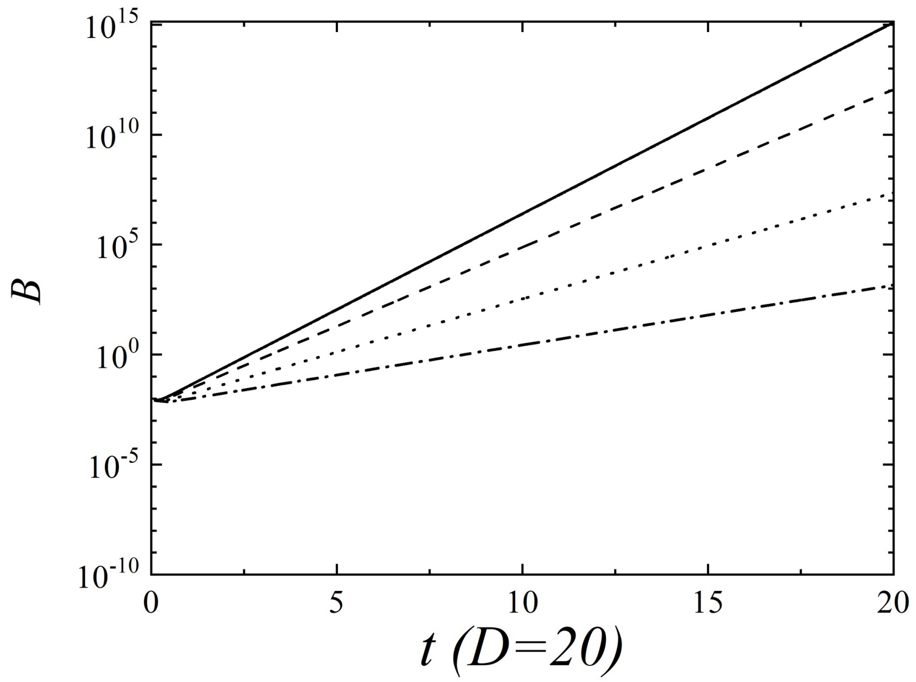

Figure 1. Magnetic field evolution for $D = 20$. Solid line shows $\chi = 1$, dashed line – $\chi = 0.7$, dotted line – $\chi = 0.3$, dot-dashed line – $\chi = 0$,

3. Thick disc model

As for the thick disc, the field will also depend on the distance from the equatorial plane, and the equation will be (Mikhailov & Pashentseva (Frolova), 2022a):

$$\frac{\partial B_r}{\partial t} = -R_\alpha B_\varphi - \chi R_\alpha z \frac{\partial B}{\partial z} + \lambda^2 \left(\frac{\partial^2 B_r}{\partial z^2} + \frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2} \right);$$

$$\frac{\partial B_\varphi}{\partial t} = -R_\omega B_r + \lambda^2 \left(\frac{\partial^2 B_r}{\partial z^2} + \frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2} \right);$$

with boundary conditions:

$$B_r|_{r=0} = B_r|_{r=1} = B_\varphi|_{r=0} = B_\varphi|_{r=1} = 0;$$

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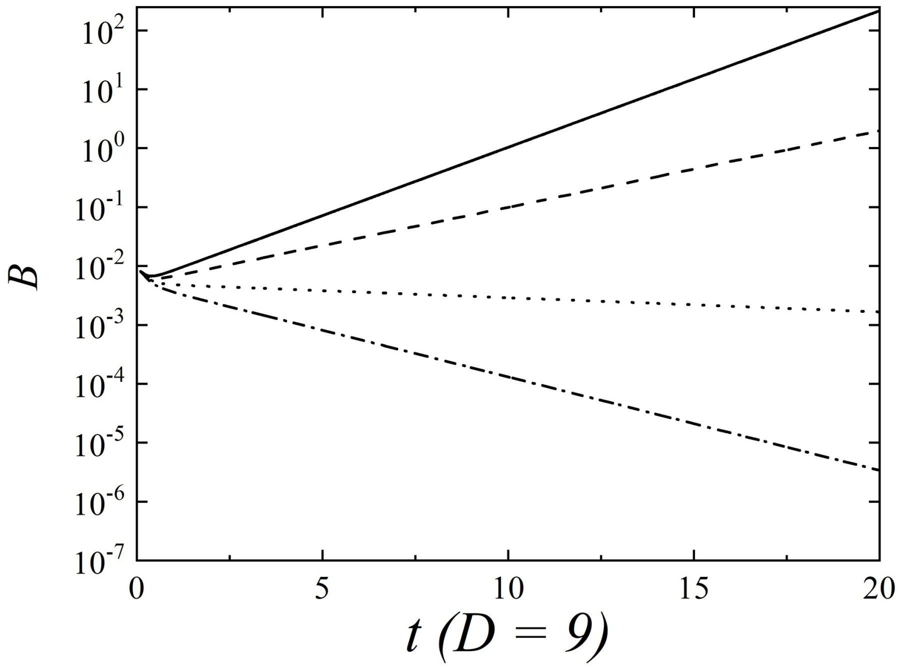

Figure 2. Magnetic field evolution for $D = 9$. Solid line shows $\chi = 1$, dashed line – $\chi = 0.7$, dotted line – $\chi = 0.3$, dot-dashed line – $\chi = 0$,

$$B_r|_{z=-\lambda} = B_r|_{z=\lambda} = B_\varphi|_{z=-\lambda} = B_\varphi|_{z=\lambda} = 0.$$

Here χ characterizes the role of vertical flows. The problem in the eigenvalue form is (Mikhailov & Pashentseva 2023):

$$\gamma B_r = -R_\alpha B_\varphi - \chi R_\alpha z \frac{\partial B}{\partial z} + \lambda^2 \left(\frac{\partial^2 B_r}{\partial z^2} + \frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_r}{\partial \varphi^2} \right);$$

$$\gamma B_\varphi = -R_\omega B_r + \lambda^2 \left(\frac{\partial^2 B_r}{\partial z^2} + \frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2} + \frac{1}{r^2}\frac{\partial^2 B_\varphi}{\partial \varphi^2} \right);$$

Approximate value of the eigenvalue is (Mikhailov & Pashentseva (Frolova), 2022a):

$$\gamma_{nm}^{\pm} = \pm\sqrt{D} - \frac{\chi}{4}\sqrt{D} - \lambda^2 \mu_{1,n}^2 - \frac{\pi m^2}{4}$$

Eigenfunctions are the following:

$$B_{r\,nm}^{+} = \sqrt{R_\alpha} J_1(\mu_n r) \sin\left(\frac{\pi m (z-\lambda)}{2\lambda} \right);$$

$$B_{\varphi\,nm}^{+} = -\sqrt{R_\omega} J_1(\mu_n r) \sin\left(\frac{\pi m (z-\lambda)}{2\lambda} \right);$$

$$B_{r\,nm}^{-} = \sqrt{R_\alpha} J_1(\mu_n r) \sin\left(\frac{\pi m (z-\lambda)}{2\lambda} \right);$$

$$B_{\varphi\,nm}^{-} = \sqrt{R_\omega} J_1(\mu_n r) \sin\left(\frac{\pi m (z-\lambda)}{2\lambda} \right).$$

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4. Magnetic field growth and critical value

As it can be seen, the main effect is connected with the role of the parameter χ. The critical dynamo number will be:

$$D_{cr} = \frac{4}{4-\chi}\left(\frac{\pi^2}{4} + \lambda^2\mu_{1,n}^2\right).$$

The field can grow if $D > D_{cr}$, when $\gamma_{nm}^{\pm} > 0$.

As for the growing magnetic field, we can check the approximate values numerically. The results are shown on figure 1.

5. Conclusion

We have studied the magnetic field growth for different models of the field growth in galactic discs. It has been shown that for thin and thick discs the results are different (Mikhailov & Pashentseva (Frolova), 2022a). We have checked the approximate analytical results using numerical modelling. It is shown that larger values of parameter χ makes the field growth rate larger. Also, the critical dynamo number becomes smaller.

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