

# Influence of gas-dynamic flows on the evolution of line spectra in a medium with non-stationary energy sources

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## Abstract

The theory we developed in previous papers on temporal variations in the spectral line profiles formed in the media with nonstationary energy sources is generalized to take into account the influence of stochastic velocity fields. The radiation transfer problem is treated for the case of completely incoherent scattering. The limiting regimes of micro- and macro-turbulence are considered under the assumption that the gas-dynamic velocity is described by a Markov process. Comparison of the characteristic features of the evolution of spectral line profiles in these two cases with that observed in the absence of turbulence allows us to make an idea of the nature of turbulence and the values of various parameters describing the primary energy sources and optical properties of the medium.

## 1. Introduction

Many of the astrophysical objects of considerable interest are known to be far from the state of equilibrium. A great variety of nonstationary phenomena is observed, differing from each other both in their scales and durations. A far from exhaustive list of such objects includes evolution of the shells formed by Novas and Supernovas bursts, solar CMEs and chromospheric outbursts, flare stars, and FUor-like phenomena. Interest in this kind of phenomena is increasing in view of the emergence of new more precise observational capabilities providing sufficiently informative photometric and spectrophotometric data. Spectroscopic studies of sources exhibiting nonstationary manifestations are crucial, as the line profile shapes offer rich information about the physical processes that are occurring at their current stage of evolution.

In a series of papers (Nikoghossian, 2021a,b, 2022a,b, 2023) we considered a range of different physical factors which can make some contribution in the course of the lines evolution such as inhomogeneity of the medium, localization of primary energy sources, scattering and absorption in the continuum, frequency redistribution, etc. In all of these problems, the non-stationarity was assumed to be due to time variations of the energy sources illuminating the medium. The temporal characteristics of the incident radiation were given by the functional forms of either the Dirac delta function or the Heaviside H-function. The importance of the relationships of these two types is obvious: the first one simulates phenomena of short-term explosive character of the type of Nova explosions, and the second one mimics the evolutionary path of the equilibrium state establishment at the inclusion of a more or less constantly acting energy source of the type of FUor-like phenomena or the case of Her Nova 1934 (Gorbatskii & Minin, 1963, Sobolev, 1985). In all of the mentioned cases the active medium is supposed to be static, i.e., it is considered that the structure and dynamics of the medium do not change under the influence of primary energy sources. Meanwhile, in astrophysical context, it is reasonable to assume that at the specified temporal characteristics, assuming a sudden crossing of the boundary of the medium-energy front, one should lead to the appearance of gas-dynamic currents and a field of fluctuating velocities in it, which is actually equivalent to the appearance of turbulent motions. The latter certainly affects the shape of the observed spectrum and its evolution. This paper

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is devoted to the study of these questions. We abstract questions related to energy dissipation or to possible changes in the physical characteristics of the medium under consideration and its dynamics during the evolution of the spectra.

We start with the formulation of the time-dependent model problem under consideration. The basic equations are given with a description of the solution methods used. The next section presents numerical results. Their physical interpretation and consequences are given in the final section.

## 2. Statement of the problem. Solution methods

As in recent works, we consider, for expository reasons, a 1D absorbing and scattering atmosphere of finite optical thickness  $\tau_0$  calculated in the center of a given spectral line. One of its boundaries, e.g.,  $\tau = 0$ , is illuminated by the continuous radiation flux  $I_0(t)$ , which varies in time according to the above-mentioned laws of  $\delta(t)$  or  $H(t)$  functional forms. When passing through the medium, the radiation in the spectral line is scattered with the probability  $\lambda$  and is also absorbed in the continuous spectrum. The probability of this latter process is described by the value of  $\beta$  given by the ratio of the absorption coefficient in the continuous spectrum to that in the center of the spectral line. The scattering is assumed to be completely incoherent and the frequency redistribution law may be written in the form

$$r(x', x) = \alpha_0(x') \alpha_0(x), \quad (1)$$

where  $\alpha_0(x) = \pi^{-1/4} \alpha(x)$ , and  $\alpha(x)$  is the profile of the absorption coefficient of the spectral line.

In the present work, calculations are carried out for the Doppler profile of the absorption coefficient  $(1/\sqrt{\pi}) \exp(-x^2)$ . The classical problem of diffuse reflection and transmission is posed and solved under the assumption of homogeneous turbulence in the medium (Bellman & Wing, 1973). We are interested in the line-transfer problem to find out the influence of different types of gas-dynamic motions on the shapes of spectral line profiles observed in the resulting reflection and absorption spectra and on the character of their evolution with time. To this end, it is sufficient to limit ourselves to considering two extreme mutually opposite regimes of micro- and macro-turbulence and to compare the results obtained with those obtained in the absence of turbulence. Consideration of a more general problem at intermediate values of the correlation length, which leads to some mathematical complications and was solved in Nikoghossian (2017), is not essential for our goal, especially since often the transition between two limiting regimes in some cases occurs in a sufficiently narrow interval of change of the specified length (Nikoghossian, 2007).

The evolution of spectral lines in the absence of turbulent motions in the medium under the assumption of complete redistribution over frequencies, we considered in Nikoghossian (2022a). Therefore, the results obtained there will serve as a basis for identifying effects due to gas-dynamic motions. At the same time, as in the above work, we use the same method based on the construction of Neumann series to solve the problems. The method dates back to the works of Ganapol (1979), Matsumoto (1967, 1974). We will see below that the Neumann series allows one to construct easily solutions of the corresponding more general problems taking into account turbulence. This is an additional advantage of the method, which is due to the fact that in both cases one starts with the same Neumann expansion for the stationary problem constructed in the above cited paper. In the homogeneous problems it is a series in powers of  $\lambda$  so that for the reflection and transmission coefficients  $\rho(x', x, \tau_0)$  and  $q(x', x, \tau_0)$  we have

$$\rho(x', x, \tau_0) = \sum_{n=1}^{\infty} \rho_n(x', x, \tau_0) \lambda^n, \quad q(x', x, \tau_0) = \sum_{n=1}^{\infty} q_n(x', x, \tau_0) \lambda^n. \quad (2)$$

where  $x'$  and  $x$  are the dimensionless frequencies (measured in terms of Doppler widths) of incident quantum and reflected or transmitted quanta. Each term of the Neumann expansions obviously have a probabilistic meaning similar to that of reflection and transmission coefficients, relating however to a certain fixed number of scattering events  $n$ . Recall that the coefficients in the Neumann expansions we find using the invariant imbedding technique (Bellman & Wing, 1973, Casti & Kalaba, 1976)

and the recurrence method for their determination we suggested in [Nikoghossian \(1984\)](#). In the case of a medium of finite optical thickness we consider, each stage of application of the above method mathematically is reduced to solving some initial-value problem. The discussion of the accuracy of the construction of the corresponding series in [Nikoghossian \(2022a\)](#) remains valid in our problems, and we will not dwell on it here.

The corresponding Eq.(2) Neumann series for spectral line profiles in the absence of turbulence is written in the form

$$R(x, \tau_0) = \sum_{n=1}^{\infty} R_n(x, \tau_0) \lambda^n, \quad Q(x, \tau_0) = \sum_{n=1}^{\infty} Q_n(x, \tau_0) \lambda^n. \quad (3)$$

In the limiting case of micro-turbulence, when the correlation length tends to zero, the absorption coefficient profile in the spectral line  $\alpha(x)$  is transformed as follows.

$$\omega(x) = \int_{-\infty}^{\infty} \alpha(x-u)P(u) du, \quad (4)$$

where  $\omega(x)$  and  $\alpha$  are the profile of the absorption coefficients correspondingly in turbulent and static atmospheres,  $P(u)$  is the distribution function of turbulent velocities  $u$  which under the assumption of a Gaussian velocity distribution has the form

$$P(u) = \frac{1}{\sqrt{\pi}u_t} \exp(-u/u_t)^2, \quad (5)$$

where  $u_t$  is the mean hydrodynamic velocity measured in terms of thermal speed. In the case of Dopplerian profile, Eq.(1) leads to known result which is the sum of two variances (see also, [Gray, 1978](#)).

$$\omega(x) = \frac{1}{\sqrt{1+u_t^2}} \exp\left(-\frac{x^2}{1+u_t^2}\right), \quad (6)$$

Thus, the whole course of reasoning carried out in [Nikoghossian \(2022a\)](#), remains in force in the case of micro-turbulence with the replacement of  $\alpha(x)$  by  $\omega(x)$ , while an additional parameter  $u_t$  appears. For each of given values of this parameter, the Neumann series is constructed as has been done in the mentioned paper. By introducing the notations  $R_n^{mic}$  and  $Q_n^{mic}$  for the obtained coefficients, similarly to Eq.(3) we can write

$$R_{mic}(x, \tau_0) = \sum_{n=1}^{\infty} R_n^{mic}(x, \tau_0) \lambda^n, \quad Q_{mic}(x, \tau_0) = \sum_{n=1}^{\infty} Q_n^{mic}(x, \tau_0) \lambda^n. \quad (7)$$

Turning to the limiting case of infinite correlation length (macro-turbulence), we note that now it is the spectral line profile that undergoes transformation Eq.(3) (see, for instance, [Bellman & Wing, 1973](#))

$$R_n^{mac}(x, \tau_0) = \int_{-\infty}^{\infty} R_n(x-u)P(u) du, \quad Q_n^{mac}(x, \tau_0) = \int_{-\infty}^{\infty} Q_n(x-u)P(u) du, \quad (8)$$

and

$$R_{mac}(x, \tau_0) = \sum_{n=1}^{\infty} R_n^{mac}(x, \tau_0) \lambda^n, \quad Q_{mac}(x, \tau_0) = \sum_{n=1}^{\infty} Q_n^{mac}(x, \tau_0) \lambda^n. \quad (9)$$

From the point of view of further exposition, the frequency distribution of the coefficients in the Neumann series for spectral line profiles can be obtained from Figs. (1), (2) for lines formed by reflection from the medium and Figs. (3), (4) - for lines formed as a result of passing through it. In fact, the curves in the figures are profiles formed after a certain fixed number of scattering events in

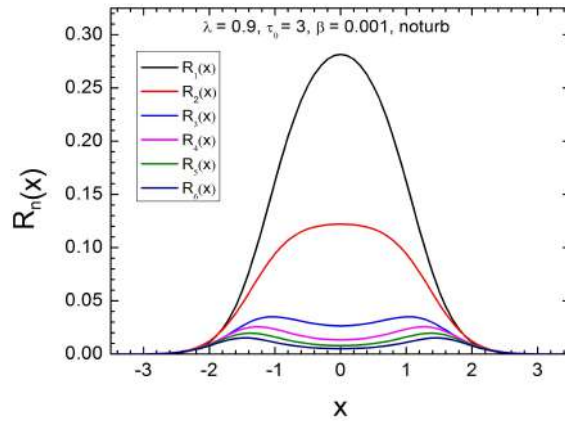


Figure 1. Line profiles formed as a result of reflection after fixed number of scattering events in the medium in the absence of turbulent motions.

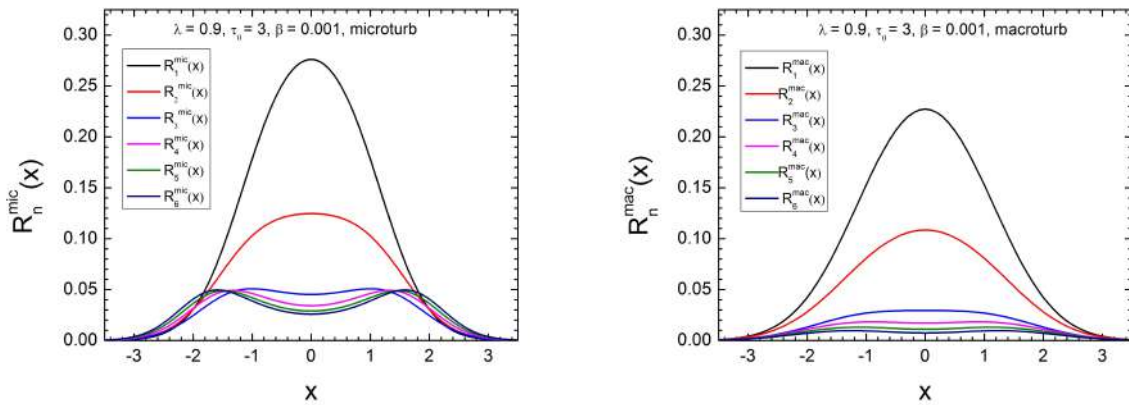


Figure 2. The same as in Fig.1 for the developed micro- (left panel) and the macro-turbulence (right panel) and  $u_t = 1$ .

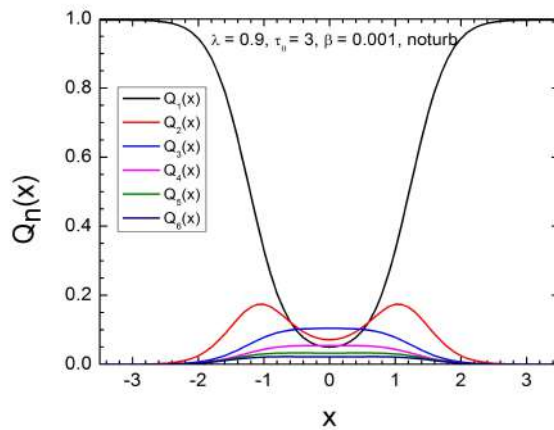


Figure 3. Line profiles formed as a result of transmission through the medium after fixed number of scattering events in the absence of turbulent motions.

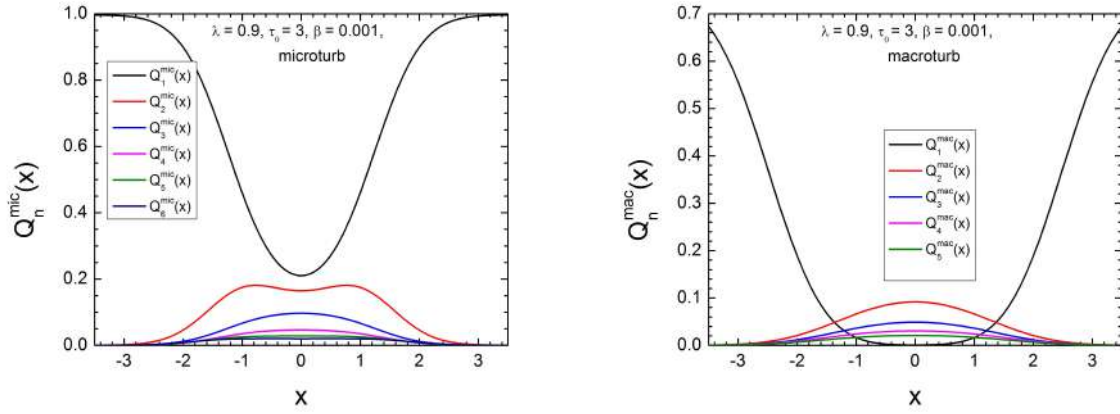


Figure 4. The same as in Fig.3 for the developed micro- (left panel) and the macro-turbulence (right panel) and  $u_t = 1$ .

the medium and clearly demonstrate the influence of turbulent motions depending on the degree of their development.

The characteristic features of demonstrated profiles are important for interpretation the further considered temporal changes of the discrete spectrum in non-stationary phenomena. The influence of turbulence on the reflectance of the medium is discernible. It is particularly seen that the shapes of the observed profiles of spectral lines are largely determined by processes associated with a small number of scattering acts. Special mention should be made of the rather broad bell-shaped profiles formed in the transmitted spectrum by macro-turbulence. The mechanism of their broadening is similar to that we have in the case of object rotation. Another characteristic feature is the double-peaked profiles appearing in the higher components of both the reflection and transmission functions for the cases of micro-turbulent and absence of it. Especially remarkable is the appearance of the emission double-peaked emission profile in the absorption spectrum formed in transmission of radiation through the medium. In the following, we will return to this question when studying the evolution of spectral line profiles under illumination of the medium by non-stationary energy sources.

### 3. The evolution of the line profiles

We recall that knowledge of Neumann expansions allows us to write directly the explicit expressions for the required probability density function (PDF) and the cumulative distribution function (CDF).

For the convenience of the reader, we reproduce here these relations, which are basic to the calculations

$$R(x, \tau_0, z) = \sum_{n=1}^{\infty} R_n(x, \tau_0) F_n(z) \lambda^n, \quad Q(x, \tau_0, z) = \sum_{n=1}^{\infty} Q_n(x, \tau_0) F_n(z) \lambda^n, \quad (10)$$

where the PDF functions  $R(x, \tau_0, z)$ ,  $Q(x, \tau_0, z)$  are introduced such that  $Rdz$  and  $Qdz$  are respectively the probabilities of observing the given reflected and transmitted lines profiles in the time interval  $(z, z + dz)$ , and,  $F_n$ , as we have shown in recent papers (Nikoghossian, 2021a,b), is the probability density function (PDF) of the total time taken by a quantum at a certain number of  $n$  scattering events given by

$$F_n(z) = \frac{z^{2n-1}}{(2n-1)!} e^{-z}, \quad n = 1, 2, \dots \quad (11)$$

which is obtained as a result of addition of two Erlang- $n$  distributions describing the total residence time of atoms in the excited state and the time spent by quanta between two consecutive scattering acts. It is taken into account that the number of free runs of a quantum exceeds by one the number

of its scattering events in the medium. The rates of both distributions for simplicity are assumed for simplicity's sake to be approximately equal, so that the dimensionless time  $z$  is read in units of the mean time  $\bar{t} = t_1 + t_2$  expressed in terms of  $t_1$  and  $t_2$  representing respectively the mean time for an atom to be in an excited state and the frequency-averaged mean time taken by a quantum to fly between two successive acts of scattering. Note that the value of  $t_2$  depends on the density of scattering centres, which in turn is determined by the physical conditions in the medium. In the simplest cases it is given by formula  $t_2 = 1/nkc$ , where  $n$  is the density of emitting or absorbing atoms, depending on whether the line is in emission or absorption,  $k$  is the total coefficient of a given line and  $c$  is the speed of light. Sometimes, when it comes to strong lines, usually hydrogen, it is necessary to take into account also the degree of ionisation of these atoms.

The functions  $F_n$  taking together describe temporal variations in the profiles of spectral lines formed at the boundaries of the medium when it is illuminated by a  $\delta$ -function shaped pulse in the continuous spectrum. Note that they take their maximum value at  $z = 2n - 1$ .

In addition, we also consider the establishment of the equilibrium state regime under prolonged illumination of the medium by a source of the unit jump form given by the Heaviside H-function. The evolution of the line profiles in this case is described by the cumulative distribution function (CDF) given by

$$\tilde{R}(x, \tau_0, z_0) = \sum_{n=1}^{\infty} R_n(x, \tau_0) \Phi_n(z_0) \lambda^n, \quad \tilde{Q}(x, \tau_0, z_0) = \sum_{n=1}^{\infty} Q_n(x, \tau_0) \Phi_n(z_0) \lambda^n, \quad (12)$$

where

$$\Phi_n(z_0) = e^{-z_0} \sum_{k=0}^{\infty} \frac{z_0^{2n+k}}{(2n+k)!} \quad (13)$$

In both illumination cases, the incident radiation in continuum is assumed to be of unit intensity. To identify the influence of turbulent motions, we should consider the cases of micro- and macro-turbulence separately. The reasoning used in the derivation of the basic probability density function (PDF) and cumulative distribution function (CDF) describing the evolution of the observed spectral line profiles remains valid. Now instead of Eqs.(9),(11) we have

$$R_{mic}(x, \tau_0, z) = \sum_{n=1}^{\infty} R_n^{mic}(x, \tau_0) F_n(z) \lambda^n, \quad Q_{mic}(x, \tau_0, z) = \sum_{n=1}^{\infty} Q_n^{mic}(x, \tau_0) F_n(z) \lambda^n, \quad (14)$$

$$\tilde{R}_{mic}(x, \tau_0, z) = \sum_{n=1}^{\infty} R_n^{mic}(x, \tau_0) \Phi_n(z) \lambda^n, \quad \tilde{Q}_{mic}(x, \tau_0, z) = \sum_{n=1}^{\infty} Q_n^{mic}(x, \tau_0) \Phi_n(z) \lambda^n, \quad (15)$$

and

$$R_{mac}(x, \tau_0, z) = \sum_{n=1}^{\infty} R_n^{mac}(x, \tau_0) F_n(z) \lambda^n, \quad Q_{mac}(x, \tau_0, z) = \sum_{n=1}^{\infty} \bar{Q}_n^{mac}(x, \tau_0) F_n(z) \lambda^n, \quad (16)$$

$$\tilde{R}_{mac}(x, \tau_0, z_0) = \sum_{n=1}^{\infty} R_n^{mac}(x, \tau_0) \Phi_n(z_0) \lambda^n, \quad \tilde{Q}_{mac}(x, \tau_0, z_0) = \sum_{n=1}^{\infty} \bar{Q}_n^{mac}(x, \tau_0) \Phi_n(z_0) \lambda^n, \quad (17)$$

where  $z_0 = t_0/\bar{t}$ , and  $t_0$  is the time interval of the observed variation. Formulas Eqs.(14),(15), as well as Eq.(16) and Eq.(17) allow us to construct the desired PDF and CDF distributions and trace the evolution of the observed spectral line profiles at both considered energy sources.

Turning directly to the description of the obtained results, we continue throughout to give the following figures in pairs (3 panels each) for convenience of comparison by the reader of the results for three limiting cases: absence of gas-dynamic motions, the developed micro- and macro-turbulence.

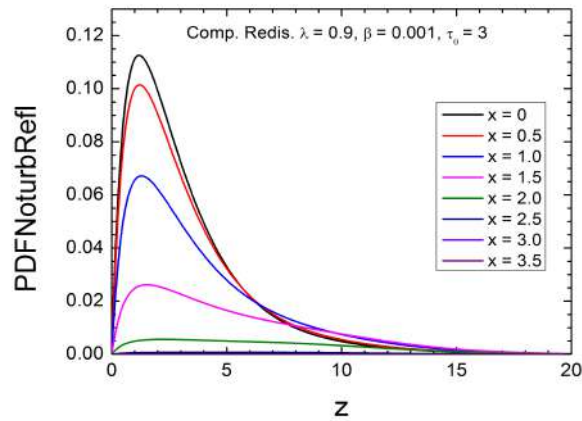


Figure 5. Probability density functions for radiation reflected from the medium of optical thickness  $\tau_0 = 3$  and  $\lambda = 0,9$  in the absence of turbulent motions.

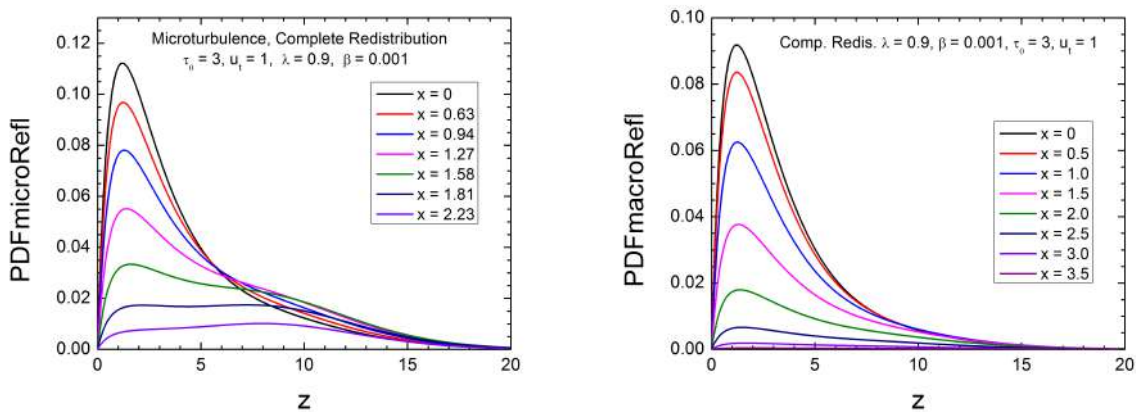


Figure 6. Probability density functions for the developed micro- (left panel) and the macro-turbulence (right panel). In both cases  $u_t = 1$

Let us first consider PDF distributions, which allow us to get a general idea of frequency-dependent evolution of spectral lines for the medium illuminated by a source of the form of the  $\delta(z)$  pulse. Figs.5,6 concern the reflection spectrum, while Figs.7,8 concern the absorption spectrum. Note that in order to achieve satisfactory accuracy in the case of a micro-turbulent medium, the zeros of the Hermite polynomial of the 49th order are chosen as frequency nodes inside the line.

The PDF distributions for both types of spectra contain rich information about the ongoing process that causes the observed changes in the line spectrum. The most important characteristics of these distributions are the location and magnitude of maxima in the centre of the lines. The location of the maxima of the central parts of the spectral lines in a small vicinity of  $z = 1$  seems somewhat unexpected in view of the values of  $z$ , where the functions  $F_n(z)$  reach their maxima. This is due to the magnitude of the functions  $R_n(x, \tau_0)$ , where the first term is much larger compared to others. Closer to the wings of the lines and in the wings themselves, the maxima are reached later at higher values of  $z$ , so that the wings of lines are formed at a later time  $z$  and the real time  $t$ . This is clearly discernible in Figs.6,8 for the lines formed as a result of reflection in the case of micro-turbulence and lines in the absorption spectrum in the case of macro-turbulence.

Of particular importance are the differences in the real time of the PDF maxima in different lines, which is largely due to the value of  $t_2$ . Knowledge of the latter allows us to determine the population of the corresponding level and estimate the optical thickness in the line. Strictly speaking, in certain cases it is necessary to take into account the physical conditions in the medium (such as, for example,

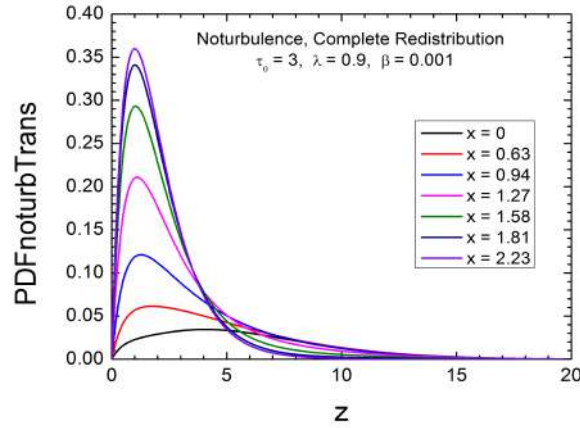


Figure 7. Probability density functions for radiation transmitted through the medium for the above adopted optical parameters in the absence of turbulent motions.

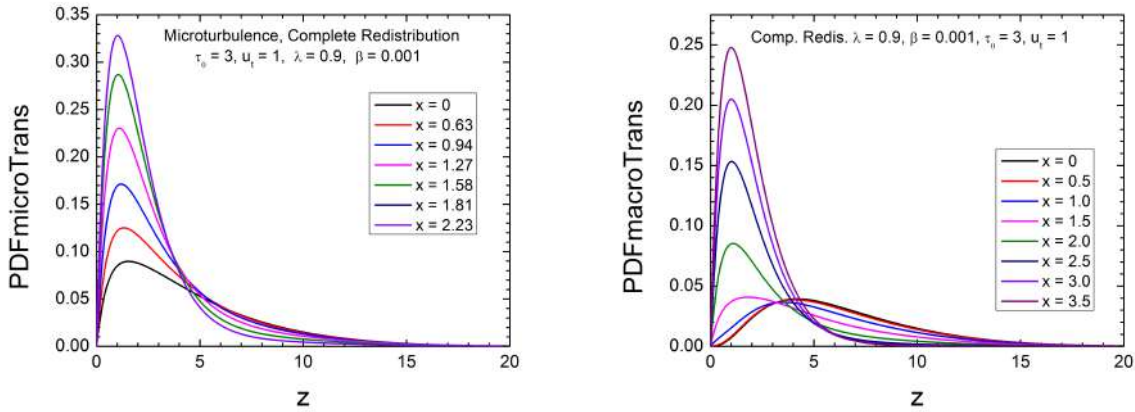


Figure 8. The same as in Fig.7 for the developed micro- (left panel) and the macro-turbulence (right panel).

the degree of ionisation of hydrogen in the case of strong hydrogen lines in the case of giants of the early spectral classes). At low densities of absorbing atoms/ions the quanta scatter less frequently but spend more time in the medium due to the increase of the path length between the scattering acts (Nikoghossian, 2021a,b), which leads to the increase of the real time to reach the intensity maximum even if the  $z_{max}$  values are not very different. Thus, the time of reaching the maximum can be regarded as an indicator of the optical thickness in the line under consideration, which, in turn, allows us to judge the chemical composition of the element in question. It should also be noted that different parts of the spectral line profile reach their maximum values at different times, which is most clearly seen in the reflection spectrum in micro-turbulence and the transmission spectrum in macro-turbulence. Usually, the wings of the lines evolve longer than their cores.

Another important characteristic of the PDF distribution is the value of the magnitude of its maximum. This value in addition to optical thickness depends also on the parameter  $\lambda$  (see Nikoghossian, 2022a) so, knowledge of the first one allows us to estimate theoretically the scattering coefficient. Further, the value of the PDF maximum corresponding to the radiation of unit intensity reflected from the medium makes it possible to calibrate the spectrum by comparing it with that for the one of observed lines supposedly possessing similar values of parameters  $\lambda$  and  $\beta$ . A similar possibility of calibration exists also in the absorption spectrum formed as a result of radiation passing through a medium. As it was shown in the above mentioned work, it is due to the asymptotic tendency of the far wings of the spectral line to a continuum, the level of which is determined mainly by the direct passage



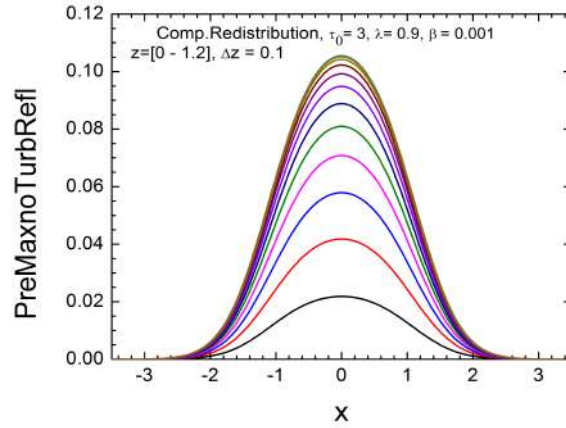


Figure 9. Evolution of the spectral line profiles formed as a result of radiation reflection from the medium until reaching the maximum of the PDF in the absence of turbulence.

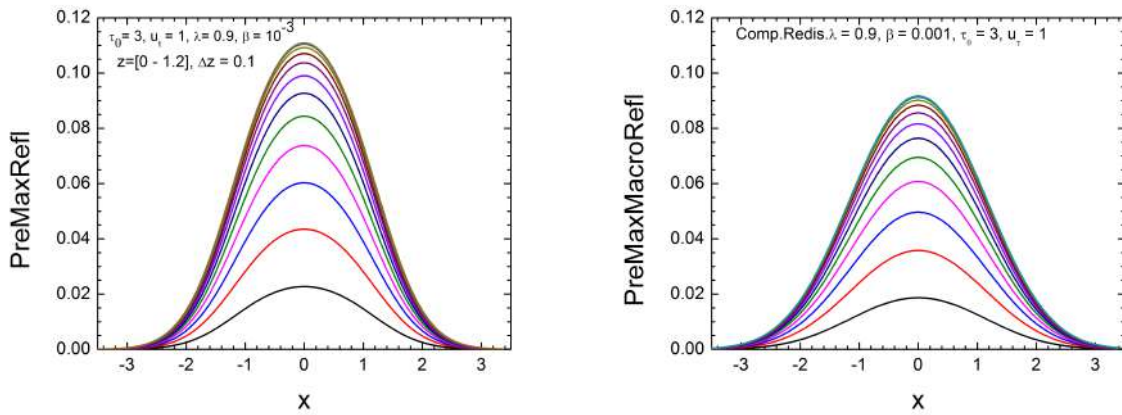


Figure 10. The same as in Fig.9 for the cases of micro-turbulence (left panel) and macro-turbulence (right panel).

of the incident radiation through the medium and is approximately equal to  $e^{-1} = 0.35$ . Such calibrations using different lines create a fundamental possibility to study these sources not only in different frequency ranges, but also, importantly, as a function of time. The next characteristic parameter of the PDF distribution is the average duration of line evolution. A comparison of the above plots shows that in the reflection spectrum the line evolves longer in the case of micro-turbulence, while in the absorption spectrum formed as a result of transmission through the medium, the evolution in time is longer in the case of macro-turbulence.

Obviously, the picture of the processes in both reflection and transmission cases depends on both the optical thickness of the medium and the value of the scattering coefficient (the role of  $\beta$  becomes noticeable for strong resonant lines at values of the scattering coefficient sufficiently close to unity). Previously, we have shown that the duration of diffusion in the frequencies of the spectral line can increase if the scattering occurs with redistribution over frequencies (Nikoghossian, 2022b, 2023). The absorption lines produced by the passage of radiation through a medium have a faster time course, which is due to the possibility of direct transmittance without scattering.

Of some interest are the fractions of energy emitted in the lines before they reach their maxima. They are of the order of 20 per cent of the total energy radiated in them during the whole time of their evolution and mainly depend on the optical thickness in the line.

A visual representation of the evolution of spectral line profiles before maximum is given by Figs.9,10 for lines formed by reflection and Figs.11,12 for lines observed as a result of passing through

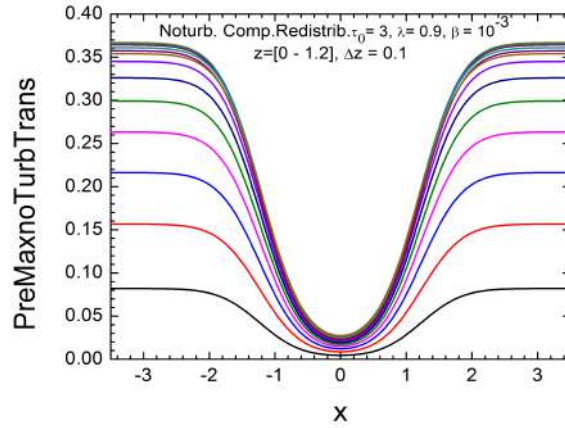


Figure 11. Evolution of the spectral line profiles formed as a result of radiation passing through the medium until reaching the maximum of the PDF in the absence of turbulence.

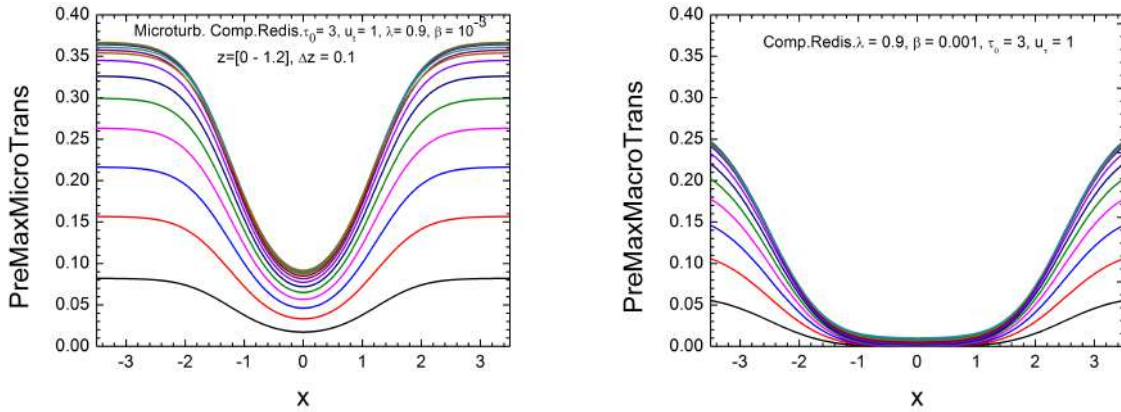


Figure 12. The same as in Fig.11 for micro- and macro-turbulence.

the medium. The time variable  $z$  grows in the figures with the increment  $\Delta z = 0.1$  varying from bottom to top. When approaching maximum, changes in line profiles slow down, after which the lines slowly weaken, which is shown in Figs.13,14 and 15,16 where  $\Delta z = 1$  and  $z$  grows from top to bottom. The figures show that the effect of gas-dynamic flows is strongest in the limit of macro-turbulence, which manifests itself in a sharp broadening of the spectral line and the appearance of large wings. In both extreme turbulent regimes, equality was assumed between the mean turbulent and thermal velocities was assumed in order to make the effect of turbulence more discernible.

It draws attention to the fact that many times after cessation of illumination of one of the boundaries of the medium some number of quanta continue to diffuse in it and leave the medium through both of its boundaries. The difference between these quantities gradually decreases, as a result of which weak, sometimes two-peaked emission lines are observed at both boundaries of the medium (Figs.13,14) so that they can appear also in the absorption spectrum (Figs.15,16).

#### 4. Illumination by a stationary source. Evolution to the steady-state

In this section we will consider another problem, important for astrophysical applications, when the medium, starting from a certain moment of time, begins to be illuminated by external energy sources and remains under their influence for any length of time until the steady state field of radiation is established. It is assumed that the temporal action of the sources is described by the Heviside unit-step function. We are interested in the evolution of emission and absorption spectral lines formed

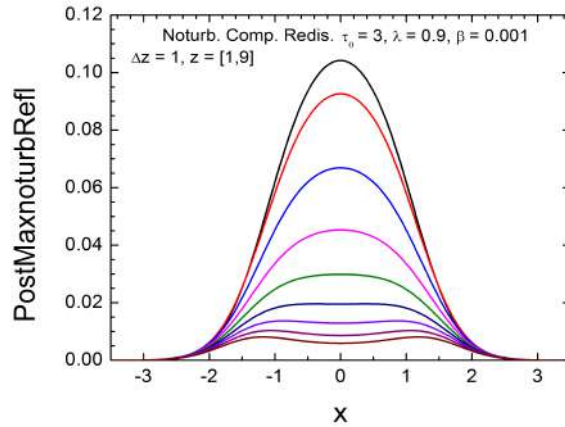


Figure 13. Evolution of the spectral line profiles formed as a result of radiation reflected from the medium after reaching the maximum of the PDF in the absence of turbulence.

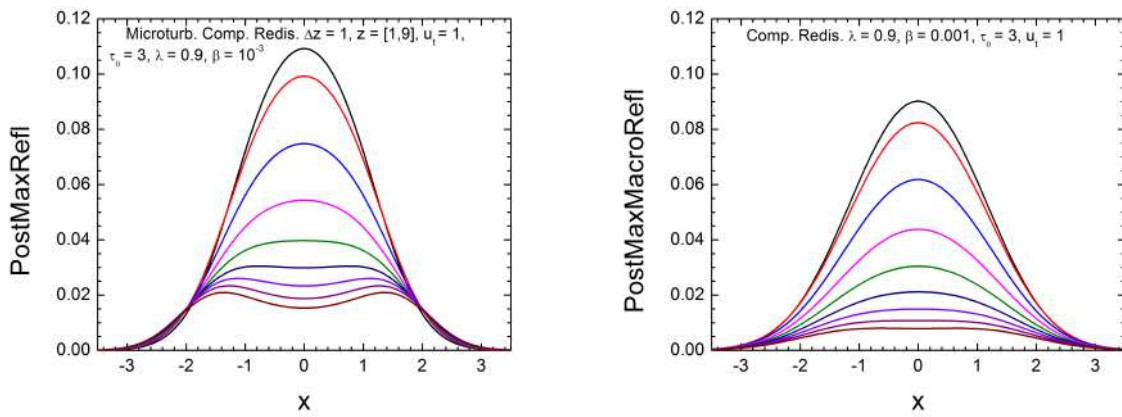


Figure 14. The same as in Fig.13 for micro- and macro-turbulence.

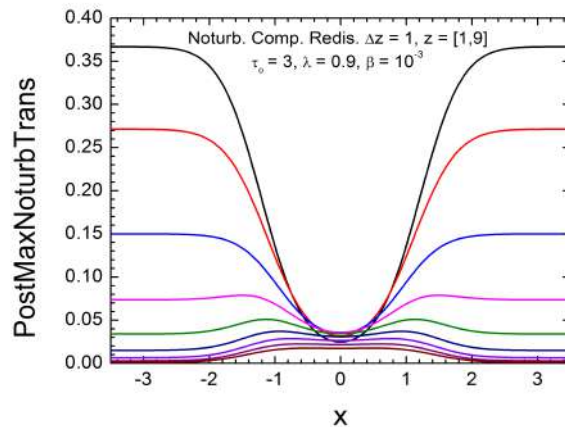


Figure 15. The transmitted spectra lines profiles evolution after PDF maximum in the absence of turbulence.

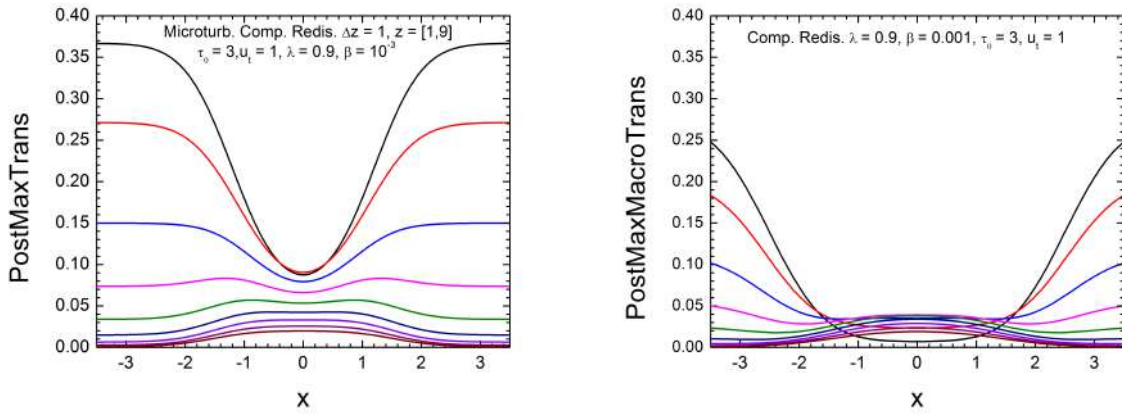


Figure 16. The same as in Fig.15 for micro- and macro-turbulence.

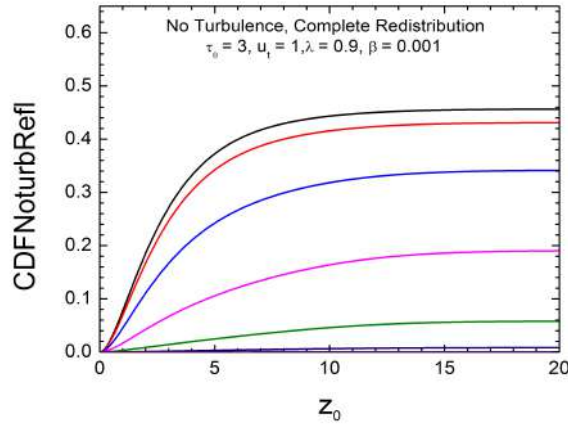


Figure 17. The cumulative distribution functions for the lines formed as a result of reflection from the medium in the absence of turbulence.

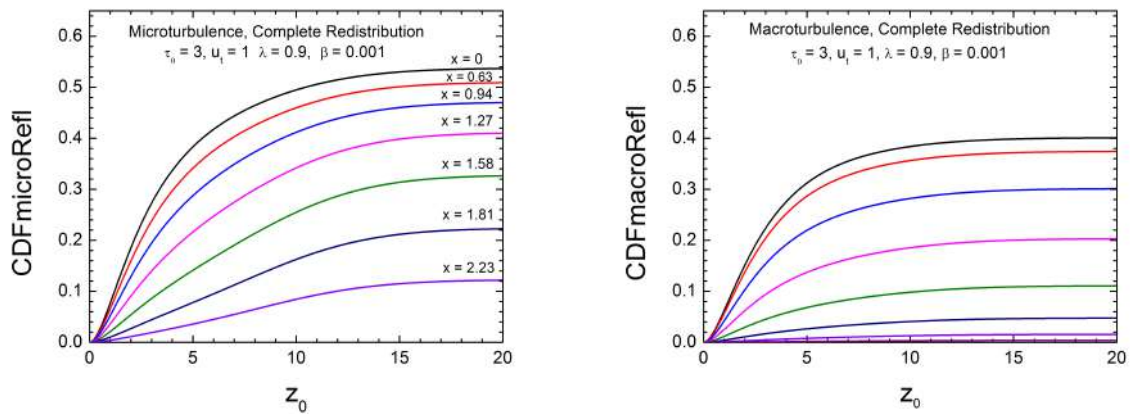


Figure 18. The same as in Fig.17 for the micro- and macro-turbulence.

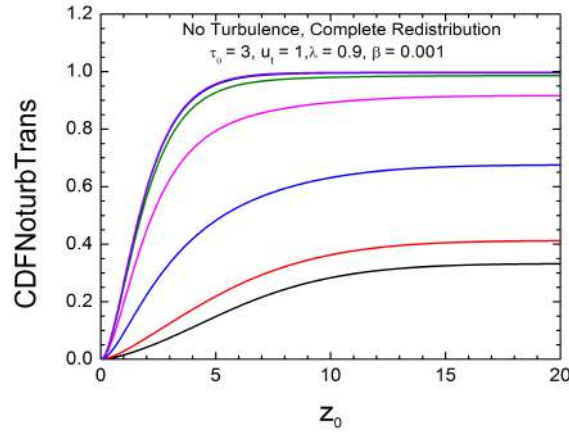


Figure 19. The cumulative distribution functions for the lines formed as a result of transmission through the medium in the absence of turbulence.

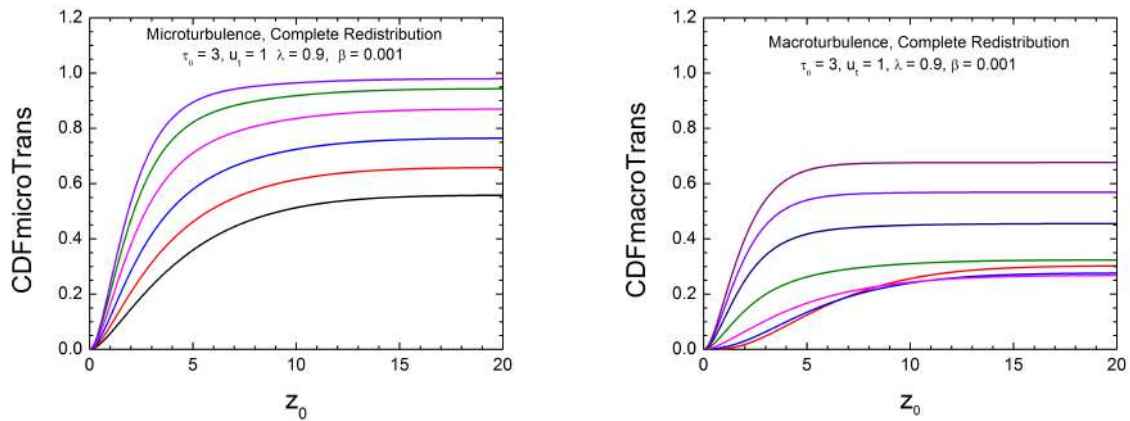


Figure 20. The same as in Fig.19 for the micro- and macro-turbulence.

as a result of reflection and transmission on the boundaries of the medium. The time dependence of the line profiles are described by cumulative distribution functions (CDF) given by Eqs.(12),(15) and (17).

Here, in addition to the initial phase of the appearance of the line spectra, we will be interested in their evolution and the rate at which the steady state is established, which depends both on the thickness of the medium and on the value of the scattering coefficient. Typical curves showing the variation of the spectral line profiles before reaching the stationary regime are shown in Figs.17-20. They are characterised by a plateau due to an asymptotic tendency of the line radiation towards their limiting values. As should be expected, the rate of the tendency towards stationary regime in the optically thin media we consider is sufficiently high. In fact, the curves achieve their limit values within the time interval  $[5 \leq z_0 \leq 10]$ . The illustrations above clearly show that the absorption lines formed as a result of transmission through the medium are established faster than the emission lines in the reflected spectrum. Another feature of the processes under study is their longer duration when the role of multiple scattering in the medium increases, i.e. at larger values of  $\lambda$ .

The absorption spectrum, as can also be seen in Figs.9-11, provides rich material for estimating both the optical characteristics of the line and the intensity of falling continuum radiation. The central residual line intensities together with equivalent widths provide important time-dependent information about these quantities and, as a consequence, provide an opportunity to study the physics of the observed phenomenon.

After reaching a plateau, the profiles of the observed spectral lines can be analysed using solutions

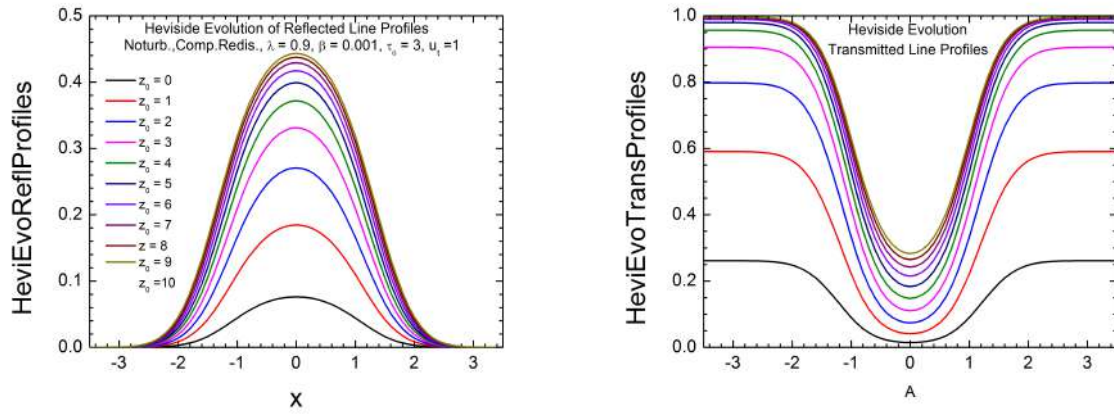


Figure 21. Evolution of the reflection and transmission lines arising under prolonged illumination by the energy sources in the absence of turbulence.

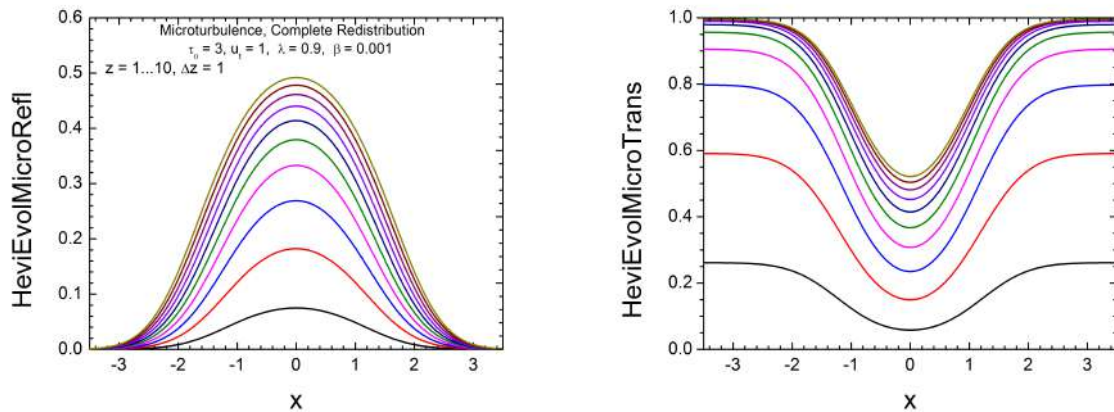


Figure 22. The same as in Fig.21 for micro-turbulence.

of classical stationary problems of the theory of radiation transfer on the line spectra formation. This will make it possible to draw conclusions about the lines themselves and the chemical content of elements, as well as about the characteristics of energy sources.

The identification of the spectral line and its assignment to a certain chemical element together with the dynamics of change and saturation rate reveal the physical picture of the dynamic process under study.

At the same time, as it is evident from Figs.17-20, the shape of the CDF distributions depends to a large extent on the nature and degree of turbulence development in the medium. This is also demonstrated in Figs.21-23, which show the evolution of spectral lines with the increment  $\Delta z = 1$  before reaching a plateau. Particularly striking are the differences in central residual intensities and line widths depending on the nature and degree of turbulence in the medium.

## 5. Concluding remarks

The paper continues our study on the evolution of line spectra under the influence of non-stationary energy sources. One of the important factors acting on the observed pattern of changes in the line spectra are gas-dynamic flows, which almost always arise at the time-varying energy sources of the forms we consider. To identify them, we treated the problem of spectral line formation in a finite scattering and absorbing medium under two limiting regimes of micro- and macro-turbulence. The values of the optical parameters of the medium, such as the optical thickness of the medium, the scattering coefficient, and the parameter  $\beta$ , are chosen in such a way in order to describe a sufficiently

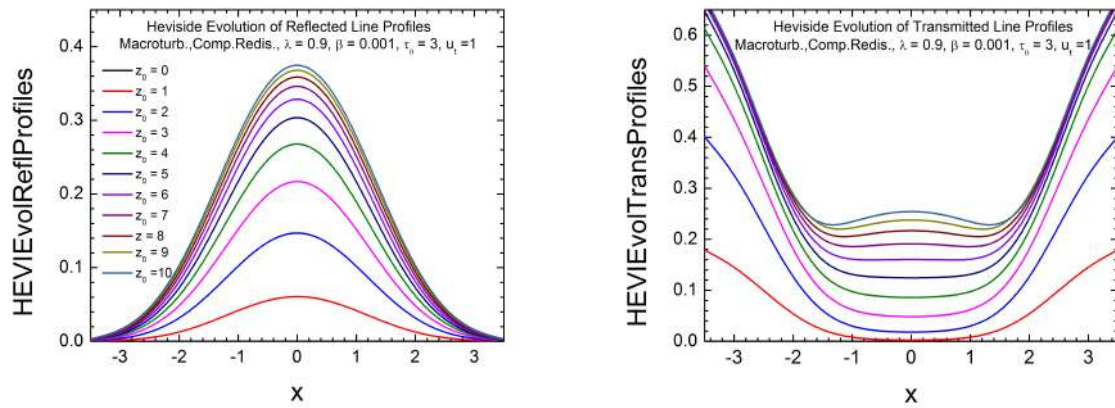


Figure 23. The same as in Figs.21,22 for macro-turbulence.

wide range of cases studied in practice. The results obtained concerning the evolutionary pattern of spectral lines arising both as a result of reflection from the medium and its passage in the two limiting regimes, as might be expected, are quite distinct, as well as differ from the case when turbulence is absent. These differences are of a general nature and are not limited to the Gaussian probability distribution of turbulent velocities considered in the paper. Note also that the value of the mean turbulent velocity  $u_t = 1$  chosen in the paper seems to be rather high to be applicable in practice to macro-turbulent flows in view of the extreme broadening of spectral lines.

It should also be noted that the assumption of approximate equality of the two possible causes of time wasting adopted in this paper may be rather crude in many situations. A more general problem with an arbitrary ratio of these two quantities will be the subject of a future paper by the author.

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