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The Master-Space Teleparallel Supergravity: Accelerated frames

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Abstract

Using Palatini's formalism extended in a plausible fashion to the recent $\widetilde{MS}_p$-Supergravity (Ter-Kazarian, 2023c, 2024b), subject to certain rules, we reinterpret a flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as the quantum field theory of Master-Space Teleparallel Supergravity ($\widetilde{MS}_p$-TSG), having the gauge *translation* group in tangent bundle. Here the spin connection represents only *inertial effects*, but not gravitation at all. In order to recover the covariance, we introduce a 1-form of the Yang–Mills connection assuming values in the Lie algebra of the translation group. The Hilbert action vanishes and the gravitino action loses its spin connections, so that the accelerated reference frame has Weitzenböck torsion induced by gravitinos. Due to the soldered character of the tangent bundle, torsion presents also the anholonomy of the translational covariant derivative. The gauge invariance of the tetrad provides torsion invariance under gauge transformations. The role of the Cartan-Killing metric usually comes, when it exists, from its being invariant under the group action. Here it does not exist, but we use the invariant Lorentz metric of Minkowski spacetime in its stead. The action of $\widetilde{MS}_p$-TSG is invariant under local translations, under local super symmetry transformations and by construction is invariant under local Lorentz rotations and under diffeomorphisms. So that this action is invariant under the Poincaré supergroup and under diffeomorphisms. We show the equivalence of the Teleparallel Gravity action with Hilbert action, which proves that the immediate cause of the *fictitious* Riemann curvature for the Levi-Civita connection arises entirely due to the inertial properties of the Lorentz-rotated frame of interest. The curvature of Weitzenböck connection vanishes identically, but for a tetrad involving a non-trivial translational gauge potential, the torsion is non-vanishing. We consider Weitzenböck connection a kind of dual of the Levi-Civita connection, which is a connection with vanishing torsion, and non-vanishing *fictitious* curvature. The Weitzenböck connection defines the acceleration through force equation, with torsion (or contortion) playing the role of force.

Keywords: *Supergravity–Teleparallel Supergravity–Accelerated frames*

1. Introduction

In a recent papers (Ter-Kazarian, 2023c, 2024b) we developed a quantum field theory of $\widetilde{MS}_p$-Supergravity as a *local* extension of the theory of *global Master space* (MS$_p$)-SUSY (Ter-Kazarian, 2023a, 2024a). The letter is the microscopic theory of deformed Lorentz symmetry and deformed geometry induced by foamy effects at the Planck scale, which tested in recent ultra-high energy experiments of cosmic rays (UHECRs) and astrophysical TeV-γ photons. They reflect the expectation that the solutions to this mystery appear to require new physics (see e.g. Batista & et al. (2019, 2023), Mattingly (2005) and references therein). These astrophysical experiments measure quantum gravity (QG) effects within a two or three orders of magnitude of Planck length $\ell_P \approx 1.62\times 10^{-33}$cm and Planck time t_P/c. One of the most important efforts of this type has historically been the search for a violation of the standard Lorenz code (SLC) of motion in ultra-high energy experiments. To this aim, as a guiding principle to make the rest of paper understandable, in the appendices we necessarily recount succinctly some of the highlights behind of *global* MS$_p$-SUSY (Ter-Kazarian, 2023a, 2024a) and *local* $\widetilde{MS}_p$-SUSY (Ter-Kazarian, 2023c, 2024b) theories, which are in use throughout the present article.

In (Ter-Kazarian, 2023a, 2024a) (see Appendix A for brief outline), we developed a microscopic theory of deformed Lorentz symmetry and deformed geometry induced by foamy effects at the Planck scale, and

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tested in ultra-high energy experiments. This theory, among other things, actually explores the first part of the phenomenon of inertia, which refers to *inertial uniform motion along rectilinear timelike world lines*. The phenomenon of inertia may be the most profound mystery in physics, and it is still the most important incomprehensive problem that needs to be solved. Today there is no known feasible way to account for credible explanation of this problem. With this perspective in sight, the *local* extension of MS_p-SUSY is the next necessary step.

In (Ter-Kazarian, 2023c, 2024b) (see Appendix B for brief outline), we conceived local $\widetilde{MS}_p$-SUSY as a quantum field theory of $\widetilde{MS}_p$-Supergravity (SG), in which we review the accelerated motion of a particle in a new perspective of local $\widetilde{MS}_p$-SUSY transformations. That is, a *creation* of a particle in curved Master space $\widetilde{MS}_p \equiv \underline{V}_2$ (2D semi-Riemannian space) means its transition from initial state defined on the background semi-Riemannian 4D-space V_4 into intermediate state defined on $\underline{V}_2$, while an *annihilation* of a particle in $\underline{V}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in V_4. The net result of each atomic double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$ to $\underline{V}_2$ and back is as if we had operated with *a local space-time translation* with acceleration, $\vec{a}$, on the original space V_4. Accordingly, the acceleration, $\vec{\underline{a}}$, holds in $\underline{V}_2$ at $\underline{V}_2 \rightleftharpoons V_4$. So, the accelerated motion of boson $A(\widetilde{x})$ in V_4 is a chain of its sequential transformations to the Weyl fermion $\underline{\chi}(\widetilde{\underline{x}})$ defined on $\underline{V}_2$ (accompanied with the auxiliary fields $\widetilde{\underline{F}}$) and back, and the same interpretation holds for fermion $\chi(\widetilde{x})$. A curvature of $\widetilde{MS}_p$ within $\widetilde{MS}_p$-SG theory arises entirely due to the inertial properties of the Lorentz-rotated frame of interest.

Using Palatini's formalism generalized for the $\widetilde{MS}_p$-SG, in present article we reinterpret a flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as a theory of $\widetilde{MS}_p$-TSG having the gauge *translation* group in tangent bundle. An important property of Teleparallel Gravity is that its spin connection is related only to the inertial properties of the frame, not to gravitation. Whereas the Hilbert action vanishes and the gravitino action loses its spin connections, so we find that the accelerated reference frame has Weitzenböck torsion induced by gravitinos.

We proceed according to the following structure. To start with, in Section 2 we discuss the Palatini's formalism and flat $\widetilde{MS}_p$-SG with torsion. Section 3 is devoted to the $\widetilde{MS}_p$-TSG with the translation group. In Section 4 we turn to the simple Newtonian gravitational field and 'absolute' acceleration. In Section 5 we derive the homogeneous acceleration field. As concluding remarks, we list in section 6 of what we think is the most important that distinguish a theory of $\widetilde{MS}_p$-TSG. In appendices A and B, we briefly revisit the global MS_p-SUSY and $\widetilde{MS}_p$-SG without going into the subtleties, respectively, as a guiding principle to make the paper understandable. We revisit the simple $(N = 1)$ $\widetilde{MS}_p$ - SG without auxiliary fields in B.1. On these premises, we discuss the velocity and acceleration in M_4. Throughout we will use the 'two-in-one' notation of a theory MS_p-SUSY (Appendix A), implying that any tensor (W) or spinor (Θ) with indices marked by 'hat' denote

$$W^{\hat{\mu}_1\cdots\hat{\mu}_m}_{\hat{\nu}_1\cdots\hat{\nu}_n} := W^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} \oplus W^{\underline{\mu}_1\cdots\underline{\mu}_m}_{\underline{\nu}_1\cdots\underline{\nu}_n},$$
$$\Theta^{\hat{\alpha}} := \theta^{\alpha} \oplus \underline{\theta}^{\alpha}, \quad \bar{\Theta}_{\hat{\dot{\alpha}}} := \bar{\theta}_{\dot{\alpha}} \oplus \underline{\bar{\theta}}_{\dot{\alpha}}. \tag{1}$$

This corresponds to the action of supercharge operators $Q \equiv (\text{either}\, q \,\text{or}\, \underline{q})$ (see (86), (87)), which is due to the fact that the framework of $\widetilde{MS}_p$-SG combines bosonic and fermionic states in V_4 and $\underline{V}_2$ on the same base rotating them into each other under the action of operators $(q, \underline{q})$. The α are all upper indices, while $\dot{\alpha}$ is a lower index. For brevity, whenever possible undotted and dotted spinor indices often can be ruthlessly suppressed without ambiguity. Unless indicated otherwise, the natural units, $h = c = 1$ are used throughout.

2. Palatini's formalism and Flat $\widetilde{MS}_p$-SG with torsion

The method of finding the dependence of spin connection ω on other fields by first treating it as an independent field in the action and then solving its (always nonpropagating) field equation is known in standard supergravity as the Palatini's formalism. This leads to the simplest description of supergravity. So, solving from (114) (thus without matter) the field equation for ω, one finds $\omega = \omega(e)$. We will use the techniques of (van Nieuwenhuizen, 1981) but extended in a plausible fashion to the $\widetilde{MS}_p$-SG. We will find the spin connection as a function of tetrad and gravitino. Whereas the Hilbert action vanishes and the gravitino action loses its spin connections, and thus we will find torsion induced by gravitinos.

Palatini's formalism can be implemented as follows. Varying the spin connection in Hilbert action (118),

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which can conveniently be rewritten in the form

$$\mathcal{L}^{(2)} = -\tfrac{1}{16}\varepsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma}\varepsilon_{\hat a\hat b\hat c\hat d}e^{\hat a}_{\hat\mu}e^{\hat b}_{\hat\nu}R_{\hat\rho\hat\sigma}{}^{\hat c\hat d}(\omega), \tag{2}$$

we obtain

$$\delta\mathcal{L}^{(2)} = -\tfrac{1}{4}\varepsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma}\varepsilon_{\hat a\hat b\hat c\hat d}(\mathcal{D}_{\hat\sigma}e^{\hat a}_{\hat\mu})e^{\hat b}_{\hat\nu}\delta\omega^{\hat c\hat d}_{\hat\rho}, \tag{3}$$

where $\mathcal{D}e^{\hat a} = de^{\hat a} + \omega^{\hat a\hat b}e_{\hat b}$. Varying then the spin connection in the Rarita-Schwinger action (119), it yields

$$\delta\mathcal{L}^{(3/2)} = 2\varepsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma}(\bar\Psi_{\hat\mu}\gamma_{\hat 5}\gamma_{\hat\nu}\sigma_{\hat c\hat d}\Psi_{\hat\sigma})(\delta\omega^{\hat c\hat d}_{\hat\rho}), \tag{4}$$

which can be recast in the form

$$\delta\mathcal{L}^{(3/2)} = \varepsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma}\varepsilon_{\hat b\hat c\hat d\hat a}(\bar\Psi_{\hat\mu}\gamma^{\hat a}\Psi_{\hat\sigma})e^{\hat b}{}_{\hat\nu}\delta\omega^{\hat c\hat d}_{\hat\rho}. \tag{5}$$

For the field equation of the spin connection, comparison of (2) and (5) gives

$$\mathcal{D}_{\hat\mu}e^{\hat a}{}_{\hat\nu} - \mathcal{D}_{\hat\nu}e^{\hat a}{}_{\hat\mu} = \tfrac{1}{2}\bar\Psi_{\hat\mu}\gamma^{\hat a}\Psi_{\hat\nu}. \tag{6}$$

Solving this equation, we as usual introduce the contorsion tensor $K^{\hat a\hat b}$ (Aldrovandi & Pereira, 1995),

$$\omega^{\hat a\hat b} = \omega^{\hat a\hat b}(e) + K^{\hat a\hat b}. \tag{7}$$

By virtue of the first tetrad postulate (which is a definition of $\omega(e)$), we obtain

$$\begin{aligned}&\partial_{\hat\mu}e^{\hat a}{}_{\hat\nu} + \omega_{\hat\mu}{}^{\hat a}{}_{\hat\nu} - (\mu\leftrightarrow\nu) = 0,\\ &K_{\hat\mu\hat a\hat\nu} - K_{\hat\nu\hat a\hat\mu} = -\bar\Psi_{\hat\mu}\gamma_{\hat a}\Psi_{\hat\nu}.\end{aligned} \tag{8}$$

By substituting the second equation of (8) into the identity

$$(K_{\hat\mu\hat a\hat\nu} - K_{\hat\nu\hat a\hat\mu}) + (K_{\hat a\hat\mu\hat\nu} - K_{\hat\nu\hat\mu\hat a}) + (K_{\hat a\hat\nu\hat\mu} - K_{\hat\mu\hat\nu\hat a}) = 2K_{\hat\mu\hat a\hat\nu}, \tag{9}$$

and using the second tetrad postulate,

$$\partial_{\hat\mu}e^{\hat a}{}_{\hat\nu} + \omega_{\hat\mu}{}^{\hat a}{}_{\hat\nu} - \Gamma\,{}^{\hat a'}_{\hat\nu\hat\mu}e^{\hat a}{}_{\hat a'} = 0, \tag{10}$$

one obtains an expression for antisymmetric part $\Gamma^{\hat a}{}_{[\hat\mu\hat\nu]}$,

$$2T^{\hat a}{}_{\hat\mu\hat\nu} = \Gamma^{\hat a}{}_{[\hat\mu\hat\nu]} = -K_{[\hat\mu}{}^{\hat a}{}_{\hat\nu]}, \tag{11}$$

with the torsion tensor $T^{\hat a}$,

$$T^{\hat a} = \tfrac{1}{2}\bar\Psi\gamma^{\hat a}\Psi. \tag{12}$$

Thus, the invariance of the total action (117) under local supersymmetry transformation requires the vanishing of the supertorsion $\tilde T^{\hat a} = 0$, which means that the connection is no longer an independent variable. The same supertorsion-free condition is necessary for the invariance of the action under local Poincaré translations, because of the variation (115) in case of an independent ω. An effect of the supertorsion-free condition on the local Poincaré superalgebra is that all commutators on $\delta e^{\hat a}$ and $\delta\Psi$ close except the commutator of two local supersymmetry transformations on the gravitino. For this commutator on the vierbein one finds

$$\begin{aligned}&[\delta_{\epsilon_1(X)}, \delta_{\epsilon_2(X)}]e^{\hat a}(X) = \delta e^{\hat a}(X) = \mathcal{D}\rho^{\hat a}(X)\\ &= \tfrac{1}{2}\bar\epsilon_2(X)\gamma^{\hat a}\mathcal{D}\epsilon_1(X) - \tfrac{1}{2}\bar\epsilon_1(X)\gamma^{\hat a}\mathcal{D}\epsilon_2(X) = \tfrac{1}{2}\mathcal{D}(\bar\epsilon_2(X)\gamma^{\hat a}\epsilon_1(X)),\end{aligned} \tag{13}$$

so that $\rho^{\hat a}(X) = \frac{1}{2}\bar\epsilon_2(X)\gamma^{\hat a}\epsilon_1(X)$ in accord with (104). The dependence of $\omega^{\hat a\hat b}$ on the tetrad field then reads

$$\begin{aligned}&\omega_{\hat\mu\hat a\hat b}(e,\Psi) = \tfrac{1}{2}(R_{\hat\mu\hat b,\hat a} - R_{\hat\mu\hat a,\hat b} + R_{\hat a\hat b,\hat\mu}),\\ &R_{\hat\mu\hat\nu,\hat a} = -\partial_{\hat\mu}e_{\hat a\hat\nu} + \partial_{\hat\nu}e_{\hat a\hat\mu} - (\bar\Psi_{\hat\mu}\gamma_{\hat a}\Psi_{\hat\nu}),\end{aligned} \tag{14}$$

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provided, $R_{\hat{\mu}\hat{b},\hat{a}} = e^{\hat{\nu}}_{\hat{b}} R_{\hat{\mu}\hat{\nu},\hat{a}}$. Since $\omega(e,\Psi)$ is an extremum of the action, then according to the *Theorem (I)* in (?), under transformation $\omega^{\hat{a}\hat{b}} \to \omega^{\hat{a}\hat{b}} + \tau^{\hat{a}\hat{b}}$, for arbitrary $\tau^{\hat{a}\hat{b}} = -\tau^{\hat{b}\hat{a}}$, the symmetry

$$\begin{array}{l} \mathcal{L}^{(2)}(e,\omega(e,\Psi)+\tau) + \mathcal{L}^{(3/2)}(e,\Psi,\omega(e,\Psi)+\tau)) \\ = \mathcal{L}^{(2)}(e,\omega(e,\Psi)) + \mathcal{L}^{(3/2)}(e,\Psi,\omega(e,\Psi)) \\ -\frac{1}{4}(\tau_{\hat{\mu}\hat{\nu}\hat{\rho}}\tau^{\hat{\rho}\hat{\nu}\hat{\mu}} - (\tau^{\hat{\lambda}}_{\ \hat{\lambda}\hat{\mu}})^2) + \text{total derivative}, \end{array} \tag{15}$$

holds because terms linear in τ cancel out. In particular case if $\tau^{\hat{a}\hat{b}} = -\omega^{\hat{a}\hat{b}}$, the Hilbert action with the Weitzenböck curvature vanishes, and the gravitino action loses its spin connections. So we may henceforth reinterpret flat $\widetilde{MS}_p$-SG with Weitzenböck torsion as a teleparallelism theory - a theory that involves only torsion, where a parallel transport is defined over finite distances and not only in an infinitesimal neighborhood. This theory represented an alternative way of including torsion to the scheme previously provided by a Einstein-Cartan theory. Using then the result for $\omega^{\hat{a}\hat{b}}$ in terms of the curls

$$\partial_{\hat{\mu}} e^{\hat{a}}_{\ \hat{\nu}} - \partial_{\hat{\nu}} e^{\hat{a}}_{\ \hat{\mu}} + \bar{\Psi}_{\hat{\mu}}\gamma^{\hat{a}}\Psi_{\hat{\nu}}, \tag{16}$$

in (14), we may rewrite the action of a $\widetilde{MS}_p$-TSG theory:

$$\begin{array}{l} \mathcal{L}_{MS-TSG} = 4\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\bar{\Psi}_{\hat{\mu}}\gamma_{\hat{5}}\gamma_{\hat{\nu}}\partial_{\hat{\rho}}\Psi_{\hat{\sigma}} \\ -R^2_{\hat{\mu}\hat{\nu}\hat{a}} - R_{\hat{\mu}\hat{\nu}\hat{a}}R^{\hat{a}\hat{\nu}\hat{\mu}} + \frac{1}{2}R^2_{\hat{\nu}\hat{\lambda}\hat{\lambda}}, \end{array} \tag{17}$$

where the objects $R_{\hat{\mu}\hat{\nu}\hat{a}}$ is interpreted as the supercovariantized torsion tensor

$$-\mathcal{D}_{\hat{\mu}} e_{\hat{a}\hat{\nu}} + \mathcal{D}_{\hat{\nu}} e_{\hat{a}\hat{\mu}} - (\bar{\Psi}_{\hat{\mu}}\gamma_{\hat{a}}\Psi_{\hat{\nu}}). \tag{18}$$

3. The $\widetilde{MS}_p$-TSG with the translation group

In Teleparallel Gravity, the spin connection represents only *inertial effects*, but not gravitation at all. All quantities related to Teleparallel Gravity will be denoted with an over 'dot'. The spin connection reads

$$\dot{\omega}^{\hat{a}}_{\ \hat{b}\hat{\mu}} = L^{\hat{a}}_{\ \hat{d}}\partial_{\hat{\mu}} L_{\hat{b}}^{\ \hat{d}}, \tag{19}$$

and the energy-momentum density of the *inertial* or *fictitious* forces is

$$\dot{i}_{\hat{a}}^{\ \hat{\rho}} = \frac{1}{k}\dot{\omega}^{\hat{c}}_{\ \hat{a}\hat{\sigma}}\dot{S}_{\hat{c}}^{\ \hat{\rho}\hat{\sigma}}, \tag{20}$$

where $\dot{S}_{\hat{c}}^{\ \hat{\rho}\hat{\sigma}}$ is the so called superpotential (see (38)). Teleparallel Gravity is a gauge theory for the *translation* group (?). The $\widetilde{MS}_p$-TSG theory, therefore, has the gauge *translation* group in tangent bundle. Namely, at each point p of coordinates X of the base space $(V_4 \oplus \underline{V}_2)$, there is attached a Minkowski tangent-space (the fiber) $T_p(V_4 \oplus \underline{V}_2) = T_{X^{\hat{\mu}}}(V_4 \oplus \underline{V}_2)$, on which the point dependent gauge transformations,

$$X'^{\hat{a}} = X^{\hat{a}} + \varepsilon^{\hat{a}}(X), \tag{21}$$

take place. Under an infinitesimal tangent space translation, it transforms according to

$$\delta\Phi(X^{\hat{a}}(X^{\hat{\mu}})) = -\varepsilon^{\hat{a}}\partial_{\hat{a}}\Phi(X^{\hat{a}}(X^{\hat{\mu}})). \tag{22}$$

The generators of this group satisfy the Lie algebra $[P_{\hat{a}}, P_{\hat{b}}] = 0$. In order to recover the covariance, it is necessary to introduce a 1-form of the Yang–Mills connection assuming values in the Lie algebra of the translation group:

$$B = e^{\hat{a}} P_{\hat{a}}, \tag{23}$$

with gauge field $e^{\hat{a}}$. Introducing the covariant derivative

$$\dot{\mathcal{D}}_{\hat{\mu}} X^{\hat{a}} = \partial_{\hat{\mu}} X^{\hat{a}} + \dot{\omega}^{\hat{a}}_{\ \hat{b}\hat{\mu}} X^{\hat{b}}, \tag{24}$$

the tetrad, which is invariant under translations, becomes

$$\dot{e}^{\hat{a}}_{\ \hat{\mu}} = \dot{\mathcal{D}}_{\hat{\mu}} X^{\hat{a}} + \dot{\omega}^{\hat{a}}_{\ \hat{b}\hat{\mu}}. \tag{25}$$

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In this new class of frames, the gauge field transforms according to $\delta e^{\hat{a}}{}_{\hat{\mu}} = -\dot{\mathcal{D}}_{\hat{\mu}}\varepsilon^{\hat{a}}$. Thus the covariant derivative, $\dot{\mathcal{D}} = d + B$, with Yang–Mills connection reads

$$\dot{\mathcal{D}}_{\hat{\mu}} = (\delta^{\hat{a}}_{\hat{\mu}} + e_{\hat{\mu}}{}^{\hat{a}})\partial_{\hat{a}} = (\partial_{\hat{\mu}} X^{\hat{a}} + e_{\hat{\mu}}{}^{\hat{a}})\partial_{\hat{a}} = \dot{e}_{\hat{\mu}}{}^{\hat{a}}\partial_{\hat{a}}. \tag{26}$$

The curvature of the Weitzenböck connection

$$\dot{\Gamma}^{\hat{\rho}}{}_{\hat{\nu}\hat{\mu}} = \dot{e}_{\hat{a}}{}^{\hat{\rho}}\dot{\mathcal{D}}_{\hat{\mu}}\dot{e}^{\hat{a}}{}_{\hat{\nu}}, \tag{27}$$

vanishes identically, while for a tetrad $\dot{e}^{\hat{a}}$ with $e^{\hat{a}}{}_{\hat{\mu}} \neq \dot{\mathcal{D}}_{\hat{\mu}}\varepsilon^{\hat{a}}$, the torsion 2-form - the field strength (here we re-instate the factor $\wedge$),

$$\dot{T}^{\hat{a}} = d\dot{e}^{\hat{a}} = \tfrac{1}{2}\dot{T}^{\hat{a}}{}_{\hat{b}\hat{c}}\dot{e}^{\hat{b}} \wedge \dot{e}^{\hat{c}} = \dot{K}_{\hat{c}}{}^{\hat{a}} \wedge \dot{e}^{\hat{c}}, \tag{28}$$

is non-vanishing:

$$\dot{T}^{\hat{a}}{}_{\hat{\mu}\hat{\nu}} = \dot{\mathcal{D}}_{\hat{\mu}}\dot{e}^{\hat{a}}{}_{\hat{\nu}} - \dot{\mathcal{D}}_{\hat{\nu}}\dot{e}^{\hat{a}}{}_{\hat{\mu}} = \dot{\Gamma}^{\hat{a}}{}_{[\hat{\mu}\hat{\nu}]} = \dot{\mathcal{D}}_{\hat{\mu}}e^{\hat{a}}{}_{\hat{\nu}} - \dot{\mathcal{D}}_{\hat{\nu}}e^{\hat{a}}{}_{\hat{\mu}} \neq 0. \tag{29}$$

Here $\dot{K}^{\hat{a}\hat{b}}$ is the contorsion tensor, and we also taken into account the vanishing torsion, $[\dot{\mathcal{D}}_{\hat{\mu}}, \dot{\mathcal{D}}_{\hat{\nu}}]X^{\hat{a}} = 0$, of *inertial* tetrad, $\dot{e}^{\hat{a}}{}_{\hat{\mu}} = \dot{\mathcal{D}}_{\hat{\mu}}X^{\hat{a}}$. Hence

$$[\dot{e}_{\hat{\mu}}, \dot{e}_{\hat{\nu}}] = \dot{T}_{\hat{\mu}\hat{\nu}} = \dot{T}^{\hat{a}}{}_{\hat{\mu}\hat{\nu}}P_{\hat{a}}. \tag{30}$$

Due to the soldered character of the tangent bundle, torsion presents also the anholonomy of the translational covariant derivative:

$$[\dot{e}_{\hat{\mu}}, \dot{e}_{\hat{\nu}}] = \dot{T}_{\hat{\mu}\hat{\nu}} = \dot{T}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}P_{\hat{\rho}}. \tag{31}$$

The gauge invariance of the tetrad provides torsion invariance under gauge transformations. As a gauge theory for the translation group, the action (17) of the $\widetilde{MS}_p$-TSG theory can be recast in the form (see also Salgado et al. (2005))

$$\begin{aligned} \dot{\mathcal{L}}_{MS-TSG} &= \tfrac{1}{4}tr\left(\hat{\dot{T}} \wedge \star\hat{\dot{T}}\right) - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}} \\ &= \tfrac{1}{4}\eta_{\hat{a}\hat{b}}\dot{T}^{\hat{a}} \wedge \star\dot{T}^{\hat{b}} - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}}, \end{aligned} \tag{32}$$

where (we re-instate the factor $\wedge$) the torsion 2-form reads

$$\hat{\dot{T}} = \tfrac{1}{2}\dot{T}^{\hat{a}}{}_{\hat{\mu}\hat{\nu}}P_{\hat{a}}dX^{\hat{\mu}} \wedge dX^{\hat{\nu}}, \tag{33}$$

and

$$\star\hat{\dot{T}} = \tfrac{1}{2}\left(\star\dot{T}^{\hat{a}}{}_{\hat{\rho}\hat{\sigma}}\right)P_{\hat{a}}dX^{\hat{\rho}} \wedge dX^{\hat{\sigma}}. \tag{34}$$

Here $\star$ denotes the Hodge dual. That is, let Ω^p be the space of p-forms on an n-dimensional manifold $\mathbf{R}$ with metric. Since vector spaces Ω^p and Ω^{n-p} have the same finite dimension, they are isomorphic. The presence of a metric renders it possible to single out an unique isomorphism, called Hodge dual. Using a coordinate basis, the Hodge dual map $\star : \Omega^p \to \Omega^{n-p}$ is the C^∞-linear map $\star : \Omega^p \to \Omega^{n-p}$, which acts on the wedge product monomials of the basis 1-forms as

$$\star(\vartheta^{a_1\cdots a_p}) = \varepsilon^{a_1\cdots a_n}e_{a_{p+1}\cdots a_n}, \tag{35}$$

where e_{a_i} $(i = p+1, ..., n)$ are understood as the down indexed 1-forms $e_{a_i} = o_{a_i b}\,\vartheta^b$ and $\epsilon^{a_1\ldots a_n}$ is the total antisymmetric pseudo-tensor. The operator $\star$ satisfies the property

$$\star\star(\vartheta^{a_1\cdots a_p}) = (-1)^{p(n-p)+(n-s)/2}\star(\vartheta^{a_1\cdots a_p}) \tag{36}$$

where s is the metric signature. Its inverse is

$$\star_{-1} = (-1)^{p(n-p)+(n-s)/2}\star. \tag{37}$$

A further relation involving Hodge duality reads ${}^\star(\alpha \wedge e_a) = (e_a\rfloor{}^\star\alpha)$, while for differential forms α, β of the same degree p, equation ${}^\star(\alpha \wedge \beta = {}^\star(\beta \wedge \alpha)$ holds.

In (32) we taken into account that the translation group is abelian, therefore its Cartan-Killing bilinear form is degenerate and cannot be used as a metric. Recall that the group manifold of translations is just the Minkowski spacetime $\mathcal{M}$, the quotient space between the Poincaré, $\mathcal{P}$, and the Lorentz, $\mathcal{L}$, groups:

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$\mathcal{M} = \mathcal{P}/\mathcal{L}$. Namely, $\mathcal{M}$ is homogeneous or transitive under X-space translations, which means that there is only one group element that moves a point of the $\mathcal{M}$ space into another given point of the $\mathcal{M}$ space. The role of the Cartan-Killing metric comes, when it exists, from its being invariant under the group action. Here it does not exist, but we can use the invariant Lorentz metric $\eta_{\hat{a}\hat{b}}$ of $\mathcal{M}$ in its stead. Defining the tensor of superpotential

$$\dot{S}_{\hat{a}}{}^{\hat{\rho}\hat{\sigma}} = -\dot{S}_{\hat{a}}{}^{\hat{\sigma}\hat{\rho}} := \dot{K}^{\hat{\rho}\hat{\sigma}}{}_{\hat{a}} - \dot{e}_{\hat{a}}{}^{\hat{\sigma}}\dot{T}^{\hat{c}\hat{\rho}}{}_{\hat{c}} + \dot{e}_{\hat{a}}{}^{\hat{\rho}}\dot{T}^{\hat{c}\hat{\sigma}}{}_{\hat{c}}, \tag{38}$$

the dual torsion can be rewritten in the form

$$\star\dot{T}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}} = \tfrac{\dot{e}}{2}\varepsilon_{\hat{\mu}\hat{\nu}\hat{\lambda}\hat{\sigma}}\dot{S}^{\hat{\rho}\hat{\lambda}\hat{\sigma}}, \tag{39}$$

with $\dot{e} = det\,\dot{e}^{\hat{a}}{}_{\mu}(X) = \sqrt{-g}$, and hence

$$\dot{\mathcal{L}}_{MS-TSG} = \tfrac{\dot{e}}{8}\dot{T}_{\hat{\rho}\hat{\mu}\hat{\nu}}\dot{S}^{\hat{\rho}\hat{\mu}\hat{\nu}} - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}}. \tag{40}$$

An entire torsion tensor can be written through the components of 'vector torsion' ($\dot{\mathcal{V}}_{\hat{\mu}} = \dot{T}^{\hat{\nu}}{}_{\hat{\mu}\hat{\nu}}$), 'axial torsion' ($\dot{\mathcal{A}}^{\hat{\mu}} = \frac{1}{6}\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\lambda}}\dot{T}_{\hat{\nu}\hat{\rho}\hat{\lambda}}$), and 'pure tensor torsion' ($\dot{\mathcal{T}}_{\hat{\rho}\hat{\mu}\hat{\nu}}$) (a tensor with vanishing vector and axial parts) as

$$\dot{T}_{\hat{\rho}\hat{\mu}\hat{\nu}} = \tfrac{2}{3}(\dot{\mathcal{T}}_{\hat{\rho}\hat{\mu}\hat{\nu}} - \dot{\mathcal{T}}_{\hat{\rho}\hat{\nu}\hat{\mu}}) + \tfrac{1}{3}(g_{\hat{\rho}\hat{\mu}}\dot{\mathcal{V}}_{\hat{\nu}} - g_{\hat{\rho}\hat{\nu}}\dot{\mathcal{V}}_{\hat{\mu}}) + \varepsilon_{\hat{\rho}\hat{\mu}\hat{\nu}\hat{\lambda}}\dot{\mathcal{A}}^{\hat{\lambda}}. \tag{41}$$

Making use of the identity $\dot{T}^{\hat{\mu}}{}_{\hat{\mu}\hat{\rho}} = \dot{K}^{\hat{\mu}}{}_{\hat{\rho}\hat{\mu}}$, the action (32) becomes

$$\dot{\mathcal{L}}_{MS-TSG} = \tfrac{\dot{e}}{4}\left(\tfrac{1}{4}\dot{T}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}\dot{T}_{\hat{\rho}}{}^{\hat{\mu}\hat{\nu}} + \tfrac{1}{2}\dot{T}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}\dot{T}^{\hat{\nu}\hat{\mu}}{}_{\hat{\rho}} - \dot{T}^{\hat{\rho}}{}_{\hat{\mu}\hat{\rho}}\dot{T}^{\hat{\nu}\hat{\mu}}{}_{\hat{\rho}}\right) - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}}, \tag{42}$$

which consequently can be recast in the form

$$\dot{\mathcal{L}}_{MS-TSG} = -\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\dot{K}^{\hat{a}}{}_{\hat{e}}\dot{K}^{\hat{e}\hat{b}}\dot{e}^{\hat{c}}\dot{e}^{\hat{d}} - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}}, \tag{43}$$

or

$$\dot{\mathcal{L}}_{MS-TSG} = -\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\dot{K}^{\hat{a}\hat{b}}\dot{T}^{\hat{c}}\dot{e}^{\hat{d}} - 4\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{d}}\mathcal{D}\Psi\dot{e}^{\hat{d}} + \text{surface term}. \tag{44}$$

This action is invariant under local translations, under local super symmetry transformations and by construction is invariant under local Lorentz rotations and under diffeomorphisms (see Salgado et al. (2003, 2005), Stelle & West (1980)). In other words, the action (42) is invariant under the Poincaré supergroup and under diffeomorphisms.

It remains to see the equivalence of the Teleparallel Gravity action $\dot{\mathcal{L}}^{(2)}$ in (42) with Hilbert action $\mathcal{L}^{(2)}$ in (118), which will prove that the immediate cause of the *fictitious* Riemann curvature (R) for the Levi-Civita connection (Γ) is the acceleration. The curvature ($\dot{R}$) of Weitzenböck connection ($\dot{\Gamma}$) vanishes identically, but for a tetrad involving a non-trivial translational gauge potential ($\dot{e}_{\hat{\mu}}{}^{\hat{a}} \neq \dot{\mathcal{D}}_{\hat{\mu}}\varepsilon^{\hat{a}}$), the torsion ($\dot{T}$) is non-vanishing. The connection ($\dot{\Gamma}$) can be considered a kind of dual of the Levi-Civita connection (Γ), which is a connection with vanishing torsion (T), and non-vanishing *fictitious* curvature (R). Provided, the following relations hold:

$$\begin{aligned}\dot{\Gamma}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}} &= \Gamma^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}} + \dot{K}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}},\\ \dot{R}^{\hat{\rho}}{}_{\hat{\lambda}\hat{\mu}\hat{\nu}} &= R^{\hat{\rho}}{}_{\hat{\lambda}\hat{\mu}\hat{\nu}} + \dot{Q}^{\hat{\rho}}{}_{\hat{\lambda}\hat{\mu}\hat{\nu}} \equiv 0,\end{aligned} \tag{45}$$

where $\dot{Q}$ is a 2-form assuming values in the Lie algebra of the Lorentz group,

$$\dot{Q} = \tfrac{1}{2}S_{\hat{a}}{}^{\hat{b}}\dot{Q}^{\hat{a}}{}_{\hat{b}\hat{\mu}\hat{\nu}}dX^{\hat{\mu}} \wedge dX^{\hat{\nu}}, \tag{46}$$

with the components

$$\dot{Q}^{\hat{a}}{}_{\hat{b}\hat{\mu}\hat{\nu}} = \mathcal{D}_{\hat{\mu}}\dot{K}^{\hat{a}}{}_{\hat{b}\hat{\nu}} - \mathcal{D}_{\hat{\nu}}\dot{K}^{\hat{a}}{}_{\hat{b}\hat{\mu}} + \dot{K}^{\hat{a}}{}_{\hat{c}\hat{\nu}}\dot{K}^{\hat{c}}{}_{\hat{b}\hat{\mu}} - \dot{K}^{\hat{a}}{}_{\hat{c}\hat{\mu}}\dot{K}^{\hat{c}}{}_{\hat{b}\hat{\nu}}. \tag{47}$$

The scalar version of second relation in (45) is

$$-\dot{R} = \dot{Q} \equiv \left(\dot{K}^{\hat{\mu}\hat{\nu}\hat{\rho}}\dot{K}_{\hat{\nu}\hat{\mu}\hat{\rho}} - \dot{K}^{\hat{\mu}\hat{\rho}}{}_{\hat{\mu}}\dot{K}^{\hat{\nu}}{}_{\hat{\rho}\hat{\nu}}\right) + \tfrac{2}{\dot{e}}\partial_{\hat{\mu}}\left(\dot{e}\dot{T}^{\hat{\nu}\hat{\mu}}{}_{\hat{\nu}}\right), \tag{48}$$

which proves that the immediate cause of the *fictitious* Riemann curvature (R) is the acceleration. Comparing with (42), this actually proves the equivalence of the Teleparallel Gravity action $\dot{\mathcal{L}}^{(2)}$ in (42) with Hilbert action $\mathcal{L}^{(2)}$ in (118):

$$\dot{\mathcal{L}}^{(2)} = \mathcal{L}^{(2)} + \text{surface term}. \tag{49}$$

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The dynamical aspects of particle mechanics involve derivatives with respect to proper time along the particle worldline, which is the line element written in frame (25):

$$ds^2 = \eta_{\hat{a}\hat{b}}\dot{e}^{\hat{a}}\dot{e}^{\hat{b}} = \eta_{\hat{a}\hat{b}}\dot{e}^{\hat{a}}{}_{\hat{\mu}}\dot{e}^{\hat{b}}{}_{\hat{\mu}}dX^{\hat{\mu}}dX^{\hat{\nu}} \equiv \eta_{\hat{\mu}\hat{\nu}}dX^{\hat{\mu}}dX^{\hat{\nu}}. \tag{50}$$

A worldline C of a particle, parametrized by proper time as $C(s) = X^{\hat{\mu}}(s)$, will have as six-velocity the vector of components $u^{\hat{\mu}} = dX^{\hat{\mu}}/ds$ and $u^{\hat{a}} = \dot{e}^{\hat{a}}_{\hat{\mu}}u^{\hat{\mu}}$, which are the particle velocity along this curve respectively in the holonomic and anholonomic bases in the X-space. The proper time can be written in the form $ds = u_{\hat{\mu}}dX^{\hat{\mu}} = u_{\hat{a}}\dot{e}^{\hat{a}}$. The equation of motion in the X-space is written as

$$\tfrac{du^{\hat{a}}}{ds} = \left(\dot{K}^{\hat{a}}{}_{\hat{b}\hat{\rho}} - \dot{\omega}^{\hat{a}}{}_{\hat{b}\hat{\rho}}\right)u^{\hat{b}}u^{\hat{\rho}}. \tag{51}$$

This equation can be rewritten in a purely spacetime form

$$\tfrac{du^{\hat{\rho}}}{ds} = \left(\dot{K}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}} - \dot{\Gamma}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}\right)u^{\hat{\mu}}u^{\hat{\nu}}. \tag{52}$$

The corresponding acceleration cannot be given a covariant meaning without a connection, while each different connection $\Gamma^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}$ will deffne a different acceleration. The Weitzenböck connection, which defines the Fock-Ivanenko derivative $\dot{\mathcal{D}}_{\hat{\mu}}$ written in terms of covariant derivative $\dot{\nabla}_{\hat{\mu}}$:

$$\dot{\mathcal{D}}_{\hat{\mu}}\Phi^{\hat{a}} = \dot{e}^{\hat{a}}{}_{\hat{\rho}}\dot{\nabla}_{\hat{\mu}}\Phi^{\hat{\rho}}, \tag{53}$$

will define the acceleration too

$$\begin{aligned}\dot{a}^{\hat{\rho}} &= \tfrac{\dot{\nabla}u^{\hat{\rho}}}{\dot{\nabla}s} = u^{\hat{\nu}}\dot{\nabla}_{\hat{\nu}}u^{\hat{\rho}} = \tfrac{du^{\hat{\rho}}}{ds} + \dot{\Gamma}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}u^{\hat{\mu}}u^{\hat{\nu}} \\ &= \dot{K}^{\hat{\rho}}{}_{\hat{\mu}\hat{\nu}}u^{\hat{\mu}}u^{\hat{\nu}} = \dot{T}^{\hat{\rho}}{}_{\hat{\nu}\hat{\mu}}u^{\hat{\mu}}u^{\hat{\nu}}.\end{aligned} \tag{54}$$

This is a force equation, with torsion (or contortion) playing the role of force. To transform the tetrad field into a reference frame in X-space with an observer attached to it, we may "attach" $\dot{e}_{\hat{0}}$ to the observer by identifying $u = \dot{e}_{\hat{0}} = \frac{d}{ds}$ with components $u^{\hat{\mu}} = \dot{e}_{\hat{0}}{}^{\hat{\mu}}$, such that $\dot{e}_{\hat{0}}$ will be the observer velocity. The Weitzenböck connection, $\dot{\Gamma}$, will attribute to the observer an acceleration

$$\dot{a}^{\hat{a}}_{(f,\dot{\Gamma})} = \dot{\omega}^{\hat{a}}{}_{\hat{0}\hat{0}} + \dot{K}^{\hat{a}}{}_{\hat{0}\hat{0}}, \tag{55}$$

seen by that very observer. Whereas,

$$\dot{\omega}^{\hat{a}}{}_{\hat{b}\hat{c}} = \dot{e}^{\hat{a}}{}_{\hat{\mu}}\dot{\nabla}_{\dot{e}_{\hat{c}}}, \dot{e}_{\hat{b}}{}^{\hat{\mu}}, \tag{56}$$

which literarily means the covariant derivative of $\dot{e}_{\hat{b}}$ along $\dot{e}_{\hat{c}}$, projected along $\dot{e}_{\hat{a}}$. As $\dot{a}^{\hat{\rho}}$ (54) is orthogonal to $u^{\hat{\rho}}$, its vanishing means that the $u^{\hat{\rho}}$ keeps parallel to itself along the worldline.

Another transport, distinct from parallel transport, is the Fermi-Walker transport, which absorbs the acceleration. Fermi-Walker transport for a vector **V** along a worldline with unit tangent **U** is characterized by a boost that compensates the acceleration $\dot{\mathbf{U}} \equiv \mathbf{U}/Ds$. The temporal-part of **V** remains unchanged, while the spatial-part becomes subject to a rotation preserving the length of the spatial-part of the vector **V** and, thus, its four-dimensional length. In particular,

$$\dot{\nabla}^{(FW)}_{\dot{e}_{\hat{0}}}\dot{e}_{\hat{0}}{}^{\hat{\rho}} = \dot{\nabla}_{\dot{e}_{\hat{0}}}\dot{e}_{\hat{0}}{}^{\hat{\rho}} - \dot{a}^{\hat{\rho}}, \tag{57}$$

implies that $\dot{e}_{\hat{0}}$, by this transport, is kept tangent along its own integral curve. Along the radial geodesics, however, Fermi-Walker transport becomes Levi-Civita's parallel transport.

4. A Newtonian gravitational field: 'absolute' acceleration

A Newtonian limit is obtained by assuming that the translational gauge field is stationary and weak. This means respectively that the time derivative of $e^{a}{}_{\mu}$ in M_4 vanishes, and that$|e^{a}{}_{\mu}| \ll 1$ (Aldrovandi & Pereira, 2013). In accord, all the particles are supposed to move in M_4 with a sufficient small velocity so that $u^i \ll u^0$. The equation of motion (52) rewritten purely in the terms of M_4 spacetime is then reduced to

$$\tfrac{du^{\rho}}{ds} + \dot{\Gamma}^{\rho}{}_{00}u^0u^0 = \dot{T}_0{}^{\rho}{}_0u^0u^0. \tag{58}$$

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In the class of frames in which the teleparallel spin connection $\dot{\omega}^{\hat{a}}{}_{\hat{b}\hat{\rho}}$ vanishes, and choosing a translational gauge in which $\partial_\mu x^a = \delta^a_\mu$, the tetrad has the form

$$\widetilde{e}^a{}_\mu = \delta^a{}_\mu + e^a{}_\mu.$$

Up to first order in the field $e^a{}_\mu$, we obtain

$$\dot{\Gamma}^\rho{}_{\mu\nu} \equiv \partial_\nu e^\rho{}_\mu,$$

where

$$e^\rho{}_\mu = \delta^\rho_a e^a{}_\mu.$$

The (58) becomes

$$\frac{d^2x^\rho}{ds^2} = \partial^\rho e_{00} u^0 u^0.$$

Substituting $u^0 = cdt/ds$, we obtain

$$\frac{d^2x^\rho}{ds^2} = \partial^\rho e_{00} c^2 (\frac{dt}{ds})^2.$$

For the temporal component this gives

$$\frac{d^2x^0}{ds^2} \equiv c^2 \frac{d^2t}{ds^2} = 0,$$

the solution of which is the constant dt/ds. Then the spatial components of d^2x^ρ/ds^2 reads

$$\frac{dv^i}{dt} = c^2 \partial^i e_{00},$$

where v^i is the particle three-velocity. If we identify $c^2 e_{00} = \Phi$, with the potential of the *fictitious* gravitational force (the inertial force):

$$F = -\nabla\Phi,$$

we obtain

$$\vec{a} = -\vec{F}.$$

Here we have taken into account that the components of $\vec{x}$ are given by $x^i = -x_i$, so that $\partial^i = -\partial_i$. As expected, due to the equivalence with the geodesic equation, the force equation (54) also can be approximated to the limit of a Newtonian gravitational field.

Now consider the Newtonian field with acceleration $-\underline{a}_{(c)}$, in $\underline{M}_2$, in the $\underline{x}^{1}$-direction of zero velocity and at $\underline{x}^{0} = 0$. Let the parallel worldlines $\underline{x} = const$ be the geodesics. For $\underline{x}^{0} = 0$ and $\underline{x}^{1} = 1/v^{\underline{0}}\underline{a}_{(c)}$, the parallel to the $\underline{x}^{0}$-axis through this point is touched by the worldline of an observer with constant intrinsic acceleration

$$\underline{a}_{(c)} = \sqrt{2}(a^{(+)}_{(c)} a^{(-)}_{(c)})^{1/2} = 2(\underline{\theta}_1\,\bar{\underline{\theta}}_1\underline{\theta}_2\,\bar{\underline{\theta}}_2)^{1/2}\, \frac{d^2\tau}{d\underline{s}^2}\Big|_{const}, \tag{59}$$

which is the hyperbola (see next subsect.) parameterized with proper time $\underline{s}$,

$$\underline{x}^{0} = \underline{\rho} \sinh \frac{\underline{s}}{\underline{\rho}}, \quad \underline{x}^{1} = \underline{\rho} \cosh \frac{\underline{s}}{\underline{\rho}}, \quad \underline{\rho} = \frac{1}{v^{\underline{0}}\underline{a}_{(c)}}. \tag{60}$$

Let this observer measure the distance $\underline{\eta}(\underline{s})$ to the straight line $\underline{x} = \underline{\rho}$ orthogonal to his worldline:

$$\underline{\eta}(\underline{s}) = \underline{\rho} - \frac{\underline{\rho}}{\cosh \frac{\underline{s}}{\underline{\rho}}} = \frac{1}{2\underline{\rho}}\underline{s}^2 + \mathcal{O}(\underline{s}^4). \tag{61}$$

The acceleration then reads

$$\underline{a} = \frac{d^2\underline{\rho}}{\underline{s}^2} = \frac{1}{2\underline{\rho}} = v^{\underline{0}}\underline{a}_{(c)}. \tag{62}$$

Since the velocity $\underline{\eta}/\underline{s}$ increases, the acceleration points into the negative $\underline{x}^{1}$-direction. Further, according to (126), the *absolute* acceleration in M_4 becomes

$$\vec{a}_{abs.} = -\underline{a} = -v^0\underline{a}_{(c)} = -4(\underline{\theta}_1\,\bar{\underline{\theta}}_1\underline{\theta}_2\,\bar{\underline{\theta}}_2)\frac{d\tau}{d\underline{s}}\,\frac{d^2\tau}{d\underline{s}^2}\Big|_{const}, \tag{63}$$

pointing into the negative $\vec{n} = \vec{x}/|\vec{x}|$-direction. It is for this special case: against *absolute* space, in the same event, and at the same velocity, an inertial observer and the *absolutely accelerated* observer can interpret gravity as acceleration and vice versa. Acceleration against Newton's absolute space now is no longer so implausible as it appeared a century ago. The vacuum of spacetime is filled with the fluctuations of all fields and a metric too. Absolute space is realized by a local inertial system.

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5. The homogeneous acceleration field

The torsion in affine connected master space $\underline{M}_2$ is described by the vector-valued 2-form $\underline{T}$ as

$$\underline{T} = d(e_{\underline{\mu}} e^{\underline{\mu}}), \tag{64}$$

where d denotes the operator of exterior covariant derivation. Let us now to study the accelerated frame of reference described by the constant torsion (written with re-instated factor $\wedge$) - the *fictitious* gravitational field strength,

$$T^{\underline{\mu}} = de^{\underline{\mu}} = \tfrac{1}{2} T^{\underline{\mu}}_{\underline{\lambda}\,\underline{\nu}} e^{\underline{\lambda}} \wedge e^{\underline{\nu}}, \tag{65}$$

in the $\underline{M}_2$. The metric of a *homogeneous* manifold is invariant under a *transitive* group of motions, i.e. if its action maps any point of the manifold into any other point (Schucking & Surowitz, 2007). For gravitational fields, the group of motions has to be *simply transitive*, i.e. there is only one group element that moves a point of the manifold into another given point of the manifold. The *homogeneity* group in the master space $\underline{M}_2$ is characterized by the *invariant differential* 1*-forms* $e^{\underline{\mu}}$ that determine the coframes. Namely, $\underline{M}_2$ is homogeneous if the metric is given in terms of *invariant differential* 1*-forms* $e^{\underline{\mu}} = e^{\underline{\mu}}_{\ \underline{\nu}} d\underline{x}^{\underline{\nu}}$ for a *simply transitive group*. Therefore the $e'^{\underline{\mu}}_{\ \underline{\nu}}$ and the $e^{\underline{m}}_{\ \underline{\nu}}$ are the same functions of their arguments $e'^{\underline{\mu}}_{\ \underline{\nu}}(\underline{x}'^{\underline{\lambda}}) = e^{\underline{\mu}}_{\ \underline{\nu}}(\underline{x}^{\underline{\lambda}})$, provided, $det[e'^{\underline{\mu}}_{\ \underline{\nu}}] \neq 0$ and $det[e^{\underline{\mu}}_{\ \underline{\nu}}] \neq 0$. In this case, the specific coframe vectors ($e^{\underline{\mu}}$),

$$\begin{array}{l} e^{(+)} = \frac{d\underline{x}^{(+)}}{\underline{a}_{(c)}\underline{x}^{(+)}}, \quad e^{(-)} = \underline{a}_{(c)}\underline{x}^{(+)} d\underline{x}^{(-)}, \\ d\underline{s}^2 = 2e^{(+)}e^{(-)}, \end{array} \tag{66}$$

with constant acceleration $\underline{a}_{(c)}$ (59), are independent and yield the components of constant and thus homogeneous torsion (65):

$$T^{(+)}_{(+)\,(-)} = 0, \quad T^{(-)}_{(+)\,(-)} = \underline{a}_{(c)}. \tag{67}$$

The embedding map (100) gives

$$e^{(\pm)} = \frac{1}{\sqrt{2}}(d\underline{x}^{\underline{0}} \pm d\underline{x}^{\underline{1}}) = \frac{1}{\sqrt{2}}(dx^0 \pm d|\vec{x}|),$$

so that $d\underline{s}^2 = ds^2$. Using the relation $e^{\underline{\mu}} \rfloor e_{\underline{\lambda}} = \delta^{\underline{\mu}}_{\underline{\lambda}}$, the frame vectors $e_{(\pm)}$ read

$$e_{(+)} = \underline{a}_{(c)} x^{(+)} \partial_{(+)}, \quad e_{(-)} = (\underline{a}_{(c)} x^{(+)})^{-1} \partial_{(-)}. \tag{68}$$

The covariant components $a_{\underline{j}}$ of the constant acceleration vector, from the (54), can be written

$$a_j = T_{jkl} v^k v^l, \tag{69}$$

which is the geodesic acceleration adapted to frame (68), with

$$\begin{array}{l} a_{(+)} = T_{(+)\,(-)\,(+)} v^{(-)} v^{(+)} = \underline{a}_{(c)} v^{(-)} v^{(+)}, \\ a_{(-)} = T_{(-)\,(+)\,(+)} v^{(+)} v^{(+)} = -\underline{a}_{(c)} \left(v^{(+)}\right)^2, \end{array} \tag{70}$$

where $v^{(\pm)} = \frac{1}{\sqrt{2}}(v^0 \pm |\vec{v}|)$. The contravariant components then are

$$a^{(+)} = -\underline{a}_{(c)} \left(v^{(+)}\right)^2, \quad a^{(-)} = \underline{a}_{(c)} v^{(-)} v^{(+)}. \tag{71}$$

Thus the acceleration four-vector reads

$$\mathbf{a} = a^{\underline{\mu}} e_{\underline{\mu}} = -\left(\underline{a}_{(c)} v^{(+)}\right)^2 x^{(+)} \partial_{(+)} + \frac{v^{(-)} v^{(+)}}{x^{(+)}} \partial_{(-)}. \tag{72}$$

This vector is orthogonal to the four-velocity
$a_j v^j = 0$, and tangent to the curves

$$\begin{array}{l} \frac{dx^{(+)}}{d\sigma} = -\left(\underline{a}_{(c)} v^{(+)}\right)^2 x^{(+)}, \\ \frac{dx^{(-)}}{d\sigma} = \frac{v^{(-)} v^{(+)}}{x^{(+)}}, \end{array} \tag{73}$$

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described by the parameter σ. Integration gives

$$x^{(+)} = C_1^{(+)} e^{-\left(\underline{a}_{(c)} v^{(+)}\right)^2 \sigma}, \qquad x^{(-)} - C_1^{(-)} = \frac{v^{(-)} v^{(+)} e^{\left(\underline{a}_{(c)} v^{(+)}\right)^2 \sigma}}{\left(\underline{a}_{(c)} v^{(+)}\right)^2 C_1^{(+)}}, \tag{74}$$

with constants $C_1^{(\pm)}$. By eliminating these constants and using the relation

$$2v^{(+)} v^{(-)} = \left(v^0\right)^2 - |\vec{v}|^2 = 1, \tag{75}$$

we obtain

$$x^{(-)} - C_1^{(-)} = \frac{1}{2x^{(+)}\left(\underline{a}_{(c)} v^{(+)}\right)^2} \tag{76}$$

These curves can be obtained from the hyperbola

$$x^{(+)} x^{(-)} = (x^{(0)})^2 - |\vec{x}|^2 = \frac{1}{2(v^{(+)}\underline{a}_{(c)})^2}, \tag{77}$$

by translation in the $x^{(-)}$-direction. If we choose the velocity as a constant, say, $v^{(+)} = const > 0$, it follows from (75) that $v^{(-)}$ is also constant and the velocity vector field

$$\mathbf{v} = v^{\underline{\mu}} e_{\underline{\mu}} = v^{(+)}\, \underline{a}_{(c)} x^{(+)} \partial_{(+)} + v^{(-)} \frac{1}{\underline{a}_{(c)} x^{(+)}} \partial_{(-)}, \tag{78}$$

is invariant in the M_4. Since $v^{(+)}$ is constant, the vectors are parallel in the usual affine sense along lines $x^{(+)} = const$. In the same manner as above, we write

$$\frac{dx^{(+)}}{ds} = v^{(+)}\, \underline{a}_{(c)} x^{(+)}, \qquad \frac{dx^{(-)}}{ds} = \frac{1}{2v^{(+)}\, \underline{a}_{(c)} x^{(+)}}, \tag{79}$$

so that integration gives

$$x^{(+)} = C_2^{(+)}\, e^{v^{(+)}\, \underline{a}_{(c)} s}, \qquad x^{(-)} - C_2^{(-)} = -\frac{e^{v^{(+)}\, \underline{a}_{(c)} s}}{2C_2^{(+)} (v^{(+)} \underline{a}_{(c)})^2}, \tag{80}$$

with constants $C_2^{(\pm)}$. By eliminating $C_2^{(+)}$ and proper time s, we obtain a set of identical hyperbolae

$$x^{(-)} - C_2^{(-)} = -\frac{1}{2x^{(+)}(v^{(+)}\underline{a}_{(c)})^2}, \tag{81}$$

that are obtained from the hyperbola

$$x^{(+)} x^{(-)} + \frac{1}{2(v^{(+)}\underline{a}_{(c)})^2} = 0, \tag{82}$$

or

$$(x^{(0)})^2 - |\vec{x}|^2 + \frac{1}{2(v^{(+)}\underline{a}_{(c)})^2} = 0, \tag{83}$$

by translation in the $x^{(-)}$-direction. The question is whether two events in M_4 are connected by a straight timelike line or by a hyperbolic path can easily be solved because the hyperbolic path has a smaller proper time. Thus, the results obtained clearly show that the accelerated frame of reference should be described as a frame with torsion. Whereas the frames expressing linear and rotational acceleration can be interpreted—via torsion—as an invariant property of spacetime.

6. Concluding remarks

In this section we highlight a few points of the $\widetilde{MS}_p$-TSG theory and discuss issues to be studied in subsequent paper.

(I) Using Palatini's formalism extended in a plausible fashion to the $\widetilde{MS}_p$ - SG, we reinterpret a flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as a $\widetilde{MS}_p$-TSG theory, having the gauge *translation* group in tangent bundle. Whereas the Hilbert action vanishes and the gravitino action loses its spin connections, so we find torsion induced by gravitinos. The accelerated reference frame has Weitzenböck torsion.

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(II) The spin connection represents only *inertial effects*, but not gravitation at all. The action of a $\widetilde{MS}_p$-TSG theory is invariant under the Poincaré supergroup and under diffeomorphisms.

(III) The translation group is abelian, therefore its Cartan-Killing bilinear form is degenerate and cannot be used as a metric. Recall that the group manifold of translations is just the Minkowski spacetime $\mathcal{M}$, the quotient space between the Poincaré, $\mathcal{P}$, and the Lorentz, $\mathcal{L}$, groups: $\mathcal{M} = \mathcal{P}/\mathcal{L}$. Namely, $\mathcal{M}$ is homogeneous or transitive under X-space translations, which means that there is only one group element that moves a point of the $\mathcal{M}$ space into another given point of the $\mathcal{M}$ space. The role of the Cartan-Killing metric comes, when it exists, from its being invariant under the group action. Here it does not exist, but we can use the invariant Lorentz metric $\eta_{\hat{a}\hat{b}}$ of $\mathcal{M}$ in its stead.

(IV) The action of $\widetilde{MS}_p$-TSG is invariant under local translations, under local super symmetry transformations and by construction is invariant under local Lorentz rotations and under diffeomorphisms. So that this action is invariant under the Poincaré supergroup and under diffeomorphisms.

(V) We show the equivalence of the Teleparallel Gravity action with Hilbert action, which will prove that the immediate cause of the *fictitious* Riemann curvature for the Levi-Civita connection is the acceleration. The curvature of Weitzenböck connection vanishes identically, but for a tetrad involving a non-trivial translational gauge potential, the torsion is non-vanishing. We consider Weitzenböck connection a kind of dual of the Levi-Civita connection, which is a connection with vanishing torsion, and non-vanishing *fictitious* curvature. The Weitzenböck connection, which defines the Fock-Ivanenko derivative written in terms of covariant derivative, will define the acceleration through force equation, with torsion (or contortion) playing the role of force.

(VI) The Weitzenböck connection ($\dot{\Gamma}$), which defines the Fock-Ivanenko derivative ($\dot{\mathcal{D}}_{\hat{\mu}}$) written in terms of covariant derivative ($\dot{\nabla}_{\hat{\mu}}$), defines the acceleration too. By means of it, we derive a force equation, with torsion (or contortion) playing the role of force. The connection ($\dot{\Gamma}$) can be considered a kind of dual of the Levi-Civita connection (Γ), which is a connection with vanishing torsion (T), and non-vanishing *fictitious* curvature (R). Using this, we prove the equivalence of the Teleparallel Gravity action ($\dot{\mathcal{L}}^{(2)}$) in (42) with Hilbert action ($\mathcal{L}^{(2)}$) in (118).

(VII) We complement a theory of $\widetilde{MS}_p$-TSG with implications for special cases. In particular, we discuss the Newtonian limit, and describe the homogeneous acceleration field.

(VIII) Further studies on the $\widetilde{MS}_p$-TSG are warranted with special emphasis on the general deformation of MS_p induced by external force exerted on a particle and inertial effects, the hypothesis of locality, which will essentially improve the framework of present paper. Actually,

a) As we emphasized already essential difference arisen between the standard supergravity theories and some rather unusual properties of a $\widetilde{MS}_p$-SG theory is as follows. In the framework of the standard supergravity theories, as in GR, a curvature of the space-time acts on all the matter fields. The source of graviton is the energy-momentum tensor of matter fields, while the source of gravitino is the spin-vector current of supergravity. The gauge action of simple $\widetilde{MS}_p$-SG is the sum of the Hilbert action for the tetrad field - *fictitious* graviton, and the Rarita-Schwinger action for the *fictitious* gravitino field. Instead we argue that a deformation of MS_p is the origin of these fields. They refer to the particle of interest itself, without relation to other matter fields, so that these fields can be globally removed by appropriate coordinate transformations. With these physical requirements, a standard coupling of supergravity with matter superfields evidently no longer holds. Instead we should look for an alternative way of implications of $\widetilde{MS}_p$-SG for the model of accelerated motion and inertial effects. We, therefore, would work out the theory of a general deformation of MS_p induced by external force exerted on a particle, in order to show that in the $\widetilde{MS}_p$-TSG theory the occurrence of the *absolute* and *inertial* accelerations, and the *inertial* force are obviously caused by this. In the same time, the *relative* acceleration (in Newton's terminology) (both magnitude and direction), to the contrary, has nothing to do with a deformation of $\underline{M}_2$ and, thus, it cannot produce the inertia effects.

b) In standard framework of the construction of reference frame of an accelerated observer, the hypothesis of locality holds for huge proper acceleration lengths and that represents strict restrictions, because it approximately replaces a noninertial frame of reference $\widetilde{S}_{(2)}$, which is held stationary in the deformed space $\mathcal{M}_2 \equiv \underline{V}_2^{(\varrho)}$ $(\varrho \neq 0)$, where $\underline{V}_2$ is the 2D semi-Riemannian space, with a continuous infinity set of the inertial frames $\{S_{(2)}, S'_{(2)}, S''_{(2)}, ...\}$ given in the flat $\underline{M}_2$ $(\varrho = 0)$. In this situation the use of the hypothesis of locality is physically unjustifiable. In this study, therefore, it is worthwhile to take into account a deformation $\underline{M}_2 \longrightarrow \underline{V}_2^{(\varrho)}$, which will essentially improve the standard framework. The above mentioned problems (a,b) will be topic for research in subsequent paper (Ter-Kazarian, 2024c).

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Appendices

Appendix A A global MS$_p$-SUSY

As a guiding principle to make the present paper understandable, in the next two sections we necessarily recount succinctly some of the highlights behind of *global* MS$_p$-SUSY (Ter-Kazarian, 2023a, 2024a) and *local* $\widetilde{MS}_p$-SUSY theories.

The flat MS$_p$ is the 2D composite space

$$MS_p \equiv \underline{M}_2 = \underline{R}^1_{(+)} \oplus \underline{R}^1_{(-)}, \tag{84}$$

with Lorentz metric. The ingredient 1D-space $\underline{R}^1_{\underline{m}}$ is spanned by the coordinates $\eta^{\underline{m}}$. The following notational conventions are used throughout this paper: all quantities related to the space $\underline{M}_2$ will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, ... = (\pm)$ denote the world indices related to $\underline{M}_2$.

Suppose the position of the particle is specified by the coordinates $x^m(s)$ $(x^0 = t)$ in the basis e_m (m=0,1,2,3) at given point in the background M_4 space. A smooth map $f: \underline{M}_2 \longrightarrow M_4$ is defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{e}_0 = e_0, \quad \underline{x}^0 = x^0, \quad \underline{e}_1 = \vec{n}, \quad \underline{x}^1 = |\vec{x}|, \tag{85}$$

where $\vec{x} = e_i x^i = \vec{n}|\vec{x}|$ $(i = 1, 2, 3)$. Given the inertial frames $S_{(4)}, S'_{(4)}, S''_{(4)}, ...$ in unaccelerated uniform motion in M_4, we may define the corresponding inertial frames $\underline{S}_{(2)}$, $\underline{S}'_{(2)}$, $\underline{S}''_{(2)}$,... in $\underline{M}_2$, which are used by the non-accelerated observers for the positions $\underline{x}^{\underline{r}}, \underline{x}'^{\underline{r}}, \underline{x}''^{\underline{r}}, ...$ of a free particle in flat $\underline{M}_2$. According to (85), the time axes of the two systems $\underline{S}_{(2)}$ and $S_{(4)}$ coincide in direction, and the time coordinates are taken the same. All the fermionic and bosonic states taken together form a basis in the Hilbert space. The basis vectors in the Hilbert space composed of $H_B \otimes H_F$ is given by

$$\{|\underline{n}_b > \otimes |0>_f,\ |\underline{n}_b > \otimes f^\dagger\, |0>_f\},$$

or

$$\{|n_b > \otimes |\underline{0}>_f,\ |n_b > \otimes \underline{f}^\dagger\, |\underline{0}>_f\},$$

where we consider two pairs of creation and annihilation operators $(b^\dagger,\, b)$ and $(f^\dagger,\, f)$ for bosons and fermions, respectively, referred to the background space M_4, as well as $(\underline{b}^\dagger,\, \underline{b})$ and $(\underline{f}^\dagger,\, \underline{f})$ for bosons and fermions, respectively, as to background master space $\underline{M}_2$. Accordingly, we construct the quantum operators, $(q^\dagger,\, \underline{q}^\dagger)$ and $(q,\, \underline{q})$, which replace bosons by fermions and vice versa:

$$\begin{aligned} q\,|\underline{n}_b,\, n_f > &= q_0\sqrt{\underline{n}_b}\,|\underline{n}_b - 1,\, n_f + 1>, \\ q^\dagger\,|\underline{n}_b,\, n_f > &= q_0\sqrt{\underline{n}_b + 1}\,|\underline{n}_b + 1,\, n_f - 1>, \end{aligned} \tag{86}$$

and that

$$\begin{aligned} \underline{q}\,|n_b,\, \underline{n}_f > &= q_0\sqrt{n_b}\,|n_b - 1,\, \underline{n}_f + 1>, \\ \underline{q}^\dagger\,|n_b,\, \underline{n}_f > &\, q_0\sqrt{n_b + 1}\,|n_b + 1,\, \underline{n}_f - 1> . \end{aligned} \tag{87}$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators q and $\underline{q}$. So, we may refer the action of the supercharge operators q and $q^\dagger$ to the background space M_4, having applied in the chain transformations of fermion χ (accompanied with the auxiliary field F as it will be seen later on) to boson $\underline{A}$, defined on $\underline{M}_2$. Respectively, we may refer the action of the supercharge operators $\underline{q}$ and $\underline{q}^\dagger$ to the $\underline{M}_2$, having applied in the chain transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field $\underline{F}$) to boson A, defined on the background space M_4. While all the particles are living on M_4, their superpartners can be viewed as living on MS$_p$. In MS$_p$-SUSY theory, obviously as in standard unbroken SUSY theory, the vacuum zero point energy problem, standing before any quantum field theory in M_4, is solved. The particles in M_4 themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4 \oplus$ MS$_p$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal $(q_0^2 \equiv \nu_b = \nu_f)$, because the fermion field has a negative zero point energy while the boson field has a positive zero point energy.

The odd part of the supersymmetry algebra is composed entirely of the spin-1/2 operators $Q_\alpha{}^i,\ Q_\beta{}^j$. In order to trace a maximal resemblance in outward appearance to the standard SUSY theories, here we set

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one notation $\hat{m} = (m \quad \text{if} \quad Q = q, \text{ or } \underline{m} \quad \text{if} \quad Q = \underline{q})$, and as before the indices α and $\dot{\alpha}$ run over 1 and 2. The underlying algebraic structure of MS_p-SUSY generators closes with the algebra of *translations* on the original space M_4 in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra those of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to M_4. Moreover, if there are several spinor generators $Q_\alpha{}^i$ with $i = 1, ..., N$ - theory with $N-$extended supersymmetry, can be written as a graded Lie algebra of SUSY field theories, with commuting and anticommuting generators. The anticommuting (Grassmann) parameters $\epsilon^\alpha(\xi^\alpha, \underline{\xi}^\alpha)$ and $\bar{\epsilon}^\alpha(\bar{\xi}^\alpha, \bar{\underline{\xi}}^\alpha)$:

$$\{\epsilon^\alpha, \epsilon^\beta\} = \{\bar{\epsilon}^\alpha, \bar{\epsilon}^\beta\} = \{\epsilon^\alpha, \bar{\epsilon}^\beta\} = 0, \quad \{\epsilon^\alpha, Q_\beta\} = \cdots = [p_{\hat{m}}, \epsilon^\alpha] = 0, \tag{88}$$

allow us to write the algebra (**??**) for $(N = 1)$ entirely in terms of commutators:

$$[\epsilon Q, \bar{Q}\bar{\epsilon}] = 2\epsilon\sigma^{\hat{m}}\bar{\epsilon}p_{\hat{m}}, \quad [\epsilon Q, \epsilon Q] = [\bar{Q}\bar{\epsilon}, \bar{Q}\bar{\epsilon}] = [p^{\hat{m}}, \epsilon Q] = [p^{\hat{m}}, \bar{Q}\bar{\epsilon}] = 0. \tag{89}$$

For brevity, here the indices $\epsilon Q = \epsilon^\alpha Q_\alpha$ and $\bar{\epsilon}\bar{Q} = \bar{\epsilon}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$ will be suppressed unless indicated otherwise. This supersymmetry transformation maps tensor fields $\mathcal{A}(A, \underline{A})$ into spinor fields $\psi(\chi, \underline{\chi})$ and vice versa. From the algebra (89) we see that Q has mass dimension $1/2$. Therefore, as usual, fields of dimension ℓ transform into fields of dimension $\ell + 1/2$ or into derivatives of fields of lower dimension.

In the framework of standard generalization of the coset construction, we will take $G = G_q \times G_{\underline{q}}$ to be the supergroup generated by the MS_p-SUSY algebra (**??**). Let the stability group $H = H_q \times H_{\underline{q}}$ be the Lorentz group (referred to M_4 and $\underline{M}_2$), and we choose to keep all of G unbroken. Given G and H, we can construct the coset, G/H, by an equivalence relation on the elements of G: $\quad \Omega \sim \Omega h$, where $\Omega = \Omega_q \times \Omega_{\underline{q}} \in G$ and $h = h_q \times h_{\underline{q}} \in H$, so that the coset can be pictured as a section of a fiber bundle with total space, G, and fiber, H. So, the Maurer-Cartan form, $\Omega^{-1}d\Omega$, is valued in the Lie algebra of G, and transforms as follows under a global G transformation,

$$\begin{array}{l} \Omega \longrightarrow g\Omega h^{-1}, \\ \Omega^{-1}d\Omega \longrightarrow h(\Omega^{-1}d\Omega)h^{-1} - dh\, h^{-1}, \end{array} \tag{90}$$

with $g \in G$.

The `superspace´ is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ (spanned by the 6D-coordinates $X^{\hat{m}} = (x^m, \underline{\eta}^{\underline{m}})$ by the inclusion of additional 8D-fermionic coordinates $\Theta^\alpha = (\theta^\alpha, \underline{\theta}^\alpha)$ and $\bar{\Theta}_{\dot{\alpha}} = (\bar{\theta}_{\dot{\alpha}}, \bar{\underline{\theta}}_{\dot{\alpha}})$. Then the net result of sequential atomic double transitions induce the inhomogeneous Lorentz group, or Poincaré group, and that the unitary linear transformation $|x, t > \to U(L, a)|x, t >$ on vectors in the physical Hilbert space.

To study the effect of supersymmetry transformations, we consider

$$g(0, \epsilon, \bar{\epsilon})\, \Omega(X, \Theta, \bar{\Theta}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})}\, e^{i(-X^{\hat{m}}p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}})}. \tag{91}$$

the transformation (91) induces the motion:

$$g(0, \epsilon, \bar{\epsilon})\, \Omega(X^{\hat{m}}, \Theta, \bar{\Theta}) \to (X^{\hat{m}} + i\,\Theta\,\sigma^{\hat{m}}\,\bar{\epsilon} - i\,\epsilon\,\sigma^{\hat{m}}\,\bar{\Theta},\ \Theta + \epsilon,\ \bar{\Theta} + \bar{\epsilon}), \tag{92}$$

namely,

$$\begin{array}{l} g_q(0, \xi, \bar{\xi})\, \Omega_q(x, \theta, \bar{\theta}) \to (x^m + i\,\theta\,\sigma^m\,\bar{\xi} - i\,\xi\,\sigma^m\,\bar{\theta},\ \theta + \xi,\ \bar{\theta} + \bar{\xi}), \\ g_{\underline{q}}(0, \underline{\xi}, \bar{\underline{\xi}})\, \Omega_{\underline{q}}(\underline{\eta}, \underline{\theta}, \bar{\underline{\theta}}) \to (\underline{\eta}^{\underline{m}} + i\,\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}} - i\,\underline{\xi}\,\sigma^{\underline{m}}\,\bar{\underline{\theta}},\ \underline{\theta} + \underline{\xi},\ \bar{\underline{\theta}} + \bar{\underline{\xi}}). \end{array} \tag{93}$$

The spinors $\theta(\underline{\theta}, \bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta}, \bar{\underline{\theta}})$ satisfy the embedding relations $\Delta\underline{x}^0 = \Delta x^0$ and $\Delta\underline{x}^2 = (\Delta\vec{x})^2$, so from (93) we obtain

$$\underline{\theta}\,\sigma^0\,\bar{\underline{\xi}} - \underline{\xi}\,\sigma^0\,\bar{\underline{\theta}} = \theta\,\sigma^0\,\bar{\xi} - \xi\,\sigma^0\,\bar{\theta}, \quad (\underline{\theta}\,\sigma^3\,\bar{\underline{\xi}} - \underline{\xi}\,\sigma^3\,\bar{\underline{\theta}})^2 = (\theta\,\vec{\sigma}\,\bar{\xi} - \xi\,\vec{\sigma}\,\bar{\theta})^2. \tag{94}$$

The *atomic displacement* caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ reads

$$\Delta\underline{x}_{(a)} = \underline{e}_{\underline{m}}\Delta\underline{x}^{\underline{m}}_{(a)} = \underline{u}\tau, \tag{95}$$

where the components $\Delta\underline{x}^{\underline{m}}_{(a)}$ are written

$$\Delta\underline{x}^{\underline{m}}_{(a)} = (\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\theta}})\tau. \tag{96}$$

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Hence the velocities of light in vacuum, $\underline{v}^{\underline{0}} = c$, and of a particle ,$\vec{\underline{v}}_{\underline{1}} = \underline{e}_{\underline{1}}\underline{v}^{\underline{1}} = \vec{n}|\vec{v}| = \vec{v}$ $(|\vec{v}| \leq c)$, are

$$\begin{aligned} \underline{v}^{\underline{0}} &= \underline{\theta}\,\sigma^{\underline{0}}\,\bar{\underline{\theta}} = (\underline{\theta}_1\,\bar{\underline{\theta}}_1 + \underline{\theta}_2\,\bar{\underline{\theta}}_2) = \underline{\theta}\,\bar{\underline{\theta}}, \\ \underline{v}^{\underline{1}} &= \underline{\theta}\,\sigma^{\underline{1}}\,\bar{\underline{\theta}} = (\underline{\theta}_1\,\bar{\underline{\theta}}_1 - \underline{\theta}_2\,\bar{\underline{\theta}}_2). \end{aligned} \tag{97}$$

Thus, we achieve the desired goal to derive the SLC in terms of spinors $(\underline{\theta}, \bar{\underline{\theta}})$ and period (τ) of superoscillations referred to the master space $\underline{M}_2$. This allows to introduce the physical finite *relative time* interval between two events as integer number of the *own atomic duration time* of double transition of a particle from M_4 to MS_p and back.

Thereby we consider the spaces M_4 and $\underline{M}_2$ as mathematical prior the *motion*, devoid of any sense of *physical space-time*, until we have to impose two specific conditions on the spinor transformation matrix M acting in $\underline{M}_2$ to derive the most important *inertial uniform motion* in M_4 with *physical properties of Special Relativity*. Actually, to derive the most important *relative inertial uniform motion* in mathematical Minkowski space M_4 devoid of any sense of *physical space-time*, it is necessary to impose specific conditions on the spinor transformation matrix M in $\underline{M}_2$. We are of course not limited within MS_p-SUSY to consider particular constant spinor $\underline{\theta}$ referred to $\underline{M}_2$, which yields the constant velocity $\vec{v}(\underline{\theta})$, but can choose at will any other constant spinors $\underline{\theta}'$, $\underline{\theta}''$, ... yielding respectively the constant velocities $\vec{v}'(\underline{\theta}')$, $\vec{v}\,''(\underline{\theta}'')$, ... of inertial observers that move uniformly forever on rectilinear timelike worldlines, whose transformational law on the original spinor $\underline{\theta}$ is known (first condition):

$$\underline{\theta}'_\alpha = M_\alpha{}^\beta \underline{\theta}_\beta, \quad \bar{\underline{\theta}}_{\dot\alpha} = (M^*)_{\dot\alpha}{}^{\dot\beta}\,\bar{\underline{\theta}}_{\dot\beta}, \quad \dot\alpha, \dot\beta = 1, 2, \tag{98}$$

where $M \in SL(2, C)$ is the hermitian unimodular two-by-two matrix, the matrix M^* is related by a similarity transformation to $(M^{-1})^\dagger$, i.e. $(M^\dagger)^\beta{}_\alpha = (M^*)_\alpha{}^\beta$. The (98) gives the second founding property of SR that the bilinear combinations are $c := \underline{\theta}\,\bar{\underline{\theta}} = \underline{\theta}'\,\bar{\underline{\theta}}' = \cdots = const$, which yields a second postulate of SR (Einstein's postulate). Therewith a quantity $\underline{e}_{\underline{m}}\,(\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}})$ (where $\underline{e}_{\underline{m}}$ is a basis vector, $\underline{\theta}$, $\underline{\xi}$ are Weyl spinors) is a Lorentz scalar if and only if the second condition holds:

$$\tfrac{1}{2}\,Tr\left(\sigma^{\underline{m}}\,M\sigma^{\underline{n}}\,M^\dagger\right)\sigma^{\underline{n}}_{\alpha\dot\alpha} = (M^{-1})_\alpha{}^\beta \sigma^{\underline{m}}_{\beta\dot\beta}(M^{-1})^{\dagger\dot\beta}{}_{\dot\alpha}, \tag{99}$$

where the map from $SL(2, C)$ to the Lorentz group is established through the $\underline{\sigma}$-matrices. The latter, according to embedding map, can be written in terms of $\vec{\sigma}$-Pauli spin matrices. The (99) combined with (96) give the first founding property $\Delta\underline{x}_{(a)} = \Delta\underline{x}'_{(a)} = \cdots = inv$ of SR. This calls for a complete reconsideration of our standard ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property. On these premises, we derive the two postulates on which the theory of SR is based. In the sequel, we turn to the deformation of these spinors (Ter-Kazarian, 2023b, 2024a): $\underline{\theta} \to \tilde{\underline{\theta}} = \lambda^{1/2}\,\underline{\theta}$, etc., where λ appears as a deformation scalar function of the Lorentz invariance (LIDF). This yields both the DLE and DMAV, respectively, in the form $\tilde{ds} = \lambda ds$ and $\tilde{c} = \lambda c$, provided, the invariance of DLE, and the same value of DMAV in free space hold for all inertial systems. Thus the Lorentz invariance deformation (LID)-generalization of global MS_p-SUSY theory formulates the generalized relativity postulates in a way that preserve the relativity of inertial frames, in spite of the appearance of modified terms in the LID dispersion relations. We complement this conceptual investigation with testing of various LIDFs in the UHECR- and TeV-γ threshold anomalies by implications for several scenarios: the Coleman and Glashow-type perturbative extension of SLC, the LID extension of standard model, the LID in quantum gravity motivated space-time models, the LID in loop quantum gravity models, and the LIDF for the models preserving the relativity of inertial frames.

Appendix B The $\widetilde{MS}_p$-Supergravity

A local extension of the MS_p-SUSY algebra leads to the gauge theory of *translations*. One might guess that the condition for the parameter $\partial_{\hat\mu}\epsilon = 0$ of a global MS_p-SUSY theory (Ter-Kazarian, 2023b, 2024a) should be relaxed for the accelerated particle motion, so that a global SUSY will be promoted to a local SUSY in which the parameter $\epsilon = \epsilon(X^{\hat\mu})$ depends explicitly on $X^{\hat\mu} = (\widetilde{x}^\mu, \widetilde{\underline{x}}^{\underline{\mu}}) \in V_4 \oplus \underline{V}_2$, where $\widetilde{x}^\mu \in V_4$ and $\widetilde{\underline{x}}^{\underline{\mu}} \in \widetilde{MS}_p (\equiv \underline{V}_2)$, with V_4 and $\underline{V}_2$ are the 4D and 2D semi-Riemannian spaces. This extension will address the *accelerated motion* and *inertia effects*.

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A smooth embedding map, generalized for curved spaces, becomes $\widetilde{f}: \underline{V}_2 \longrightarrow V_4$ defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{\widetilde{e}}_0 = \widetilde{e}_0, \quad \underline{\widetilde{x}}^0 = \widetilde{x}^0, \quad \underline{\widetilde{e}}_1 = \vec{\widetilde{n}}, \quad \underline{\widetilde{x}}^1 = |\vec{\widetilde{x}}|, \tag{100}$$

where $\vec{\widetilde{x}} = \widetilde{e}_i\widetilde{x}^i = \vec{\widetilde{n}}|\vec{\widetilde{x}}|$ $(i = 1, 2, 3)$ (the middle letters of the Latin alphabet $(i, j, ...)$ will be reserved for space indices). We expect the notion of a general coordinate transformation should be

$$[\delta_{\epsilon_1(X)}, \delta_{\epsilon_2(X)}]V = \tfrac{1}{2}\bar{\epsilon}_2(X)\sigma^{\hat{\mu}}\epsilon_1(X)\,\widetilde{\partial}_{\hat{\mu}}V. \tag{101}$$

On the premises of (**?**), we review the accelerated motion of a particle in a new perspective of local $\widetilde{MS}_p$-SUSY transformations that a *creation* of a particle in $\underline{V}_2$ means its transition from initial state defined on V_4 into intermediate state defined on $\underline{V}_2$, while an *annihilation* of a particle in $\underline{V}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in V_4. The net result of each atomic double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$ to $\underline{V}_2$ and back is as if we had operated with *a local space-time translation* with acceleration, $\vec{a}$, on the original space V_4. Accordingly, the acceleration, $\underline{\vec{a}}$, holds in $\underline{V}_2$ at $\underline{V}_2 \rightleftharpoons V_4$. So, the accelerated motion of boson $A(\widetilde{x})$ in V_4 is a chain of its sequential transformations to the Weyl fermion $\underline{\chi}(\underline{\widetilde{x}})$ defined on $\underline{V}_2$ (accompanied with the auxiliary fields $\underline{\widetilde{F}}$) and back,

$$\to A(\widetilde{x}) \to \underline{\chi}^{(\underline{F})}(\underline{\widetilde{x}}) \to A(\widetilde{x}) \to \underline{\chi}^{(\underline{F})}(\underline{\widetilde{x}}) \to, \tag{102}$$

and the same interpretation holds for fermion $\chi(\widetilde{x})$.

The mathematical structure of a local theory of $\widetilde{MS}_p$-SUSY has much in common with those used in the geometrical framework of standard supergravity theories. Such a local SUSY can already be read off from the algebra of a global MS$_p$-SUSY (**?**) in the form

$$[\epsilon(X)Q, \bar{Q}\bar{\epsilon}(X)] = 2\epsilon(X)\sigma^{\hat{\mu}}\bar{\epsilon}(X)\widetilde{p}_{\hat{\mu}}, \tag{103}$$

which says that the product of two supersymmetry transformations corresponds to a translation in 6D X-space of which the momentum $\widetilde{p}_{\hat{\mu}} = i\widetilde{\partial}_{\hat{\mu}}$ is the generator. We expect the notion of a general coordinate transformation should be

$$[\delta_{\epsilon_1(X)}, \delta_{\epsilon_2(X)}]V = \tfrac{1}{2}\bar{\epsilon}_2(X)\sigma^{\hat{\mu}}\epsilon_1(X)\,\widetilde{\partial}_{\hat{\mu}}V. \tag{104}$$

Then for the local $\widetilde{MS}_p$-SUSY to exist it requires the background spaces $(V_4, \underline{V}_2)$ to be curved. In order to become on the same footing with $\underline{V}_2$, the V_4 refers to the accelerated proper reference frame of a particle without relation to other matter fields. This leads us to extend the concept of differential forms to superspace. A useful guide in the construction of local superspace is that it should admit rigid superspace as a limit. The superspace is a direct sum of background semi-Riemannian 4D-space and curved Master space $V_4 \oplus \widetilde{MS}_p$, with an inclusion of additional fermionic coordinates $\Theta(\underline{\theta}, \underline{\bar{\theta}})$ and $\bar{\Theta}(\underline{\theta}, \underline{\bar{\theta}})$, which are induced by the spinors $\underline{\theta}$ and $\underline{\bar{\theta}}$ referred to $\widetilde{MS}_p$.

The multiplication of two local sequential supersymmetric transformations induces the motion (a generalization of the motion (93))

$$\begin{array}{l} g(0,\, \epsilon(X),\, \bar{\epsilon}(X))\,\Omega(X^{\hat{\mu}},\, \Theta^{\hat{\alpha}},\, \bar{\Theta}_{\hat{\dot{\alpha}}} \longrightarrow \\ (X^{\hat{\mu}} + i\,\Theta^{\hat{\alpha}}\,\sigma^{\hat{\mu}}\,\bar{\epsilon}(X) - i\,\epsilon(X)\,\sigma^{\hat{\mu}}\,\bar{\Theta}_{\hat{\dot{\alpha}}},\, \Theta + \epsilon(X), \bar{\Theta} + \bar{\epsilon}(X)), \end{array} \tag{105}$$

which gives

$$\begin{array}{l} g_q(0,\, \xi(\widetilde{x}),\, \bar{\xi}(\widetilde{x}))\,\Omega_q(\widetilde{x},\, \theta,\, \bar{\theta}) \;\to \\ (\widetilde{x}^m + i\,\theta\,\sigma^m\,\bar{\xi}(\widetilde{x}) - i\,\xi(\widetilde{x})\,\sigma^m\,\bar{\theta},\;\; \theta + \xi(\widetilde{x}),\, \bar{\theta} + \bar{\xi}(\widetilde{x})), \\ \underline{g_q}(0,\, \underline{\xi}(\underline{\widetilde{x}}),\, \underline{\bar{\xi}}(\underline{\widetilde{x}}))\,\Omega_{\underline{q}}(\underline{\widetilde{x}},\, \underline{\theta},\, \underline{\bar{\theta}}) \;\to \\ (\underline{\widetilde{x}}^{\underline{m}} + i\,\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar{\xi}}(\underline{\widetilde{x}}) - i\,\underline{\xi}(\underline{\widetilde{x}})\,\sigma^{\underline{m}}\,\underline{\bar{\theta}},\;\; \underline{\theta} + \underline{\xi}(\underline{\widetilde{x}}),\, \underline{\bar{\theta}} + \underline{\bar{\xi}}(\underline{\widetilde{x}})). \end{array} \tag{106}$$

Being embedded in V_4, the $\widetilde{MS}_p$ is the *unmanifested* indispensable individual companion of a particle of interest devoid of any matter influence. While all the particles are living on V_4, their superpartners can be viewed as living on $\widetilde{MS}_p$. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The action of simple $\widetilde{MS}_p$-SG includes the Hilbert term for a *fictitious* graviton coexisting with a *fictitious* fermionic field of, so-called, gravitino (sparticle) described by the Rarita-Scwinger kinetic term. These two particles differ in their spin: 2 for the graviton, 3/2 for the gravitino. They are the bosonic and fermionic states of a gauge particle in V_4 and $\widetilde{MS}_p$, respectively, or vice versa.

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B.1 The simple $(N=1)$ $\widetilde{MS}_p$ - SG without auxiliary fields, revisited

The generalized Poincaré superalgebra for the simple $(N=1)$ $\widetilde{MS}_p$-SG reads:

$$
\begin{array}{l}
[P_{\hat a}, P_{\hat b}] = 0, \quad [S_{\hat a\hat b}, P_{\hat c}] = (\eta_{\hat a\hat c}P_{\hat b} - \eta_{\hat b\hat c}P_{\hat a}), \\
[S_{\hat a\hat b}, S_{\hat c\hat d}] = i(\eta_{\hat a\hat c}S_{\hat b\hat d} - \eta_{\hat b\hat c}S_{\hat a\hat d} + \eta_{\hat b\hat d}S_{\hat a\hat c} - \eta_{\hat a\hat d}S_{\hat b\hat c}), \\
[S_{\hat a\hat b}, Q^{\alpha}] = \frac{1}{2}(\gamma_{\hat a\hat b})^{\alpha}_{\beta}Q^{\beta}, \\
[P_{\hat a}, Q^{\beta}] = 0, \quad [Q_{\alpha}, \bar Q_{\dot\beta}] = \frac{1}{2}(\gamma^{\hat a})_{\alpha\dot\beta}P_{\hat a}.
\end{array}
\tag{107}
$$

with $(S_{\hat a\hat b})^{\hat c}_{\hat d} = i(\delta^{\hat c}_{\hat a}\eta_{\hat b\hat d} - \delta^{\hat c}_{\hat b}\eta_{\hat a\hat d})$ (**??**) a given representation of the Lorentz generators. Using (107) and a general form for gauge transformations on B^A,

$$
\delta B = \mathcal{D}\lambda = d\lambda + [B, \lambda], \tag{108}
$$

with

$$
\lambda = \rho^{\hat a}P_{\hat a} + \tfrac{1}{2}\kappa^{\hat a\hat b}S_{\hat a\hat b} + \bar Q\varepsilon, \tag{109}
$$

we obtain that the $(e^{\hat a}, \omega^{\hat a\hat b}, \Psi)$ transform under Poincaré translations as

$$
\delta e^{\hat a} = \mathcal{D}\rho^{\hat a}, \quad \delta\omega^{\hat a\hat b} = 0, \quad \delta\Psi = 0; \tag{110}
$$

under Lorentz rotations as

$$
\delta e^{\hat a} = \kappa^{\hat a}_{\hat b}\delta e^{\hat b}, \quad \delta\omega^{\hat a\hat b} = -\mathcal{D}\kappa^{\hat a\hat b}, \quad \delta\Psi = \tfrac{1}{4}\kappa^{\hat a\hat b}\gamma_{\hat a\hat b}\Psi; \tag{111}
$$

and under supersymmetry transformation as

$$
\delta e^{\hat a} = \tfrac{1}{2}\bar\varepsilon\gamma^{\hat a}\Psi, \quad \delta\omega^{\hat a\hat b} = 0, \quad \delta\Psi = \mathcal{D}\varepsilon. \tag{112}
$$

In first-order formalism, the gauge fields $(e^{\hat a}, \omega^{\hat a\hat b}, \Psi)$, (with $\Psi = (\psi, \underline{\psi})$ a two-component Majorana spinor) are considered as an independent members of multiplet in the adjoint representation of the Poincaré supergroup of $D = 6$ $((3+1), (1+1))$ simple $(N=1)$ $\widetilde{MS}_p$-SG with the generators $(P_{\hat a}, S_{\hat a\hat b}, Q^{\alpha})$. Unless indicated otherwise, henceforth the world indices are kept implicit without ambiguity. The operators carry Lorentz indices not related to coordinate transformations. The Yang-Mills connection for the Poincare´ supergroup is given by

$$
B = B^A T_A = e^{\hat a}P_{\hat a} + \tfrac{1}{2}i\omega^{\hat a\hat b}S_{\hat a\hat b} + \Psi\bar Q. \tag{113}
$$

The field strength associated with connection B is defined as the Poincaré Lie superalgebra-valued curvature two-form R^A. Splitting the index A, and taking the $\Theta = \bar\Theta = 0$ component of R^A, we obtain

$$
\begin{array}{l}
R^{\ \hat a\hat b}(\omega) = d\omega^{\hat a\hat b} - \omega^{\hat a}{}_{\hat c}\omega^{\hat c\hat d}, \\
\tilde T^{\hat a} = T^{\hat a} - \frac{1}{2}\bar\Psi\gamma^{\hat a}\Psi, \quad \rho = \mathcal{D}\Psi,
\end{array}
\tag{114}
$$

where $\gamma^{\hat a} = (\gamma^a, \sigma^{\underline{a}})$, $R^{\ \hat a\hat b}(\omega)$ is the Riemann curvature in terms of the spin connection $\omega^{\hat a\hat b}$, and the generalized Weyl lemma (see App./(4)) requires that the, so-called, supertorsion $\tilde T^{\hat a}$ be inserted. The solution $\omega(e)$ satisfies the tetrad postulate that the completely covariant derivative of the tetrad field vanishes, therefore $R^{\hat a\hat b}(\omega) = R(\omega)e^{\hat a}e^{\hat b}$.

For the bosonic part of the gauge action (graviton of spin 2) of simple $\widetilde{MS}_p$-SG it then seems appropriate to take the generalized Hilbert action with $e = det\, e^{\hat a}{}_{\mu}(X)$. While the fermionic part of the standard gauge action (garvitino of spin 3/2), which has positive energy, is the Rarita-Schwinger action. The full nonlinear gravitino action in curved space then should be its extension to curved space, which can be achieved by inserting the Lorentz covariant derivative $\mathcal{D}\Psi = d\Psi + \frac{1}{2}\omega^{\ \hat a\hat b}\gamma_{\hat a\hat b}\Psi$. In both parts, the spin connection is considered a dependent field, otherwise in the case of an independent spin connection ω, the action will be invariant under diffeomorphism, and under local Lorentz rotations, but it will be not invariant under the neither the Poincaré translations nor the supersymmetry. In the case if spin connection is independent, we should have under the local Poincaré translations

$$
\begin{array}{l}
\delta\hat{\mathcal{L}}_{pt} = \delta\left(\varepsilon_{\hat a\hat b\hat c\hat d}e^{\hat a}e^{\hat b}R^{\hat c\hat d} + 4\bar\Psi\gamma_{\hat 5}e^{\hat a}\gamma_{\hat a}\mathcal{D}\Psi\right) \\
= 2\varepsilon_{\hat a\hat b\hat c\hat d}R^{\hat a\hat b}\tilde T^{\hat c}\rho^{\hat d} + \text{surf. term},
\end{array}
\tag{115}
$$

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and under local supersymmetry transformations

$$\delta\hat{\mathcal{L}}_{SUSY} = -4\bar{\varepsilon}\gamma_{\hat{5}}\gamma_{\hat{a}}\mathcal{D}\Psi\tilde{T}^{\hat{a}} + \text{surf. term.} \tag{116}$$

The invariance of the action then requires the vanishing of the supertorsion $\tilde{T}^{\hat{a}} = 0$, which means that the connection is no longer an independent variable. So that the starting point of our approach is the action of a simple $\widetilde{MS}_p$-SG theory written in 'two in one'-notation (1), which is invariant under the local supersymmetry transformation (112), where the Poincaré superalgebra closes off shell without the need for any auxiliary fields:

$$\mathcal{L}_{MS-SG} = \varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}e^{\hat{a}}e^{\hat{b}}R^{\hat{c}\hat{d}}(\omega) + 4\bar{\Psi}\gamma_{\hat{5}}e^{\hat{a}}\gamma_{\hat{a}}\mathcal{D}\Psi. \tag{117}$$

This is the sum of bosonic and fermionic parts with the same spin connection, where $\gamma_{\hat{a}} = (\gamma_a \oplus \sigma_{\underline{a}})$, $\gamma_{\hat{5}} = (\gamma_5 \oplus \gamma_{\underline{5}})$, $\gamma_{\underline{5}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is given in the chiral or Weyl representations, i.e. in the irreducible 2-dimensional spinor representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$, since two-component formalism works for a Weyl fermion. This is indispensable in order to solve algebraical constraints in superspace because they can be used as building blocks of any fermion field (van Nieuwenhuizen, 1981). In this representation, action of projection matrices $L = (1/2)(1 + \gamma_{\underline{5}})$ and $R = (1/2)(1 - \gamma_{\underline{5}})$ on a Dirac fermion leads to zero two lower components of the left-handed spinor and zero two upper components of the right-handed spinor, respectively. The two-component notation described above essentially does away with the vanishing components explicitly and deals only with the non-trivial ones. Taking into account that $g_{\hat{\mu}\hat{\nu}} = \eta_{\hat{a}\hat{b}}e_{\hat{\mu}}{}^{\hat{a}}e_{\hat{\nu}}{}^{\hat{b}}$ and $\gamma_{\hat{\mu}} = e_{\hat{\mu}}{}^{\hat{a}}\gamma_{\hat{a}}$, with $\eta_{\hat{a}\hat{b}} = (\eta_{ab} \oplus \underline{\eta}_{\underline{ab}})$ related to the tangent space, where $\eta_{ab} = diag(+1, -1, -1, -1)$ and $\underline{\eta}_{\underline{ab}} = diag(+1, -1)$, we can recast the generalized bosonic and fermionic actions given in (117), respectively, in the forms

$$\mathcal{L}^{(2)} = -\tfrac{1}{4}\sqrt{g}R(g, \Gamma) = -\tfrac{1}{4}eR(e, \omega), \tag{118}$$

and

$$\mathcal{L}^{(3/2)} = 4\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\bar{\Psi}_{\hat{\mu}}\gamma_{\hat{5}}\gamma_{\hat{\nu}}\mathcal{D}_{\hat{\rho}}\Psi_{\hat{\sigma}}. \tag{119}$$

Here we taken into account that $\mathcal{D}_{\hat{\rho}}\Psi_{\hat{\sigma}}$ is the curl due to the ε-symbol, and as far as $\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ is the density (which always equals $0, \pm 1$), so there is no need to put the density e in front of fermionic part.

The accelerated motion of a particle is described by the parameter $\epsilon = \epsilon(X^{\hat{\mu}})$ in (105) of local SUSY, which depends explicitly on $X^{\hat{\mu}} = (\widetilde{x}^{\mu}, \underline{\widetilde{x}}^{\underline{\mu}})$, where $\widetilde{x}^{\mu} \in V_4$ and $\underline{\widetilde{x}}^{\underline{\mu}} \in \underline{V}_2$. To be specific, let us focus for the motion on the simple case of a peculiar anticommuting spinors $(\underline{\xi}(\underline{\widetilde{x}}), \underline{\bar{\xi}}(\underline{\widetilde{x}}))$ and $(\xi(\widetilde{x}), \bar{\xi}(\widetilde{x}))$ defined as

$$\begin{aligned} \underline{\xi}^{\alpha}(\underline{\widetilde{x}}) &= i\,\tfrac{\underline{\tau}(\underline{\widetilde{x}})}{2}\,\underline{\theta}^{\alpha}, \quad & \underline{\bar{\xi}}_{\dot{\alpha}}(\underline{\widetilde{x}}) &= -i\,\tfrac{\underline{\tau}^*(\underline{\widetilde{x}})}{2}\,\underline{\bar{\theta}}_{\dot{\alpha}}, \\ \xi^{\alpha}(\widetilde{x}) &= i\tfrac{\tau(\widetilde{x})}{2}\theta^{\alpha}, \quad & \bar{\xi}_{\dot{\alpha}}(\widetilde{x}) &= -i\tfrac{\tau^*(\widetilde{x})}{2}\bar{\theta}_{\dot{\alpha}}. \end{aligned} \tag{120}$$

Here the real parameter $\tau(\widetilde{x}) = \tau^*(\widetilde{x}) = \underline{\tau}(\underline{\widetilde{x}}) = \underline{\tau}^*(\underline{\widetilde{x}})$ can physically be interpreted as the *atomic duration time* of double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$, i.e. the period of superoscillations. In this case, the *atomic displacement* caused by double transition reads

$$\Delta\underline{\widetilde{x}}_{(a)} = \underline{\widetilde{e}}_{\underline{m}}\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)} = \underline{\widetilde{u}}\tau(\underline{\widetilde{x}}), \tag{121}$$

where, according to the motion (93), the components $\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)}$ are written

$$\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)} = \underline{\widetilde{v}}^{\underline{m}}\,\tau(\underline{\widetilde{x}}) = i\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar{\xi}}(\underline{\widetilde{x}}) - i\underline{\xi}(\underline{\widetilde{x}})\,\sigma^{\underline{m}}\,\underline{\bar{\theta}}. \tag{122}$$

The corresponding acceleration reads

$$a^{(\pm)} = i\underline{\theta}\,\sigma^{(\pm)}\,\frac{d^2\underline{\bar{\xi}}}{d\underline{\widetilde{s}}^2} - i\frac{d^2\underline{\xi}}{d\underline{\widetilde{s}}^2}\,\sigma^{(\pm)}\,\underline{\bar{\theta}}, \tag{123}$$

where $\sigma^{(\pm)} = \frac{1}{\sqrt{2}}(\sigma^{\underline{0}} \pm \sigma^{\underline{1}}) = \frac{1}{\sqrt{2}}(\sigma^0 \pm \sigma^3)$ and $d\underline{\widetilde{s}}^2 = d\underline{\widetilde{x}}^{(+)}d\underline{\widetilde{x}}^{(-)}$. By virtue of (120), the (123) is reduced to

$$a^{(\pm)} = v_c^{(\pm)}\,\frac{d^2\tau}{d\underline{\widetilde{s}}^2}, \tag{124}$$

where $v_c^{(\pm)} \equiv (\underline{\theta}\,\sigma^{(\pm)}\underline{\bar{\theta}})$.

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In Van der Warden notations for the Weyl two-component formalism $(\underline{\bar{\theta}}_{\dot{\alpha}})^* = \underline{\theta}_\alpha$ and $\underline{\bar{\theta}}_{\dot{\alpha}} = (\underline{\theta}_\alpha)^*$, the (124) gives

$$\begin{array}{l} \underset{\widetilde{}}{a} = \sqrt{2}(a^{(+)}a^{(-)})^{1/2} = \sqrt{2}\underline{v}_c \frac{d^2\tau}{d\underset{\widetilde{}}{s}^2}, \\ \underline{v}_c = (v_c^{(+)}v_c^{(-)})^{1/2} = \sqrt{2}(\underline{\theta}_1\,\underline{\bar{\theta}}_1\underline{\theta}_2\,\underline{\bar{\theta}}_2)^{1/2}, \end{array} \quad (125)$$

with $v_c^{(+)} = \sqrt{2}(\underline{\theta}_1\,\underline{\bar{\theta}}_1)$ and $v_c^{(-)} = \sqrt{2}(\underline{\theta}_2\,\underline{\bar{\theta}}_2)$. The acceleration will generally remain a measure of the velocity variation over proper time $(\underset{\widetilde{}}{s})$. The (124) and (125) yield

$$\begin{array}{l} v^{(\pm)} = v_c^{(\pm)}\left(\frac{d\tau}{d\underset{\widetilde{}}{s}} + 1\right), \\ \underset{\widetilde{}}{v} = \sqrt{2}(v^{(+)}v^{(-)})^{1/2} = \sqrt{2}\underline{v}_c\left(\frac{d\tau}{d\underset{\widetilde{}}{s}} + 1\right). \end{array} \quad (126)$$

The spinors $\theta(\underline{\theta},\underline{\bar{\theta}})$ and $\bar{\theta}(\underline{\theta},\underline{\bar{\theta}})$ satisfy the embedding map (100), namely $\Delta\underset{\widetilde{}}{x}^{\underline{0}} = \Delta\tilde{x}^0$ and $(\Delta\underset{\widetilde{}}{x}^{\underline{1}})^2 = (\Delta\vec{\tilde{x}})^2$, so from (93) we obtain

$$\begin{array}{l} \underline{\theta}\,\sigma^0\,\underline{\bar{\xi}} - \underline{\xi}\,\sigma^0\,\underline{\bar{\theta}} = \theta\,\sigma^0\,\bar{\xi} - \xi\,\sigma^0\,\bar{\theta}, \\ (\underline{\theta}\,\sigma^3\,\underline{\bar{\xi}} - \underline{\xi}\,\sigma^3\,\underline{\bar{\theta}})^2 = (\theta\,\vec{\sigma}\,\bar{\xi} - \xi\,\vec{\sigma}\,\bar{\theta})^2. \end{array} \quad (127)$$

Denote

$$\begin{array}{l} \underline{v}^{\underline{0}}_{(c)} = \frac{1}{\sqrt{2}}\left(v_c^{(+)} + v_c^{(-)}\right) = (\underline{\theta}\,\underline{\bar{\theta}}), \\ \underline{v}^{\underline{1}}_{(c)} = \frac{1}{\sqrt{2}}\left(v_c^{(+)} - v_c^{(-)}\right) = (\underline{\theta}_1\,\underline{\bar{\theta}}_1 - \underline{\theta}_2\,\underline{\bar{\theta}}_2), \end{array} \quad (128)$$

then

$$\begin{array}{l} \theta_1(\underline{\theta},\,\underline{\bar{\theta}}) = \frac{1}{2}\left[\left(\underline{v}^{\underline{0}}_{(c)} + \sqrt{\frac{2}{3}}\underline{v}^{\underline{1}}_{(c)}\right)^{1/2} + \left(\underline{v}^{\underline{0}}_{(c)} - \sqrt{\frac{2}{3}}\underline{v}^{\underline{1}}_{(c)}\right)^{1/2}\right], \\ \theta_2(\underline{\theta},\,\underline{\bar{\theta}}) = \frac{1}{2}\left[\left(\underline{v}^{\underline{0}}_{(c)} + \sqrt{\frac{2}{3}}\underline{v}^{\underline{1}}_{(c)}\right)^{1/2} - \left(\underline{v}^{\underline{0}}_{(c)} - \sqrt{\frac{2}{3}}\underline{v}^{\underline{1}}_{(c)}\right)^{1/2}\right]. \end{array} \quad (129)$$

The dynamical aspects of particle mechanics involve derivatives with respect to proper time along the particle worldline, which is the line element written in frame (25):

$$ds^2 = \eta_{\hat{a}\hat{b}}\dot{e}^{\hat{a}}\dot{e}^{\hat{b}} = \eta_{\hat{a}\hat{b}}\dot{e}^{\hat{a}}{}_{\hat{\mu}}\dot{e}^{\hat{b}}{}_{\hat{\mu}}dX^{\hat{\mu}}dX^{\hat{\nu}} \equiv \eta_{\hat{\mu}\hat{\nu}}dX^{\hat{\mu}}dX^{\hat{\nu}}. \quad (130)$$

A worldline C of a particle, parametrized by proper time as $C(s) = X^{\hat{\mu}}(s)$, will have as six-velocity the vector of components $u^{\hat{\mu}} = dX^{\hat{\mu}}/ds$ and $u^{\hat{a}} = \dot{e}^{\hat{a}}_{\hat{\mu}}u^{\hat{\mu}}$, which are the particle velocity along this curve respectively in the holonomic and anholonomic bases in the X-space.

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