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Spotedness of most active flare stars detected by TESS

A.A. Akopian*

Byurakan Astrophysical Observatory, Armenia

Abstract

A new method has been proposed to determine the coverage of starspots by using periodic functions related to flare frequency. The distribution of starspots on flare stars is modeled through a von Mises circular distribution, with parameters derived from the corresponding flare frequency function. Estimates of spot coverage for the stars studied have been obtained.

Keywords: *flare stars, starspots*

1. Introduction

The idea of spots existing on stars was proposed by the French astronomer Ismaël Bouillaud (latinized as Bullialdus) in 1667 (Bullialdi, 1667) . He suggested that the variability of the star Mira (o Cet), discovered by David Fabricius in 1596, could be explained by the presence of cold, dark spots on the star's surface. Although Bouillaud's logically sound hypothesis was later proven incorrect for Mira, it found application much later, after about 300 years. In 1947, Kron (1947) reported the possible detection of spots on the surface of the star AR Lacertae B. Subsequent studies by Kron and others laid the foundation for the introduction of a new type of variable star - rotating variable stars.

The study of flare stars and closely related rotating variable stars, such as BY Dra and RS CVn, has reached a new level thanks to observational data from space telescopes such as Kepler and TESS. Data from Kepler and TESS indicate that *i*) flare stars also exhibit rotational variability, which is due to their rotation and nonuniform spot distribution across their surfaces, and *ii*) the flare-productive active regions of stars are closely associated with stellar spots. Results from the author's previous works (Akopian, 2015, 2019, 2023) suggest that the axial rotation of a star, due to the uneven distribution of spots across its surface, modulates not only the observed radiation flux but also the frequency of observed flares. Therefore, it is expected that there is a close relationship between the qualitative and quantitative characteristics of stellar spots and the flare activity of stars.

In (Akopian, 2023) data from the orbital observatory TESS (Yang et al., 2022) have been utilized to study the most active 21 flare stars , which exhibited 100 or more flares each from July 2018 to October 2020. The axial rotation periods of them were calculated (for nine stars for the first time) using two complementary algorithms: the Lomb-Scargle and the BLS (Box Least Squares).

Using the BLS algorithm, the duration of the "plateau" during the deep minimum phase was determined for all the stars, providing information on the compactness of the starspot distribution along the stellar longitude. In particular, a relationship was identified between the parameter k of the periodic functions of flare frequency and the relative duration of the "plateau" during the stars' deep minimum phases.

The calculated axial rotation periods were then used to determine the parameters of the periodic or cyclical variability in the flare frequencies of the stars. Corresponding periodic functions of flare frequency were obtained, revealing that the periods of these flare frequency functions closely matched the axial rotation periods of the stars. The expected (theoretical) phase distributions of the flares were constructed and compared with the observed flare distributions for the same time frame. This comparison, conducted using the chi-squared criterion, confirmed the periodicity of the flare frequency for all stars, without exception. It has been pointed out that the similarity of the periodic flare frequency functions to the von Mises angular distribution function is likely not coincidental.

*aakopian57@gmail.com

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This paper aims to determine the spot coverage of flare stars and the rotating variable with flare activity through the periodic flare frequency function (PFFF) determined in (Akopian, 2023).

2. Problem Statement

This work is based on the following assumptions and premises:

1) Analysis of Kepler and TESS photometric data demonstrates the presence of rotational modulation in flare stars. The observed variability aligns with a model that includes stellar rotation and an uneven distribution of starspots.

2) The basic assumption is Ambartsumian's idea that the random and independent nature of flares allows us to represent the sequence of stellar flares as a random Poisson process (Ambartsumian, 1969). Since the flare frequency is modulated by the star's rotation, it is necessary to treat the flare sequence as a Poisson process with a cyclical/periodic parameter. In previous works (Akopian, 2015, 2019, 2023) , a function of the form

$$\lambda\left(t\right)=\frac{\lambda exp\left[ksin\left(\omega_0 t+\vartheta\right)\right]}{I_0\left(k\right)}\ \left(k\geq 0,\ \omega_0\geq 0,\ 0\leq\vartheta\leq 2\pi,\ \lambda>0\ \right), \tag{1}$$

where $k,\ \omega_0,\ \vartheta,\ \lambda$ are constants and $I_0\left(k\right)=\int\limits_0^{2\pi} exp\left[ksin\left(u\right)\right]du$ where $I_0\left(k\right))$ is the modified Bessel function of the first kind of order 0, was successfully used to describe a Poisson process with a periodic parameter. This function, proposed by Kutoyants (1980) provides a variety of periodic functions depending on the value of parameter k.

3) According to (Yang et al., 2017), flare activity is positively correlated with the size of the spots ((see Figure 10 in Yang et al., 2017), which suggests a similar correlation between the spot size and the number of flares. Since the Poisson process parameter $\lambda\left(t\right)$ represents the average number of events (in this case, flares) per unit of time, it follows that the spot coverage parameter of a star can be determined by assessing the $\lambda\left(t\right)$ parameter.

4) Considering the known properties of trigonometric functions, we observe that the function $\lambda\left(t\right)$ is essentially identical to the von Mises distribution function:

$$M\left(x|\mu,\,k\right)=\frac{exp\left[kcos\left(x-\mu\right)\right]}{2I_0\left(k\right)} \tag{2}$$

where μ is the mean (simultaneously the mode and median) direction of the distribution, k is the concentration measure, $1/k$ is the variance, and $1-I_1\left(k\right)/I_0\left(k\right)$ is the circular variance. This was first pointed out in (Akopian, 2023). The von Mises distribution, also known as the circular normal distribution or Tikhonov distribution, is widely used for statistical analysis of angular (circular) data and serves as a good approximation for the wrapped normal distribution (the circular analogue of the normal distribution).

Thus, to determine the spot distribution on the hemisphere of the star facing the observer, it is necessary to first determine the PFFF of the star and represent it as a von Mises distribution function, which, due to the positive correlation between flare activity and spot size (Yang et al., 2017), will simultaneously serve as a distribution function for the effective area of spots on the star's hemisphere facing the observer, up to a constant factor.

Second, the obtained PFFF must be related to the observed light curves. This can be done by comparing the minima and maxima of the PFFF and the light curve. At the maximum of the light curve, the least spot-covered hemisphere (out of all possible) with minimal flaring activity is observed, and conversely, at the minimum of the light curve, the most spot-covered hemisphere with maximal flare activity is observed. Thus, the task is reduced to representing the ratio of fluxes at the maximum and minimum through the analogous ratio for $\lambda\left(t\right)$, which is equal

$$\frac{\lambda_{max}}{\lambda_{min}}=exp\left(2k\right).$$

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For large angular inclinations of the star's axis (60°–90°) relative to the line of sight, these hemispheres, due to the symmetry of the above functions, can be considered diametrically opposite, which allows for an estimate of the overall spot coverage of the star.

3. Determining the Spot Coverage Using PFFF and the von Mises Distribution

In this section, the three main steps for estimating the coverage of starspots (spottedness) are described. The first subsection presents a concise overview of the determination of the PFFF function. The second subsection presents how to transform PFFF functions into a circular von Mises distribution. This step seems too important because von Mises distribution is a well-known and well-studied distribution that provides many useful statistical tools.

3.1. Determining the PFFF

Let's consider the registration of n flares from a star in the time interval $(0, T)$ as a statistical event. The likelihood function of this event, given the chosen form of the PFFF (1), is represented as:

$$\exp\left(-\frac{\lambda T}{2\pi}\right)\left(\frac{\lambda}{I_0(k)}\right)^n exp\left[k\sum_{i=1}^{n} sin\left(\omega_0 t_i+\vartheta\right)\right] \tag{3}$$

where t_i are the flare moments of the star.

From the problem statement, it follows that the angular frequency ω_0 should be equal or close to the rotational frequency of the star, i.e. $\omega_0 \approx 2\pi/P_r$. To estimate the maximum likelihood for the other parameters, we have (Kutoyants, 1980):

$$\hat{\lambda}=\frac{2\pi n}{T} \tag{4}$$

$$\tan\left(\hat{\vartheta}\right)=\frac{\sum_{i=1}^{n} cos\left(\omega_0 t_i\right)}{\sum_{i=1}^{n} sin\left(\omega_0 t_i\right)} \tag{5}$$

$$\frac{dlogI_0\left(k\right)}{dk}|_{k=\hat{k}}=\frac{\sqrt{\left(\sum_{i=1}^{n} sin\left(\omega_0 t_i\right)\right)^2+\left(\sum_{i=1}^{n} cos\left(\omega_0 t_i\right)\right)^2}}{n} \tag{6}$$

Here, the time interval T should be a multiple of the period $2\pi/\omega_0$, and $\hat{\vartheta}$ is selected from the roots of the second equation according to the following rule:

$$\hat{\vartheta}=\begin{cases} \hat{\vartheta},\ if\ sgn\left(\sum\limits_{i=1}^{n} sin\left(\omega_0 t_i\right)\right)=sgn\left(\sum\limits_{i=1}^{n} cos\left(\omega_0 t_i\right)\right) \\ \hat{\vartheta}+2\pi,\ if\ \hat{\vartheta}<0\ and\ sgn\left(\sum\limits_{i=1}^{n} sin\left(\omega_0 t_i\right)\right)=sgn\left(\sum\limits_{i=1}^{n} cos\left(\omega_0 t_i\right)\right) \\ \hat{\vartheta}+\pi,\ \ in\,other\,cases. \end{cases} \tag{7}$$

By setting ω_0 and determining the parameters $\hat{\lambda}$, $\hat{\vartheta}$, $\hat{k}$, the function λ(t) is fully determined. The likelihood measure for the hypothesis of periodicity in the frequency of flares is the logarithm of the likelihood ratio between two Poisson processes: one with a periodic parameter (hypothesis H_1) and the other with a stationary parameter (hypothesis H_0):

$$\ln L_{H_1/H_0}=k\sum_{i=1}^{n} sin\left(\omega_0 t_i+\vartheta\right)-nln\left(I_0\left(k\right)\right)$$

This implies, as noted in (Lewis, 1970), that the hypothesis of stationarity can be confidently rejected if the logarithm of the likelihood ratio:

$$\ln L_{H_1/H_0}>3 \tag{8}$$

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	TIC	P_f	k	ϑ	$\lambda_{max}/\lambda_{min}$	lnL_{H_1/H_0}	χ^2	μ	μ_0
1	25118964	1.008	0.489	2.075	2.661	5.901	3.155	1.444	1.955
2	141807839	0.813	0.324	1.619	1.912	3.142	1.533	3.968	4.016
3	141914082	2.700	0.321	4.786	1.902	3.017	1.890	5.857	2.876
4	141975926	1.155	0.508	5.579	2.764	7.578	1.131	5.482	3.207
5	150359500	1.057	0.398	0.342	2.215	8.870	0.952	0.739	5.793
6	167344043	2.662	0.393	2.330	2.191	6.167	4.385	0.655	1.415
7	220432563	1.134	0.540	5.396	2.925	7.788	2.834	5.865	3.407
8	277298771	1.156	0.434	5.988	2.382	4.776	4.720	4.338	2.473
9	358176584	0.921	0.326	2.487	1.920	3.207	2.475	4.513	6.233
10	359313701	1.765	0.428	1.244	2.351	5.301	1.985	1.705	1.378
11	364588501	2.252	0.274	0.131	1.729	3.548	2.641	4.492	3.052
12	373431012	0.510	0.332	4.579	1.941	3.016	2.185	4.504	1.229
13	382258517	3.295	0.363	2.238	2.066	3.916	2.470	1.785	2.452
14	441734910	1.359	0.461	1.376	2.513	5.665	1.014	4.395	4.511

Table 1. Parameters of PFFF and von Mises distribution for selected stars

Agreement with observational data is checked by comparing the expected (theoretical) distribution of flares with the corresponding observed distribution using the chi-squared test (Pearson's goodness-of-fit test). A more detailed description of the method is provided in works (Akopian, 2019, 2023). This article uses the previously determined PFFFs in work (Akopian, 2023) for the stars that meet the condition (8). The main parameters of PFFFs are presented in columns 3-9 of the Table 1.

3.2. Representation as a von Mises Circular Distribution

To represent the results in terms of a von Mises distribution, we need only (parameter of concentration k remains unchanged) to estimate the mean $\hat{\mu}$ of the distribution as follows:

$$\tan(\hat{\mu}) = \frac{\sum_{i=1}^{n} sin(\omega_0 t_i)}{\sum_{i=1}^{n} cos(\omega_0 t_i)} \tag{9}$$

Thus, considering equation (5) $\tan\left(\hat{\vartheta}\right) = 1/tan(\hat{\mu})$. From the roots of equation (9), $\hat{\mu}$ is chosen according to the following rule:

$$\hat{\mu} = \begin{cases} \hat{\mu}, \quad if \quad \left(\sum_{i=1}^{n} cos(\omega_0 t_i)\right) > 0 \\ \hat{\mu} + \pi, \quad if \quad \left(\sum_{i=1}^{n} sin(\omega_0 t_i)\right) \geq 0 \quad and \quad \left(\sum_{i=1}^{n} cos(\omega_0 t_i)\right) < 0 \\ \hat{\mu} - \pi, \quad if \quad \left(\sum_{i=1}^{n} sin(\omega_0 t_i)\right) < 0 \quad and \quad \left(\sum_{i=1}^{n} cos(\omega_0 t_i)\right) < 0 \end{cases} \tag{10}$$

Using the relationships in (5), (7), (9), and (10) , one can establish a connection between the parameters $\hat{\mu}$ and $\hat{\vartheta}$. The main parameters $\hat{\mu}, \hat{\mu_0}$ of von Mises circular distribution presented in 9th and 10th columns of Table 1 , where $\hat{\mu_0}$ is the first bin start angle (in radians).

In Fig. 1 an example of a representation of PFFF in terms of a von Mises circular distribution is presented. On the right panel of the figure expected (theoretical) phase distribution (black bars) of flares versus observed (black dots) are given for the star TIC 358176584. On the left panel observed distribution of flares (red bars) is presented as a von Mises distribution, where the red dots around the circle indicate flares, the blue stem is the mean direction of distribution $\hat{\mu}$, blue dot - the first bin start angle $\hat{\mu}_0$. The sequence of bars from left to right on the left panel matches the clockwise order of the right panel.

3.3. Determining Spot Coverage

As noted in Section 2, the ratio of the registered flux at the minimum to the maximum flux can be expressed through the PFFF parameters, and the spot coverage of the star can be determined in this way. The relationship is as follows:

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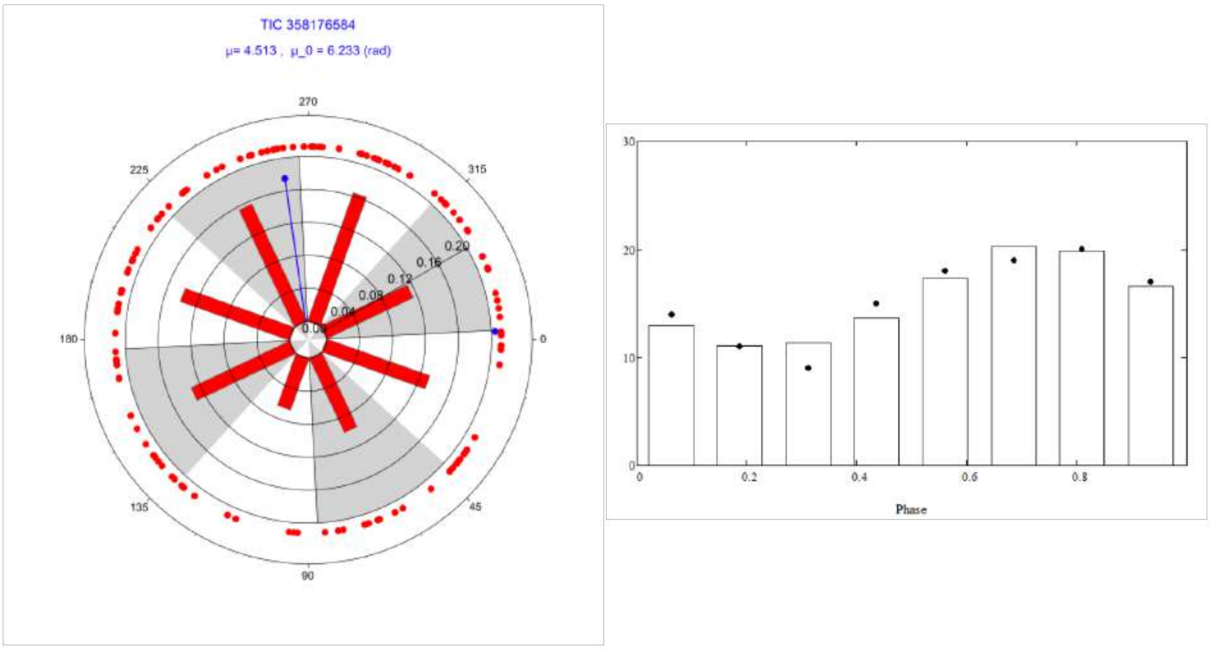

Figure 1. Phase distribution of flares/starspots

$$\frac{F_{min}}{F_{max}} = \frac{L_{st}S_{st} - S_{spmin}\left(L_{st} - L_{sp}\right)}{L_{st}S_{st} - S_{spmax}\left(L_{st} - L_{sp}\right)}] \tag{11}$$

where F_{min}, F_{max} are the registered fluxes, L_{st}, L_{sp} are the average surface brightnesses of the spotless star surface and the starspots, respectively, S_{st} is the area of the hemisphere of the star in projection, and S_{spmin}, S_{spmax} are the areas occupied by spots on this surface at the minimum and maximum fluxes.

The above-mentioned relationship between spot size and flare activity (Yang et al., 2017) can be written as:

$$\frac{S_{spmin}}{S_{spmax}} = \frac{\lambda_{max}}{\lambda_{min}} = e^{2k} \tag{12}$$

Introducing the notation $F_{min}/F_{max} \equiv F_r$, $S_{spmax/}S_{st} \equiv A$, $L_{sp}/L_{st} \equiv L_r$ after straightforward transformations, we can obtain the following expression for the spot coverage parameter A.

$$A = \frac{1 - F_r}{\left(e^{2k} - F_r\right)\left(1 - L_r\right)} \tag{13}$$

In the case where the radiation from the spots is neglected, $L_r = 0$, and thus:

$$A = \frac{1 - F_r}{e^{2k} - F_r} \tag{14}$$

In this case, the spot coverage parameter can only be estimated from below. In the approximation of spot radiation, analogous to the Sun, we can assume $L_r = 0.2 \div 0.3$, which leads to an increase in the estimate from 25% to 40%. A slight increase could also result from considering edge effects, such as the limb darkening of the stars. By definition, A is the ratio of the spot surface area to the surface area of the minimally spotted hemisphere of the star in projection. For the maximally spotted hemisphere, this ratio is increased by a factor of e^{2k}. The overall spot coverage of the star can only be estimated in cases where these hemispheres are opposite each other, which occurs when the star's rotation axis is tilted by approximately 90°. In this case, the total spot coverage parameter A_{total} is equal to:

$$A_{total} = \frac{A\left(1 + e^{2k}\right)}{2} \tag{15}$$

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-	TIC	F_r	A	Ae^{2k}	A_{total}	Sector
1	25118964	0.961	0.023	0.061	0.042	2
2	141807839	0.971	0.031	0.060	0.046	2
3	141914082	0.953	0.049	0.094	0.072	5
4	141975926	0.957	0.024	0.066	0.045	1
5	150359500	0.961	0.031	0.069	0.050	4
6	167344043	0.889	0.085	0.187	0.136	2
7	220432563	0.961	0.200	0.058	0.039	27
8	277298771	0.961	0.028	0.066	0.047	2
9	358176584	0.954	0.048	0.091	0.068	1
10	359313701	0.985	0.011	0.026	0.018	15
11	364588501	0.979	0.028	0.038	0.048	3
12	373431012	0.977	0.024	0.035	0.046	6
13	382258517	0.988	0.011	0.017	0.023	3
14	441734910	0.945	0.035	0.088	0.061	15

Table 2. Spottedness of stars

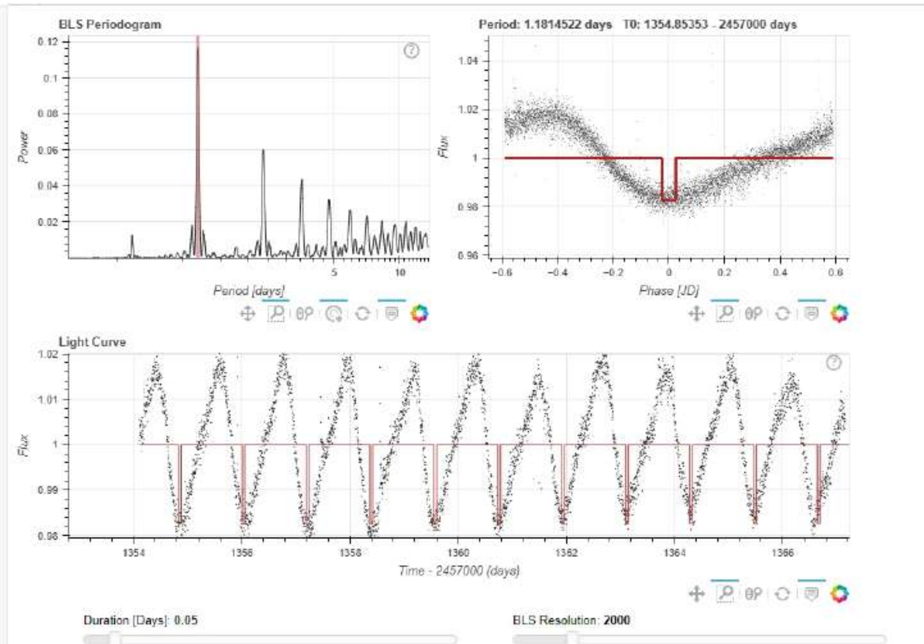

Figure 2. Illustration of the definition of the F_r ratio for the star TIC 27729877, TESS sector 2

Using the obtained expressions, the ratio F_r and the spottedness of the studied stars were determined (Table 2). The most regular light curves were selected to determine the ratio F_r. For this analysis, data from the Science Processing Operations Center (SPOC) with two-minute exposures were utilized. Column 7 ('Sector') indicates the TESS observation sector from which these light curves were obtained. An illustration is given in Fig. 2.

4. Conclusions

A novel approach has been introduced to evaluate the coverage of starspots by using periodic functions that are linked to flare frequency. The spatial distribution of starspots on flare-active stars is modeled using a von Mises circular distribution, with parameters obtained from the related flare frequency function. As a result, the selected stars' spot coverage estimates have been calculated.

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