Combining Bispectral Analysis with the Levenberg–Marquardt Algorithm for Enhanced Non-linear Signal Processing in Astronomy

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Abstract

The understanding of intricate, non-linear phenomena within astronomical data is pivotal for the progression of our knowledge of the universe. This paper introduces an innovative method that merges bispectral analysis with the Levenberg–Marquardt (LM) algorithm for processing astronomical signals. The LM algorithm is renowned for its effectiveness in non-linear parameter estimation and model fitting. At the same time, bispectral analysis is adept at identifying and measuring non-linear interactions and phase coupling within signals. By integrating these two techniques, our approach facilitates a comprehensive analysis of astronomical signals, encompassing both modeled and unmodeled non-linearities. This integrated methodology is particularly advantageous in fields such as the exploration of pulsars, binary star systems, exoplanet detection, and active galactic nuclei, where non-linear dynamics exert significant influence. The findings illustrate that this collaborative approach amplifies the identification and delineation of non-linear processes, leading to more precise models and profound insights into complex astronomical phenomena.

Keywords: *bispectral analysis, Doppler effect, Levenberg–Marquardt algorithm*

1. Introduction

Astronomical signal processing frequently involves the analysis of complex, non-linear phenomena that arise from the intricate interactions between celestial objects and their environments. Signals originating from sources such as pulsars, black holes, and stellar oscillations often contain subtle modulations and harmonics intricately embedded within background noise. To effectively uncover the hidden dynamics of these signals, the application of advanced processing techniques is crucial.

Bispectral analysis, recognized as a higher-order spectral method, has emerged as a powerful tool for identifying non-linearities and phase coupling in signals. Unlike traditional Fourier-based methods, bispectral analysis emphasizes the interrelationships between frequency components, thereby revealing interactions that linear techniques might overlook. However, extracting precise information from bispectral data requires robust optimization techniques to address the inherent complexity associated with astronomical signals.

The Levenberg–Marquardt (LM) algorithm is a hybrid optimization method that adeptly combines elements of gradient descent and least-squares approaches. Its efficacy in solving non-linear least-squares problems renders it particularly suitable for refining parameter estimates within non-linear signal models. By integrating bispectral analysis with the LM algorithm, researchers can attain enhanced accuracy in characterizing non-linear phenomena in the field of astronomy.

This paper presents an integrated approach that capitalizes on the strengths of both bispectral analysis and the Levenberg–Marquardt algorithm to improve the processing of non-linear astronomical signals. The proposed framework enhances the detection of phase-coupled frequencies and optimizes model parameters, thereby providing fresh insights into complex astrophysical processes [Randall et al. (2016), Smith & Randall (2016), Wang et al. (2019), Sivolenko et al. (2019a), Medvedev et al. (2022)].

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2. The Micro-Doppler Effect: A Deep Technical Overview

The micro-Doppler effect is a refined aspect of the Doppler effect, focusing on frequency changes due to small-scale motions within a system. Unlike the classical Doppler effect, which looks at overall motion, the micro-Doppler effect results from oscillatory movements of substructures like rotating surfaces or vibrating components. It plays a significant role in astronomy, helping to analyze the dynamics of stars, planetary systems, and compact objects.

For a source moving with velocity v along the observer's line of sight, the observed frequency shift Δf is given by:

$$f_{obs} = f_0 \times \left(1 + \frac{v}{c}\right). \tag{1}$$

Here f_0 – frequency of the emitted wave, v – radial velocity of the source, c – speed of light.

This relationship captures the bulk motion but does not account for internal dynamics, such as rotation or oscillations. For a rotating star, the surface exhibits differential velocities due to rotation. The radial velocity of a point on the star's surface relative to the observer is:

$$v_m\,(t) = R \times \omega \times \sin(\theta) \times \cos(\varphi + \omega \times t). \tag{2}$$

Where R – radius of the star, ω – angular velocity, θ – inclination angle of the rotation axis, φ – the initial phase of rotation. The resulting micro-Doppler shift is:

$$\Delta f_{micro}(t) = f_0 \times \frac{2R \times \omega \times \sin(\theta) \times \cos(\varphi + \omega \times t)}{c}. \tag{3}$$

This produces a periodic modulation in the observed frequency, which can be used to infer the star's rotational parameters and surface features (e.g., star spots).

In pulsating stars, such as Cepheids or RR Lyrae, radial oscillations of the stellar surface generate periodic velocity components:

$$v_m\,(t) = A \times \sin(2\pi \times f_p \times t). \tag{4}$$

Here A – amplitude of oscillation, f_p – frequency of the pulsation. The micro-Doppler shift is:

$$\Delta f_{micro}(t) = f_0 \times \frac{2A \times \sin(2\pi \times f_p \times t)}{c}. \tag{5}$$

This oscillatory modulation is key to asteroseismology, providing detailed information about the star's interior structure and dynamics.

In compact binaries, orbital motion introduces time-dependent micro-Doppler effects. If a binary companion induces oscillations in the primary star's radial velocity, the total velocity becomes:

$$v\,(t) = v_0 + v_{binary}\,(t) + v_{oscillation}(t). \tag{6}$$

Here, $v_{binary}\,(t)$ is the periodic velocity from orbital motion, and $v_{oscillation}(t)$ arises from internal stellar pulsations. The observed frequency shift is:

$$f_{obs}\,(t) = f_0 \times \left(1 + \frac{v_0 + v_{binary}\,(t) + v_{oscillation}(t)}{c}\right). \tag{7}$$

The combined micro-Doppler signature provides a powerful diagnostic of orbital and intrinsic stellar properties.

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3. Levenberg–Marquardt algorithm

The Levenberg–Marquardt algorithm (LMA) serves as an optimization technique utilized for addressing non-linear least-squares problems. It is particularly effective in fitting models to data when the models exhibit non-linear dependencies on their parameters. The LMA integrates the gradient-descent method with the Gauss-Newton algorithm, dynamically adjusting between these two methodologies to optimize both convergence speed and robustness.

Mathematical foundation includes non-linear last-squares problem.

$$S(\theta) = \sum_{i=1}^{n} \Big(y_i - f(x_i; \theta) \Big)^2 \tag{8}$$

where $\theta = [\theta_1, \theta_2, ..., \theta_p]$ is a vector of model parameters, $f(x_i; \theta)$ is a non-linear model function, y_i is observed data, x_i is an independent variable. The objective is to find θ that minimizes $S(\theta)$.

Using a first-order Taylor expansion, the model $f(x_i; \theta)$ around an initial estimate θ_k is approximated as:

$$f(x_i; \theta) = f(x_i; \theta_k) + \sum_{j=1}^{p} (J_{ij} \Delta\theta_j) \tag{9}$$

where $\Delta\theta = \theta - \theta_k$, $J_{ij} = \frac{\partial f(x_i;\theta)}{\partial \theta_j}$ is a Jacobian matrix of partial derivatives.

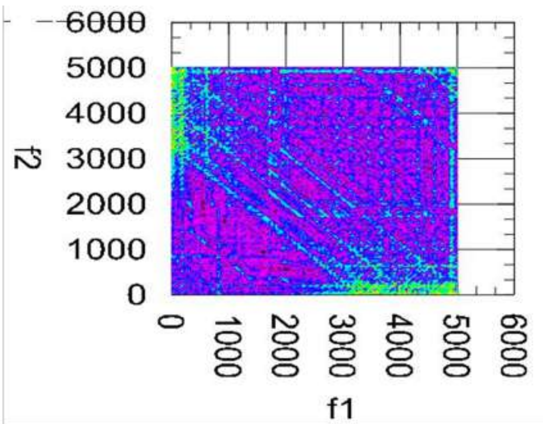
4. Higher Ordered Statistics

The utilization of Higher-Order Statistics (HOS) is essential in the detection of non-Gaussian signals in radar, sonar, and communication systems. Traditional second-order methods, which are based on mean and variance, are not effective in capturing non-Gaussian signals such as radar echoes from clutter and impulsive noise. HOS is instrumental in identifying these challenging signals.

Moreover, higher-order statistics are crucial in system identification, particularly in detecting system non-linearities. They are effective in identifying mechanical faults, such as cracks or misalignments in structures.

Source Separation: HOS methods are employed in blind source separation techniques, facilitating the separation of multiple signal sources in communication systems.

Bispectral and Trispectral Analysis: These analyses are utilized in target detection, non-linear system identification, and vibration analysis, providing a more robust characterization of signals with inherent non-linearities (see Figure 1).



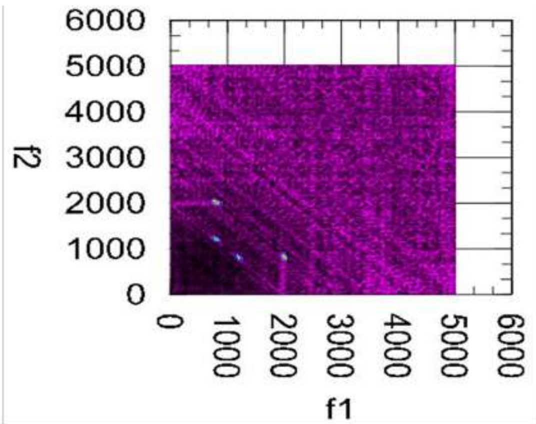


Figure 1. Quadratic phase coupling between frequencies.

With consideration of the bispectrum properties for a real-valued stationary discrete process $\{x^{(m)}(i)\}$ with finite sample number $i = 1, ..., N-1$ and with a finite set of $m = 1, 2, ..., M$ independent realizations $x^{(m)}(i)$ [Grigoryan et al. (2022)]. The autocorrelation function can be written:

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$$R_x(k) = \Big\langle \sum_{i=0}^{N-1} [x^m(i) - E][x^m(i+k) - E] \Big\rangle_\infty \tag{10}$$

where $k = -N+1, ..., N-1$ is shift index, $\Big\langle ... \Big\rangle_\infty$ denotes ensemble averaging for infinite realization number, i.e. for $M \to \infty$; $E = \Big\langle \frac{1}{N} \sum_{i=0}^{N-1} x^m(i) \Big\rangle_\infty$ is the mean value; $R_x(0) = \sigma_x^2 = \Big\langle \sum_{i=0}^{N-1} [x^m(i) - E]^2 \Big\rangle_\infty$ is the variance.

The autocorrelation function is one variable function. Spectral density $P_x(p)$ is defined from the Wiener-Khinchin theorem using direct Fourier transfer:

$$P_x(p) = \Big\langle \sum_{k=-\infty}^{k=+\infty} R_x(k) exp(-j2\pi kp) \Big\rangle_\infty \tag{11}$$

or by

$$P_x(p) = \Big\langle X_{((m)}(p) X^{(*m)}(p) \Big\rangle_\infty \tag{12}$$

where $p = -N+1, ..., N-1$ is the frequency sample index; $X_{(m)}(p) \Big\langle \sum_{i=0}^{N-1} x^{((m)}(i) exp(-j2\pi ip) \Big\rangle_\infty$ is Fourier transform for m-th realization; $*$ is complex conjugation. In equation (3) due to the multiplication of the complex conjugated functions, the phase information is lost.

$R_x(k,l)$ triple autocorrelation function and $\dot{B}_x(p,q)$ bispectrum are functions of two variables, in opposite to autocorrelation function and spectral density. $R_x(k,l)$ the triple autocorrelation function is set as:

$$R_x(k,l) = \Big\langle \sum_{i=0}^{N-1} [x^m(i) - E][x^m(i+k) - E] \times [x^m(i+l) - E] \Big\rangle_\infty \tag{13}$$

where $k = -N+1, ..., N_1$ and $l = -N+1, ..., N-1$ are the independent shift indexes. Unlike spectral density, $\dot{B}_x(p,q)$ the bispectrum is a complex-valued function of two independent frequencies p and q. It can be written as a 2-D discrete Fourier transform of triple autocorrelation function:

$$\dot{B}_x(p,q) = \Big\langle \sum_{k=-N+1}^{N-1} \sum_{l=-N+1}^{N-1} R_x(k,l) exp[-j2\pi(kp + lq)] \Big\rangle \tag{14}$$

or as

$$\dot{B}_x(p,q) = \Big\langle X^{(m)}(p) X^{(m)}(q) X^{*(m)}(p+q) \Big\rangle_\infty \Big\langle X^{(m)}(p) X^{(m)}(q) X^{*(m)}(-p-q) \Big\rangle_\infty \tag{15}$$

where $\dot{B}_x(p,q) = |\dot{B}_x(p,q)| exp[j\gamma_x(p,q)]$, $\dot{B}_x(p,q)$ and $\lambda_x(p,q)$ are the magnitude bispectrum (bimagnitude) and phase bispectrum (biphase), respectively $p = -N+1, ..., N-1$ and $q = -N+1, ..., N-1$ are the frequency indices. The power spectrum, from (3), is the ensemble averaging of the multiplication of two complex conjugated functions of one variable.

From (15), this bispectrum is a composite average of three complex-valued functions corresponding to different frequency values. As such, spectral density only provides information about amplitude, whereas bispectrum can provide information about phase and amplitude. For one of the main properties, joint phase information storage [Sivolenko et al. (2019b), Peloso & Pietroni (2013), Muthuswamy et al. (1999)], we use bispectral estimation.

5. Main Results

In various scientific disciplines, including astronomy, the complex interactions between celestial phenomena frequently exhibit non-linear characteristics in observational data. Two significant methodologies—bispectral analysis and the Levenberg–Marquardt algorithm (LMA)—provide distinct approaches for identifying and characterizing these non-linearities.

This article accentuates their comparative effectiveness, specifically highlighting scenarios where bispectral analysis encounters limitations (see Fig. 2) and where LMA demonstrates superior performance in detecting Gaussian signals (see Fig. 3).

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Table 1. Aspect comparision

Aspect	Bispectral Analysis	Levenberg–Marquardt Algorithm
Signal Type	Periodic, harmonic signals	Any signal with a defined model structure
Gaussian Signals	Ineffective (blind, as in Fig. 1)	Effective (detected in Fig. 2)
Noise Sensitivity	High (struggles with low SNR)	Moderate (handles noise with good models)
Model Dependence	No explicit model required	Requires a well-defined parametric model
Computational Efficiency	Faster (analyzes frequency relationships)	Requires a well-defined parametric model

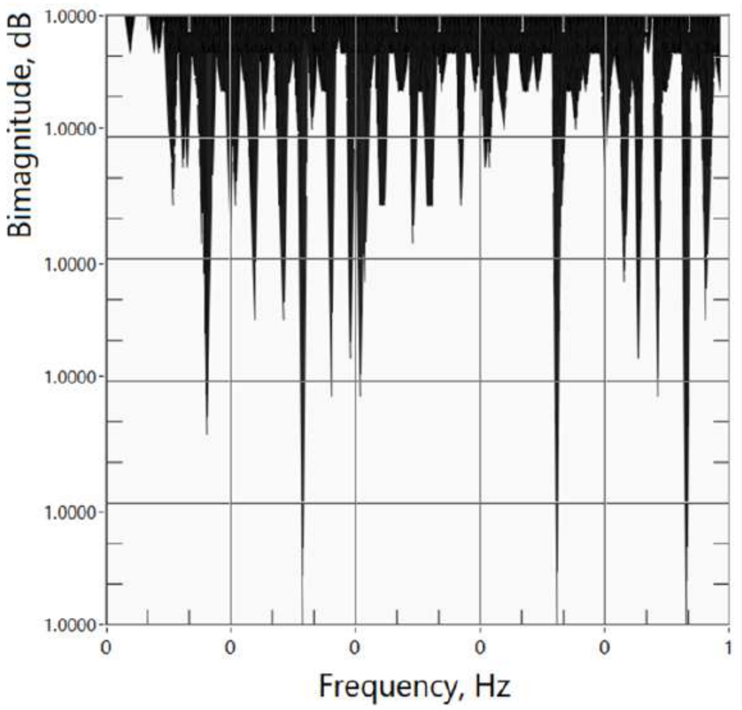

Figure 2. Bispectral analysis of Gaussian signals.

Bispectral Strengths:

- Bispectral analysis is effective for identifying phase coupling and non-linear interactions where specific harmonics interact
- It excels in detecting non-linearities in structured and periodic signals with clear frequency-phase relationships.

Bispectral Blind Spots:

- In cases of Gaussian signals or non-periodic, stochastic non-linearities (as shown in fig.2), the bispectrum often fails because it relies on coherent phase relationships. Gaussian signals, being inherently random, do not produce significant bispectral peaks.
- If the signal-to-noise ratio (SNR) is low, bispectral methods can struggle to extract weak non-linear features buried in noise.

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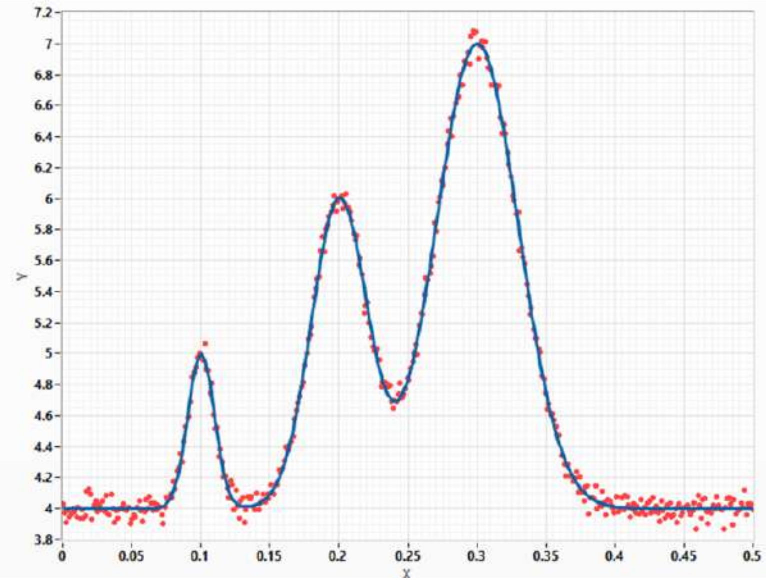

Figure 3. LMA demonstrates superior performance in detecting Gaussian signals.

LMA Strengths:

- LMA, as an iterative optimization tool, can recover non-linear signal parameters even when the bispectrum is ineffective. For instance, Gaussian signals, which appear featureless in bispectral analysis, can still be modeled if a proper parametric model is assumed.
- The LMA can adapt to non-periodic or stochastic non-linearities by fitting a model to data, as illustrated in Fig.3.
- The algorithm's dynamic damping ensures convergence even for highly non-linear problems.

LMA Limitations:

- LMA requires a good initial guess and a well-defined model structure. Without an approximate understanding of the underlying system, the algorithm may converge to a local minimum or fail to provide meaningful results.
- Computationally more expensive compared to the bispectrum, as it involves iterative Jacobian evaluations and matrix inversions. Additionally, the article presents partial code implementations to elucidate these methodologies (see Fig.4.).

6. Conclusion

A comprehensive comparative analysis of bispectral analysis and the Levenberg–Marquardt algorithm elucidates their respective strengths as well as their complementary capabilities in the realm of signal processing. Bispectral analysis is particularly adept at identifying phase-coupled harmonic nonlinearities, which are critical for understanding certain signal characteristics. However, it encounters limitations when tasked with detecting stochastic or Gaussian signals, rendering it less effective in scenarios where such signals are prominent.

In contrast, the Levenberg–Marquardt algorithm is renowned for its robustness and versatility, excelling in the modeling of a wide spectrum of nonlinearities. This algorithm is particularly valuable as it can effectively address complexities that remain undetected by bispectral analysis. By leveraging the strengths of both methodologies, researchers and practitioners can establish a more holistic analytical framework. This integration not only enhances the capability to process and interpret nonlinear signals but also contributes significantly to advancements in fields such as astronomy and various scientific disciplines where nuanced signal characteristics are pivotal.

Ultimately, the combination of bispectral analysis and the Levenberg–Marquardt algorithm presents an enriched toolkit that empowers researchers to undertake comprehensive analyses of nonlinear signals, thereby fostering deeper insights and facilitating progress in their respective fields.

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$$y = a_1 e^{-\frac{(x-m_1)^2}{2\sigma_1^2}} + a_2 e^{-\frac{(x-m_2)^2}{2\sigma_2^2}} + a_3 e^{-\frac{(x-m_3)^2}{2\sigma_3^2}} + offset$$

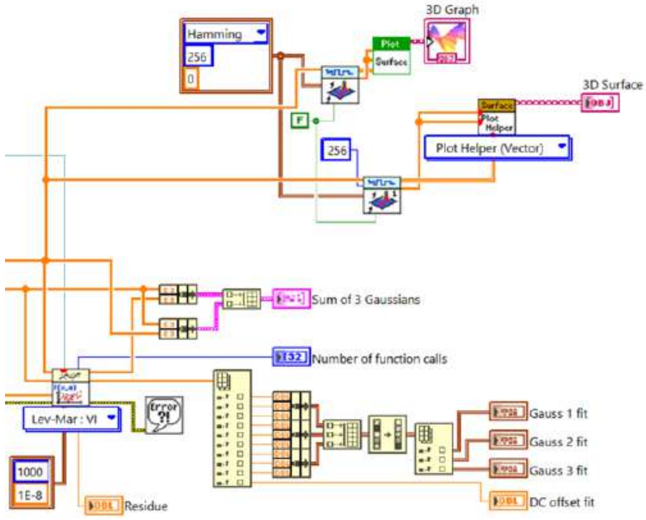

Figure 4. Implemented code in LabVIEW .

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