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The Time-dependent Problem of the Line Radiation Reflection

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Abstract

We consider the classical time-dependent problem of diffuse reflection of the line-radiation from a semi-infinite absorbing and scattering atmosphere. By the example of the simplest 1D problem it is shown how its solution is constructed in the general case, when both the photon lifetime in the absorbed state and the time of its travel between two consecutive acts of scattering are taken into account. The numerical values of the coefficients in the expansion of the reflection function in the Neumann series are given. The obtained solution is applied to the problem, in which the scattering in both the spectral line and in the continuous spectrum is taken into account.

1. Introduction

Temporal changes in the spectra of various cosmic objects are an important direction in the study of physical phenomena occurring in them. The problems arising in this case belong to the theory of the time-dependent transfer of radiation, in which the residence time of quanta in the medium is taken into account. The theory has been developed in the works of a number of authors, of which we shall mention here only the works of Ganapol (1979), Ganapol & Matsumoto (1986), Matsumoto (1967, 1974), Minin (1964, 1965, 1967), Sobolev (1963). A number of methods for solving such problems under one or another simplifying assumptions have been proposed. In particular, the first two of the mentioned authors proposed methods based on finding the Laplace transform of any characteristic of the radiation field in time directly from the corresponding characteristic in the stationary case. In this way Minin considered the problem of diffuse reflection from semi-infinite atmosphere for isotropic scattering.

Another way based on expanding the requisite quantities into Neumann series was developed in works of Matsumoto and Ganapol. Central to their theory is the analogy between the forms of time-dependent and time-independent emergent intensity when expressed in Neumann series format. This allows to obtain a factor in each term in the required time-dependent series directly from those in the time-independent problem. In the latter case the corresponding factors are determined via recursion relations.

It is known that the time spent by a quantum during multiple scattering in a medium is due to two processes: the time spent by it in the absorbed state and the time spent on the path between two consecutive acts of scattering. Depending on the particular physical problem, one of them predominates, which simplifies the corresponding theory. For instance, Sobolev (1963) considered the 1D time-dependent transfer problem for two extreme cases $t_1 \gg t_2$ and $t_2 \gg t_1$.

This paper pursue two goals. On the example of the simplest one-dimensional problem of diffuse reflection of radiation from a semi-infinite atmosphere the method of obtaining the required solution is proposed for the general case, when both above mentioned processes are taken into account.The immediate motivation for turning to this problem is our study of the changes in the spectra of giants of the early spectral classes, caused by the scattering of radiation in the continuous spectrum (Nikoghossian, 2020a,b,c).

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2. The problem of the diffuse reflection from a semi-infinite atmosphere.

In this paper, we limit ourselves to considering the simplest 1D problem when the scattering of radiation both in the line and in the continuous spectrum is assumed to be monochromatic. Problems in this approximation have at least two advantages. First, many of the features required for the study can be obtained analytically and, second, by abstracting from the frequency and direction redistribution effects, the investigated features of the spectral line variations are manifested in a pure form.

We start by considering the steady-state problem of diffuse reflection from a semi-infinite atmosphere absorbing and scattering both in the line and continuous spectrum. The atmosphere is assumed to be illuminated by the continuous radiation of unite intensity. Obviously, the reflected radiation is due to scattering in both the line and continuous spectra. If no distinction is made between which of these two processes results in the observed radiation, then, as shown in Nikoghossian (2020a,b), the reflection coefficient is given by the formula

$$\rho\left(x\right)=\frac{1}{\tilde{\lambda}}\left(2-\tilde{\lambda}-2\sqrt{1-\tilde{\lambda}}\right), \tag{1}$$

with

$$\tilde{\lambda}\left(x\right)=\frac{\lambda\alpha\left(x\right)+\gamma}{\alpha\left(x\right)+\beta+\gamma}, \tag{2}$$

where the commonly accepted notations are used: λ is the scattering coefficient in the line (the probability of reradiation of the absorbed photon by the exited atom), $\alpha(x)$ is the line absorption profile in terms of dimensionless frequency measured by displacement from the centre of the line in Doppler-widths, β and γ are correspondingly the ratios of the continuum absorption and continuum scattering coefficients χ_ν and σ to the absorption coefficient in the center of the line (k_0). Assuming that the scattering in the continuous spectrum is due to free electrons, we can write $\sigma = n_e\sigma_0$, where n_e is the electron density and

$$\sigma_0=\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2, \tag{3}$$

where m and e are the mass and the charge of electron, and c is the light speed. As it was shown in Israelian & Nikoghossian (1996), Nikoghossian (2020a,b,c), the values of parameters β and γ appeared in Eq. 2 determine whether the line will be observed in absorption, or in emission (the so called Shuster's mechanism (1905)). In the two-level isothermal atmosphere the condition of the emission line reads

$$\left(1-\lambda\right)\gamma>\lambda\beta. \tag{4}$$

The physical meaning of this inequality is not difficult to understand if one pre-tests two mutually opposite elementary processes: a photon is scattered in a continuous spectrum (on free electrons) and then is absorbed and thermalized in the spectral line, and, conversely, a photon scatters in frequencies of the spectral line and is absorbed afterwards in the continuum. The condition particularly implies that the appearance of spectral lines in emission should be expected in the case of weak lines. It should be noted that there is a simple relationship between the profile of the line formed by reflection from a homogeneous semi-infinite atmosphere R_* and that formed in a semi-infinite isothermal atmosphere R_0: $R_* + R_0 = 1$ (Arutyunyan & Nikogosyan, 1978), so the line emission profile in one of these problems corresponds to the absorption profile in the other.

Our interest in this paper is the lines formation resulting in reflection from the semi-infinite medium. The Neumann series for the reflectance $\rho(x)$ has a form

$$\rho\left(x\right)=\sum_{n=1}^{\infty}\rho_n\tilde{\lambda}^n\left(x\right), \tag{5}$$

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where $\rho_1 = 0,25$; $\rho_2 = 0,125$, and the higher order factors being found recursively according to

$$\rho_n = \frac{1}{2}\left(\rho_{n-1} + \frac{1}{2}\sum_{k=1}^{n-2}\rho_k\rho_{n-k-1}\right), \tag{6}$$

Obviously, each summand is the probability of reflection from the medium as a result of a certain number of n scattering events. Below is a brief table of these values

Table 1.

n	ρ_n	n	ρ_n	n	ρ_n	n	ρ_n
1	0.250000	11	0.014015	21	0.005562	31	0,003154
2	0.125000	12	0.012398	22	0.005200	32	0,003011
3	0,078125	13	0,011070	23	0,004875	33	0,002878
4	0,054687	14	0,009963	24	0,004583	34	0,002754
5	0,041015	15	0,009029	25	0,004318	35	0,002640
6	0,032226	16	0,008232	26	0,004078	36	0,002533
7	0,026184	17	0,007546	27	0,003860	37	0,002433
8	0,021820	18	0,006951	28	0,003660	38	0,002340
9	0,018547	19	0,006429	29	0,003477	39	0,002251
10	0,01602	20	0,005970	30	0,003309	40	0,002169

A characteristic feature of the Neumann series, as can also be seen from the table above, is its slow convergence. In this 1D problem of coherent scattering, the required factors are found easily recursively. In a more general formulation of the problem, their computation is fraught with difficulties, to overcome which asymptotic methods have been developed (see, for instance, Uesugi & Irvine, 1970).

3. Temporal picture of the reflected line formation depending on the scattering in continuum.

Turning to the non-stationary problem, we remind that in each act of scattering a photon takes time both to stay in the absorbed state and to travel to the next act of scattering. Both quantities are independent random variables distributed according to the same exponential law with mean values equal to t_1 and t_2, respectively. If the medium is taken to consist of two-level atoms, then, allowing for scattering in the continuum, we can write, $t_2 = 1/[(1+n^+/n_1)n_1k_0c]$, where n_1 is the number of atoms on the ground level, and we also assume that $n_e \approx n^+$. Instead of time t, we introduce dimensionless variables $u = t/t_1$ and $\omega = t/t_2$ correspondingly for each of the mentioned two processes. The total time it takes for a photon to stay in a medium is obviously the sum of two random quantities, each of which, in turn, is the sum of independent random quantities realized in the course of multiple scattering. It is well known that the distribution function of the sum of the number of n such quantities distributed equally according to the exponential law is given by the Erlang-n distribution. Recall that it is a two-parameter function and is a special case of the Gamma distribution,

$$\mathrm{Er}(x,k,\Lambda) = \frac{\Lambda^k x^{k-1}}{(k-1)!}\mathrm{e}^{-\Lambda x}, \tag{7}$$

Parameters k and Λ are known as the shape and rate parameters, respectively. It is important to note that the Erlang distribution is stable so that the sum and the product of such distributions yields again the Erlang distribution with proper parameters.

When applied to our problem, taking into account the dimensionless values for time introduced in it, the parameter Λ takes on the meaning of the value inverse to the mean time for each of the two elementary processes associated with the waste of time mentioned above. Then for the distribution

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function of the probability density function (PDF) of time lost by a photon to stay in the absorbed state during n-fold scattering we will have

$$f_1\left(u,n\right) = \frac{u^{n-1}}{(n-1)!}\mathrm{e}^{-u} \tag{8}$$

Similarly, for the second process associated with spending time traveling in the medium, we can write

$$f_2\left(\omega,n+1\right) = \frac{\omega^n}{n!}\mathrm{e}^{-\omega} \tag{9}$$

Here we have taken into account that the number of elementary events associated with the travel of photons in the medium exceeds the number of scatterings in the medium by one. This is due to the fact that the path taken by a photon as it falls from the outside onto the medium, and hence the time it takes to travel this path, is not related to the act of scattering.

Distributions Eq. 8 and Eq. 9 specify the PDF for the total duration of each of the two processes under consideration. In turn, the PDF of the total time spent by a photon in the medium, regardless of the type of process, is set, as we know, by the convolution of the above two PDF

$$F_n\left(z\right) = \int_0^z f_2\left(\omega,n+1\right)f_1\left(z-\omega,n\right)\mathrm{d}\omega \tag{10}$$

The resulting integral is calculated explicitly to give

$$F_n(z) = \frac{z^{2n-1}}{(2n-1)!}\mathrm{e}^{-z} \tag{11}$$

Introducing a function $\bar{\rho}(x,z)$ into consideration, such that $\bar{\rho}dz$ is the probability of reflection from the medium of quantum of frequency x in the time interval $z, z+dz$. we return to the reflection function in the stationary problem Eq. 5, and represent it in the form of a Neumann series

$$\bar{\rho}\left(x,z\right) = \sum_{n=1}^{\infty}\rho_n\tilde{\lambda}^n\left(x\right)F_n\left(z,n\right), \tag{12}$$

or

$$\bar{\rho}\left(x,z\right) = \mathrm{e}^{-z}\sum_{n=1}^{\infty}\rho_n\tilde{\lambda}^n\left(x\right)\frac{z^{2n-1}}{(2n-1)!} \tag{13}$$

Eq. 13 shows the time distribution of the probability of quanta of different frequencies leaving the medium within a spectral line. At the same time, the cumulative distribution function (CDF), which demonstrates the evolution of the spectral line profile up to a certain time point z_0, is also of great interest. Denoting it by $\mathrm{P(x, z_0)}$, from Eq. 10 we find

$$\mathrm{P}\left(x,z_0\right) = \mathrm{e}^{-z_0}\sum_{n=1}^{\infty}\rho_n\tilde{\lambda}^n\sum_{k=0}^{\infty}\frac{z^{2n+k}}{(2n+k)!} \tag{14}$$

As it follows from the physical meaning of the dimensionless time variables $z = t/\bar{t}$ and $z_0 = t_0/\bar{t}$, the scale factor $\bar{t} = t_1t_2/\left(t_1+t_2\right)$,

As an application of the results obtained, we will limit ourselves here to examining the behavior of the found temporal characteristics and for different frequencies within the spectral line. In Figures 1, 2 we present the PDF and CDF for the same line with a moderate scattering coefficient $\lambda = 0.5$, and different values of the γ parameter, characterizing the role of scattering in the continuous spectrum. Comparing the locations of the curves corresponding to the same frequency value in the above graphs, it is easy to find that they are symmetrically inverted. In one case it corresponds to the absorption line (Figure 1), while the other refer to the emission line (Figure 2). The evolution of the spectral line profiles before the stationary mode was established is also reciprocally reversed. If, in the case of

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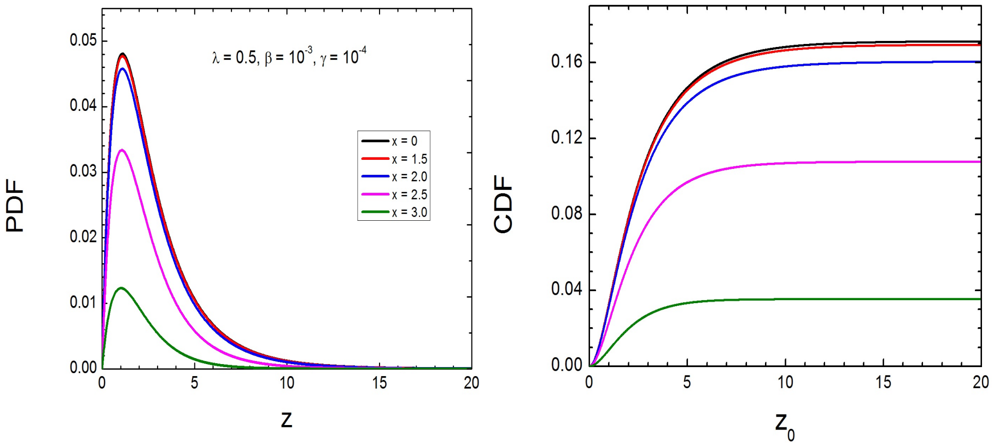

Figure 1. The frequency (center to wings) variation of the mean number of scattering events (left) and the average time the photons spent in diffusing within the atmosphere for $\lambda = 0.5$ and indicated values of γ. In both cases β is 10^{-3}.

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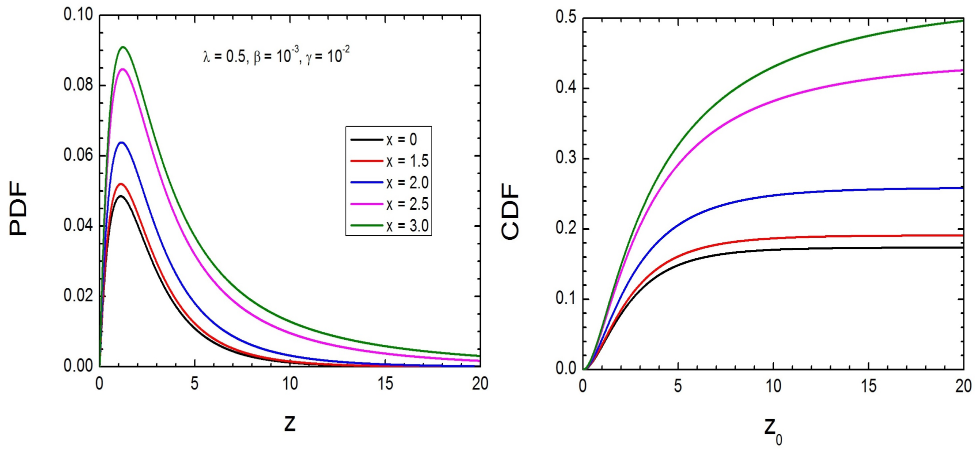

Figure 2. The mean number of scattering events underwent by reflected photons in continuous spectrum for the same as in Fig.1 values of parameters.

the absorption line, the saturation first occurs in the line core, in the case of the emission line, the wings of the lines are established first. The difference between the saturation times of the different parts of the line reaches 10 or more in the scale of units adopted in the paper, which, as we remember, depends significantly on the atmospheric density. Thus, there is a direct correlation between the role of the indicated scattering due to the presence of a certain number of free electrons in our case and the saturation time of the spectral line profiles.

The results presented in the paper show that despite the comparatively small coefficient of electron scattering, under certain physical conditions spectral lines can be observed in the emission. It is also important to note that the temporal variation of PDF obtained in this work, as one would expect, adequately describes the physical picture of the multiple scattering process in a medium, in contrast to the often used approach, which uses a single Erlang distribution over dimensionless time on a scale containing both t_1 and t_2.

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References

Arutyunyan G. A., Nikogosyan A. G., 1978, Akademiia Nauk SSSR Doklady, 242, 66

Ganapol B. D., 1979, J. Quant. Spectrosc. Radiat. Transfer. , 22, 135

Ganapol B. D., Matsumoto M., 1986, J. Quant. Spectrosc. Radiat. Transfer. , 35, 71

Israelian G., Nikoghossian A., 1996, J. Quant. Spectrosc. Radiat. Transfer. , 56, 509

Matsumoto M., 1967, Publ. Astron. Soc. Jpn. , 19, 163

Matsumoto M., 1974, Publ. Astron. Soc. Jpn. , 26, 241

Minin I. N., 1964, Soviet Physics Doklady, 9, 114

Minin I. N., 1965, Astrophysics, 1, 104

Minin I. N., 1967, Astrophysics, 3, 154

Nikoghossian A. G., 2020a, Astrophysics,

Nikoghossian A. G., 2020b, Communications of the Byurakan Astrophysical Observatory, 67, 1

Nikoghossian A. G., 2020c, Communications of the Byurakan Astrophysical Observatory, 67, 21

Schuster A., 1905, Astrophys. J. , 21, 1

Sobolev V. V., 1963, A treatise on radiative transfer.

Uesugi A., Irvine W. M., 1970, Astrophys. J. , 159, 127

doi: https://doi.org/10.52526/25792776-2021.68.1-32