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The Spectral Line Evolution in a Semi-infinite Atmosphere with Local Time-dependent Energy Sources

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Abstract

The time-dependent problem of spectral line formation in a one-dimensional semi-infinite scattering and absorbing atmosphere containing a local energy source radiating equally in both directions is considered. The energy release is assumed to be non-stationary and to be either $\delta(t)$-shaped or of the unit jump form given by the Heaviside $H(t)$ function. The main attention is paid to the temporal dependence of the observed line profiles on the depth of energy eruption in the atmosphere. The role of scattering in the continuous spectrum is emphasized.

1. Introduction

In our previous works (Nikoghossian, 2021a,b) we used a relatively simple way of finding solutions to the time-dependent problems of the line radiation transfer in 1D atmosphere. The method is based on the construction of Neumann series for the quantities sought and goes back to the works of Ganapol (1979), Matsumoto (1967, 1974). Solutions were constructed for diffuse reflection from a semi-infinite atmosphere as well as for diffuse reflection and transmittance for a medium of finite optical thickness. In both cases the scattering was assumed to be coherent. These mathematically simple model problems in the above approach have a number of advantages. The main one is that, despite the assumptions made, they allow one to clearly trace the evolution of the spectral lines formed depending on the values of various quantities determining the local optical properties of the medium and location of primary sources. In a number of cases, it is not difficult to conclude that some of detected patterns remain valid also for fairly more general problem formulations. The assumption of scattering coherence in the problem simplification is crucial, since in some cases the coefficients in the expansion of the Neumann series are constants and are calculated once and for all. Tables of such coefficients for the coefficient of reflection from a semi-infinite atmosphere are given in Nikoghossian (2021b). The frequency dependence of the reflection function, as well as the dependence on the local optical properties of the medium, appear through a certain parameter $\tilde{\lambda}$, of the form

$$\tilde{\lambda}(x) = \frac{\lambda\alpha(x) + \gamma}{\lambda\alpha(x) + \beta + \gamma} \tag{1}$$

where λ is the coefficient of the quantum re-radiation in the elementary event of scattering in the line, $\alpha(x)$ is the line absorption profile, x is the dimensionless frequency measured by displacement from the center of the line in Doppler widths, β and γ correspondingly are the ratios of the continuum absorption and scattering coefficients to that in the center of the line. It is noteworthy that, with the assumptions we have made, the role of scattering in the continuum, which is also assumed to be coherent, is taken into account quite simply. This is another advantage of the approach used. In astrophysical applications, one of the most common scattering mechanisms in the continuum spectrum is scattering on free electrons (see, for example, Nikoghossian, 2020) and the list of literature there. In this paper, we will continue the study of the temporal variations of spectral lines in various problems frequently encountered in astrophysical applications, in which multiple scattering of radiation in the medium should be taken into account. Here we will assume that the primary energy source is located inside the atmosphere at some predetermined optical depth τ_0.

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2. The line radiation transfer in a semi-infinite atmosphere with the local energy sources.

Before addressing to our problem it is expedient to present some formulas and relations being in use in solving the problems of diffuse reflection (and transmission for finite atmosphere) we have considered in the cited papers (Nikoghossian, 2021a,b). The reflection function from the semi-infinite atmosphere $\rho(x)$, as was said, is sought in the form of a Neumann series

$$\rho(x) = \sum_{n=1}^{\infty} \rho_n \tilde{\lambda}^n (x). \tag{2}$$

The coefficients ρ_n for $n > 2$ are found recurrently by

$$\rho_n = \frac{1}{2} \left(\rho_{n-1} + \frac{1}{2} \sum_{k=1}^{n-2} \rho_k \rho_{n-k-1} \right) \tag{3}$$

taking of $\rho_1 = 0.25$ and $\rho_2 = 0.125$.

Analogously, in the case of a medium of finite optical thickness τ_0 for the reflection and transmission functions are given by

$$\rho (x, \tau_0) = \sum_{n=1}^{\infty} \rho_n (x, \tau_0) \tilde{\lambda}^n (x), \qquad q (x, \tau_0) = \sum_{n=0}^{\infty} q_n (x, \tau_0) \tilde{\lambda}^n (x). \tag{4}$$

Note that the coefficients in these series depend on the frequency in the line. The first two coefficients in the series are found immediately to give

$$\rho_1 (x, \tau_0) = \frac{1}{4} \left(1 - \mathrm{e}^{-2\nu(\mathrm{x})\tau_0} \right), \qquad \rho_2 (x, \tau_0) = \frac{1}{8} \left[1 - (1 + 2\nu (x) \tau_0) \, \mathrm{e}^{-2\nu(\mathrm{x})\tau_0} \right], \tag{5}$$

$$q_0 (x, \tau_0) = \mathrm{e}^{-\nu(x)\tau_0} \qquad q_1 (x, \tau_0) = \frac{1}{2} \tau_0 \mathrm{e}^{-\nu(x)\tau_0}, \tag{6}$$

where $v(x) = \alpha(x) + \beta$. The remaining coefficients associated with the twofold reflection processes are calculated sequentially by means of the formulas

$$\rho_n (x, \tau_0) = 2\nu (x) \int_0^{\tau_0} \Phi_n (x, t) \, \mathrm{e}^{-2\nu(x)(\tau - t)} dt, \tag{7}$$

$$q_n (x, \tau_0) = \nu (x) \int_0^{\tau_0} \Psi_n (x, t) \, \mathrm{e}^{-2\nu(x)(\tau - t)} dt, \tag{8}$$

where the functions $\Phi_n(x,t)$ and $\Psi_n(x,t)$ are determined as

$$\Phi_n (x, \tau_0) = \frac{1}{2} \left[\rho_{n-1} (x, \tau_0) + \frac{1}{2} \sum_{k=1}^{n-2} \rho_k (x, \tau_0) \, \rho_{n-k-1} (x, \tau_0) \right], \tag{9}$$

$$\Psi_n (x, \tau_0) = \frac{1}{2} \left[q_{n-1} (x, \tau_0) + \frac{1}{2} \sum_{k=1}^{n-1} \rho_k (x, \tau_0) \, q_{n-k-1} (x, \tau_0) \right]. \tag{10}$$

Having now all the prerequisites we formulate problem under consideration as follows. Suppose that the primary source of energy is located at depth τ_0 in the 1D atmosphere and radiate equally in both directions (see Fig. 1). If the total intensity is taken as unity, for the probability distribution function of radiation $P(x, \tau_0)$ outgoing from the atmosphere one obviously can write

$$P (x, \tau_0) = \frac{1}{2} \frac{(1 + \rho_\infty (x)) \, q (x, \tau_0)}{1 - \rho_\infty (x) \, \rho (x, \tau_0)}. \tag{11}$$

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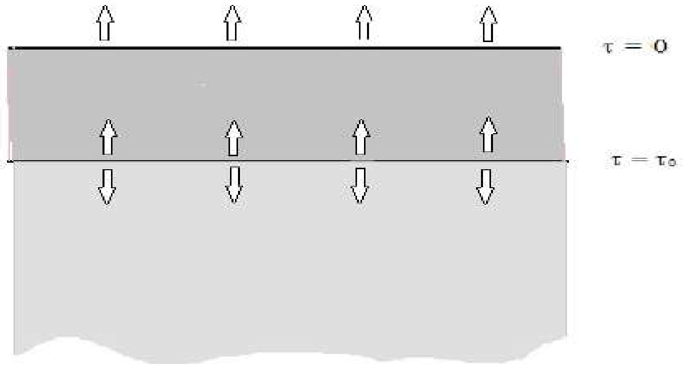

Figure 1. Schematic picture of the radiation transport in the presence of internal energy sources

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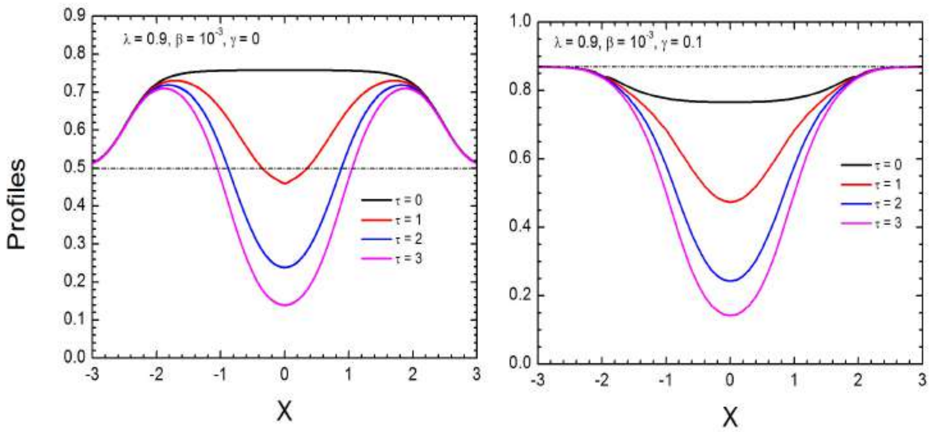

Figure 2. Profiles of spectral lines formed in the stationary regime.

For further treatment, it is convenient to replace the ratio $1/(1-\rho_\infty)\rho$ with corresponding infinite series as follows,

$$\frac{1}{1-\rho_\infty(x)\,\rho(x,\tau_0)} = \sum_{i=0}^{\infty}\left[\rho_\infty(x)\,\rho(x,\tau_0)\right]^i, \tag{12}$$

which, obviously, converges at any values of the physical parameters if only $\lambda < 1$ when $\gamma = 0, x = 0$. The convergence rate is higher at lower values of optical thickness, scattering coefficient, also in the wings of the line, where the scattering effect is weakly expressed.

Typical examples of spectral line profiles formed in the stationary regime are shown in Fig. 2. The results are shown for the case when there is no scattering in the continuum (left panel) and for the case when it is present (right panel). As could be expected with sources located inside the atmosphere, we observe in both cases absorption lines in spite of the difference in the levels of the continuous spectrum. However, in the absence of scattering in the continuum, the lines have a specific shape associated with emission components in their wings. There exists a clear correlation between the depth of the energy source and the equivalent width, as well as the central depth of the spectral line.

Fig. 3 shows the equivalent width of the resulting spectral line as a function of the depth of the internal energy source. Both cases, with and without allowance of scattering in the continuum, are depicted. It is seen that when photons are scattered both in the line and in continuous spectrum, the equivalent width of the line grows more slowly with optical depth. Thus, the shape of the spectral line together with its equivalent width provide a fundamental opportunity to judge the depth in the atmosphere where the energy release occurs. Below we will see that the time characteristics of the line appearance can provide additional information about the energy release in the atmosphere.

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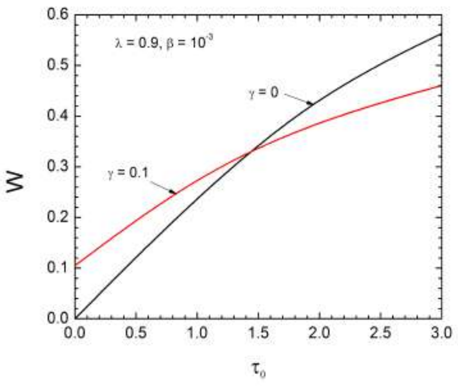

Figure 3. The depth variation of equivalent widths of the spectral line.

3. The time dependent problem of the spectral line formation in the semi-infinite atmosphere with local non-stationary sources of energy

After solving the stationary problem, it is easy to construct the solution of the corresponding time-dependent problem. To this purpose let us return to Eq.(11) and write it briefly in the operator notation

$$P = \frac{1}{2}(1 + R_\infty)Q\sum_{i=0}^{\infty}(R_\infty R)^i. \tag{13}$$

When passing to the time-dependent problem, it is necessary to know the coefficients in the expansions of each of the above operators with respect to the powers of $\tilde{\lambda}(x)$.

$$Q = \sum_{n=0}^{\infty} q_n\left(x,\tau_0\right)\tilde{\lambda}^n\left(x\right), \qquad R = \sum_{k=1}^{\infty}\rho_k\left(x,\tau_0\right)\tilde{\lambda}^k\left(x\right), \qquad R_\infty = \sum_{i=1}^{\infty}\rho_i^\infty\tilde{\lambda}^i\left(x,\right),$$

$$P = \sum_{k=0}^{\infty} p_k\left(x,\tau_0\right)\tilde{\lambda}^i\left(x\right), \qquad R_\infty R = \sum_{i=1}^{\infty}\tilde{\lambda}^i\left(x\right)\sum_{j=1}^{i}\rho_j^\infty\rho_{i-j}\left(x,\tau_0\right). \tag{14}$$

Eq.(13) shows that the calculations numerically are reduced to the multiplication of power series according to the well-known Cauchy formula, which does not meet any fundamental difficulties. In the first approximation, which neglects the multiple interactions between finite surface part $(0, \tau_0)$ of the atmosphere and its the rest infinitely deep part, we will have

$$p_n\left(x,\tau_0\right) = \frac{1}{2}\left(q_n\left(x,\tau_0\right) + \sum_{l=1}^{n}\rho_l^\infty q_{n-l}\left(x,\tau_0\right)\right), \tag{15}$$

and, in the second approximation, limiting ourselves to the linear part in the infinite series in Eq.(13),

$$2p_n\left(x,\tau_0\right) = q_n\left(x,\tau_0\right) +$$

$$\sum_{l=1}^{n}\left[\rho_l^\infty q_{n-l}\left(x,\tau_0\right) + q_l\sum_{i=1}^{l}\rho_l^\infty\rho_{l-l}\left(x,\tau_0\right) + \sum_{i=1}^{l}\rho_l^\infty q_{l-l}\left(x,\tau_0\right)\sum_{k=1}^{l}\rho_k^\infty\rho_{k-l}\left(x,\tau_0\right)\right]. \tag{16}$$

Obviously, the accuracy of the results obtained depends largely on the number of terms that have to be limited in the corresponding expansions. Difficulties are usually associated with finding the coefficients of the Neumann series for the reflection and transmittance functions of the finite atmosphere, through which the required values of $p(x,\tau)$ are expressed. However, as can be concluded from Fig. 4 and Fig. 5, in fact it is often sufficient to limit ourselves to a small number of terms in

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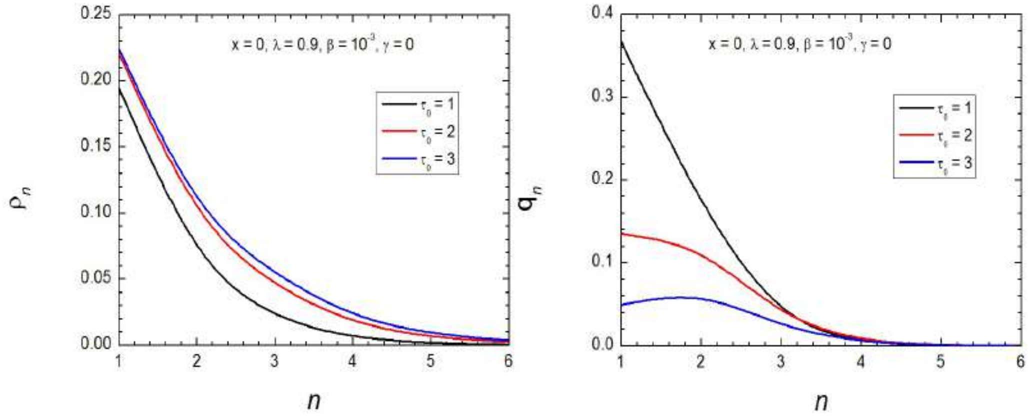

Figure 4. Coefficients in the Neymann series of reflectance and transmittance of finite media of different thicknesses and indicated values of other parameters. The data are enveloped by B-splines for illustration.

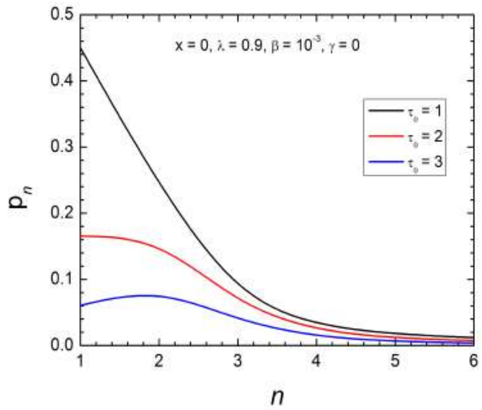

Figure 5. The same as in Fig.4 for the function $P(x, \tau_0)$.

the proper expansions in order to achieve the necessary accuracy. For clarity, in the above mentioned figures, the discrete set of coefficients is enveloped using B splines. Figures show that even for relatively large values of the scattering coefficient the required number of terms in the Neumann series is small even in the central frequencies of the spectral line.

Having the coefficients of p_n, we pass to the time-dependent problem and establish the temporal characteristics of changes in the observed spectral line. As we showed in the cited papers (Nikoghossian, 2021a,b) such transition is carried out by multiplication of coefficients in the Neumann expansion of the quantity under study by the total time taken by the quantum at the corresponding number of scattering events. Referring the reader to the above-mentioned works for the reasoning in deriving the distribution law of the total time spent by the quantum while staying in the medium, the probability distribution function (PDF) of this time is given by

$$F_n(z) = \mathrm{e}^{-z} \frac{z^{2n-1}}{(2n-1)!}, \tag{17}$$

where the time variable $z = t/\bar{t}$ and $\bar{t} = t_1 t_2/(t_1 + t_2)$. Here we used the commonly adopted notations: t_1 is the average time of the atom stay in the exited state, and t_2 is the mean time of the quantum travel between two successive events of scattering.

Hence the evolution of the observed spectral line profile for the pulse of $\delta(t)$ form of the internal

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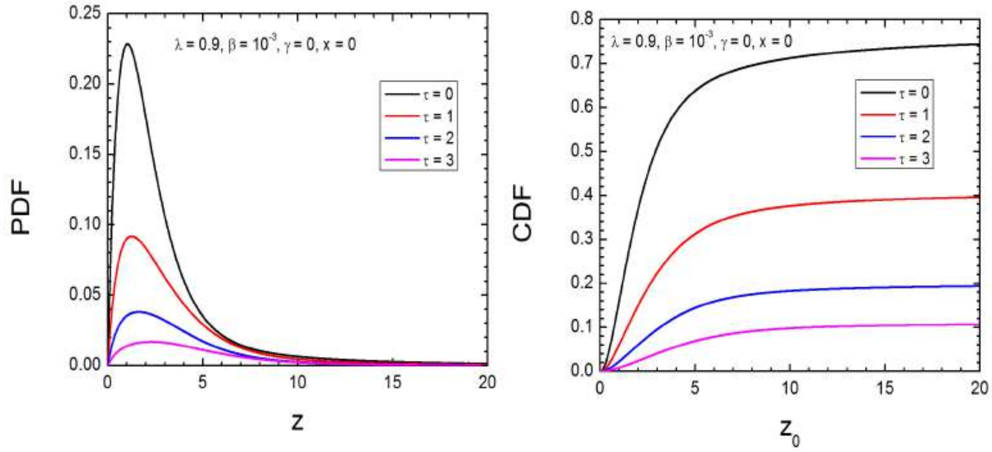

Figure 6. Typical graphs of PDF and CDF for the photons in the center of the line in the absence of continuum scattering when $\lambda = 0.9$.

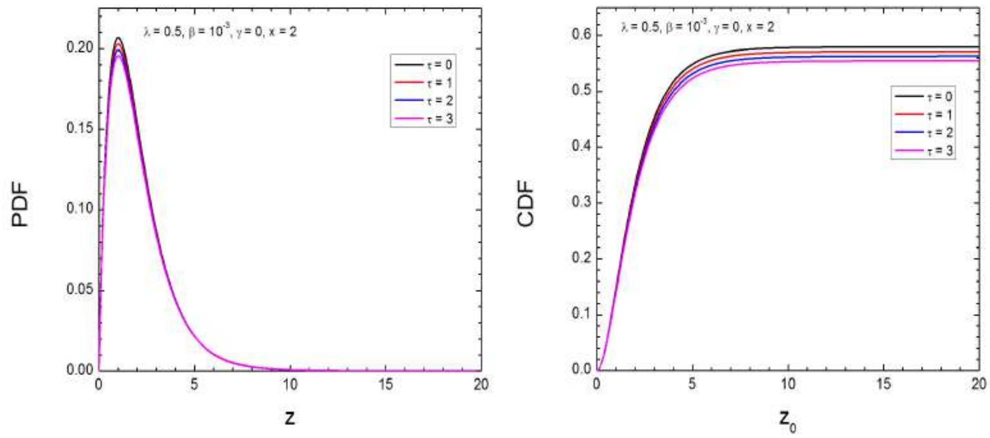

Figure 7. The same as in Fig.6 for $\lambda = 0.5$.

energy release can be written as

$$\bar{P}\left(x, \tau_0, z\right) = \mathrm{e}^{-z} \sum_{n=0}^{\infty} p_n(x, \tau_0) \tilde{\lambda}^n\left(x\right) \frac{z^{2n-1}}{(2n-1)!} \tag{18}$$

Another case of interest is the internal energy source burst of the form os unite jump $H(z)$ known as The Heviside function. Then Eq.(18) yields

$$\bar{P}\left(x, \tau_0, z_0\right) = \mathrm{e}^{-z_0} \sum_{n=0}^{\infty} p_n\left(x, \tau_0\right) \tilde{\lambda}^n\left(x\right) \sum_{k=0}^{z_0} \frac{z^{2n+k}}{(2n+k)!} \tag{19}$$

Figures 6 and 7 demonstrate the PDF (probability density function) and CDF (cumulative distribution function) of the observed spectral line for different values of optical and geometrical parameters. Note that the time reading of z, z_0 in the figures coincides with the moment of the beginning of the quanta' exit from the semi-infinite atmosphere.

The effect of radiation multiple scattering in a spectral line on their temporal characteristics is demonstrated in Figures 6 and 7, where two cases with particular values of $\lambda = 0.9$ and 0.5 are depicted. The main salient trait is that when the level of the scattering process is high, the different regions of the line are set to plateau at different time intervals, with the core of the line being set later, as a rule. Another characteristic feature of the large role of radiant energy scattering manifests itself in the dependence of the time of spectral line appearance on the depth where the primary energy release occurs. It is most discernible in the case of the $\delta(t)$-form energy release by comparing the time

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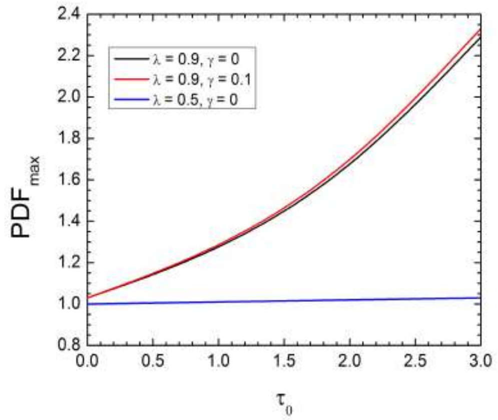

Figure 8. Variations of the maximum values of PDF with optical depth of the primary energy sources.

the observed line reaches its maximum intensity depending on the depth in the atmosphere where the primary source of energy is located. These functions are shown in Fig. 8, from which one can see that the dependence is stronger the stronger the process of radiation scattering in the line developed. At the same time, it is seen that the role of scattering in the continuous spectrum here is insignificant. It is important to note that the effect of scattering in the continuum on the evolution of the spectral line profile is negligible for both strong and weak lines and weakly depends on the value of the scattering coefficient λ.

4. Concluding remarks

The Neumann series for the reflection coefficient from a semi-infinite atmosphere, as well as for the reflectance and transmittance of a medium of finite optical thickness, constructed in our previous works, allow to turn to a time-dependent, more general classical problem in which it is assumed that the primary energy sources are located inside the atmosphere. In the problem considered here, these sources themselves also depend on time. Two possible types of non-stationarity of the sources are considered: $\delta(t)$-shaped energy emission and the form of a unit jump given by the Heaviside $H(t)$ function. The problem posed was to describe the evolution of the profile of the observed absorbtion lines and to reveal its dependence on the depth of local energy sources and the role of scattering in the continuous spectrum. The calculations showed that the influence of the source depth on the observed characteristics of the observed line is strong enough to make, in principle, an idea of the internal source depth on the base of the indicated characteristics (equivalent width, central residual intensity). The temporal characteristics of line changes provide additional information about the capacity and depth of energy released within the atmosphere. It is also shown that scattering in the continuous spectrum does not play a significant role in the evolution of the spectral line profile.

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