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Gasdynamical flows in star forming regions

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Abstract

A review of gas-dynamical flows in astrophysics is given with an emphasis on accretion flows and outflows. Analytical estimates of the dependence of the velocity and density on the radius during spherical accretion are considered, as well as the rate of mass inflow and the corresponding luminosity of the accretion disk are also estimated. Some features of gas inflow and outflow near the star are mentioned, like mass-loading flows where source functions of mass and momentum are presented and analyzed. The velocities and radii of the ionization fronts of hot stars are considered. The Sedov-Taylor analytical formula on the dependence of the radius of the fireball on time during a strong explosion is given, where the explosion energy and density of the medium act as parameters. Examples of the use of dimensional theory in astrophysics, in particular, in accretion flows used above are considered. An example of a numerical solution of the interaction of an interstellar cloud with the solar wind is also given, followed by an estimate of the inflow of the cloud's gas environment into the Earth's atmosphere, which affects the terrestrial ozone content.

Keywords: *ISM: gas-dynamical flows: dimensional analysis: star formation via mass-loading flows*

1. Introduction

Ordered motions of a mass of gas, that is, gas-dynamic flows, occur in astrophysics on all scales, which emphasizes their importance when considering many processes, especially those associated with the formation of stars. In particular, many interesting questions are connected with the reverse influence of these flows on the medium of the star formation. The present review is devoted to such flows, which are especially often formed during the collapse of clouds that give rise to stars, especially flows with loaded mass, but which also often also introduce momentum into the medium of the star formation source.

2. Star formation via accretion onto a gravitating center

Let us consider the motion of a gas towards a stationary center with mass M. The mass potential at a distance r will be respectively given by Frank et al. (2002).

$$\Phi = -GM/r, \tag{1}$$

and for the converted part of the energy

$$\Delta\Phi = GM/r, \tag{2}$$

provided sufficiently large initial distances. Since we are talking about the conversion of potential energy into kinetic energy, a unit mass will acquire speed

$$v_{ff}^2/2 = GM/r, \tag{3}$$

where v_{ff} is the speed of free fall. It is interesting that by means of formulas (2,3) one can immediately estimate the order of the largest part of the potential energy that can be converted into the kinetic energy of motion in this process (spherically symmetric accretion onto a fixed center). Assuming that the mass m has a rest energy mc^2, we obtain

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$$\frac{mGM/r}{mc^2} = (1/2)v_{ff}^2/c^2 = 1/2 \tag{4}$$

provided that the value of v_{ff} approaches the speed of light. Usually, this happens during accretion onto compact relativistic objects such as black holes and neutron stars. Recall that, for example, in thermonuclear fusion reactions, for example, helium, only $\Delta E = 0.007 \cdot mc^2$ is released, that is, accretion is indeed the most efficient energy converter in nature.

3. On the formation of massive stars

The formation of low-mass stars is widely known, and is described in detail in, for example, Bodenheimer (2011); Stahler & Palla (2005). Massive stars are more interesting in terms of environmental feedback, but there are also many unresolved problems (e.g. Louvet (2018)). Let us clarify that we will call massive stars with a mass of the order of $8M_\odot$ and more. Being powerful sources of L_c radiation and stellar wind, newly forming stars have a significant impact on the vicinity of the molecular cloud, the source of matter for forming stars, and bring a large amount of mass, momentum and energy into the interstellar medium right after formation. Note that the characteristic glow time of a star with luminosity, L_*, mass, M_*, and radius R_* is determined by the formula $t_K = (GM_*^2)/(R_* \cdot L_*)$ which can be rewritten as (Dyson (1994))

$$t_K = (GM_*^2)/(R_* \cdot L_*) = 3 \cdot 10^7 \cdot (M_*/M_\odot)^2 (R_*/R_\odot)^{-1} (L_*/L_\odot))^{-1} years, \tag{5}$$

and, since $L_* = M_*^{3.5}$ on the upper part of the main sequence (MS), this time is highly dependent on mass, and is no more than 10^5 years even for the stars with minimal for that case masses of $M_* = 10M_\odot$. To estimate the time of accumulation of matter by a protostar (accretion time), we set $\dot{M} = M_*/t$, where $t = R_*/v$, and v is the virial velocity $v^2 = (GM_*)/R_*$, that is, $\dot{M} = v^3/G$, and so (Shu et al. (1987)),

$$t_{acc} = (GM_*)/v^3, \tag{6}$$

where also $v = a$ is the isothermal speed of sound at the corresponding temperatures - the rate of transmission of gas-dynamic perturbations in this process. Comparing the estimate of t_K with the time it takes for the protostar to collect matter (t_{acc} - is the accretion time), Dyson (1994) noted that massive stars should continue to gain mass, that is, accrete, already in the nuclear reactions phase that can serve as energy sources, since this time is clearly longer:

$$t_{acc} = (GM_*)/(a^3) \approx 4 \cdot 10^4 (M_*/M_\odot)(a/(1km/s))^{(-3)} yr, \tag{7}$$

and $t_{acc} = 5 - 10 \cdot 10^5$ years, again for $M_* = 10M_\odot$.

As a result, the protostar (with the reservation about the beginning of nuclear reactions) remains immersed in a gas-dust shell (cocoon), which is very sensitive to the onset of hard radiation and the stellar wind, analyzed in detail in (Kahn (1974)).

The density distribution in this accretion phase is given by the obvious formula

$$\rho(r) = \frac{\dot{M}}{(4\pi r^2 v)} = \frac{\dot{M} r^{-3/2}}{4\pi (2GM_*)^{1/2}}, \tag{8}$$

together with (3), this formula determines the behavior of the main gas-dynamic parameters at the early stage of accretion. To describe the temperature, it is necessary to clarify which stage, isothermal or adiabatic accretion, is in question. Recall that with intensive emission, an isothermal process with a constant temperature is established, and

$$P = K\rho^{-1}, K = const., \gamma = 1. \tag{9}$$

And when energy is conserved, an adiabatic process takes place, in which

$$P = K\rho^{-\gamma}, K = const., \gamma = 5/3. \tag{10}$$

When the protostar reaches the main sequence, but still gaining mass, the star (protostar) begins to ionize the surrounding gas, and the so-called ultra-compact HII region - UCHII is formed. We will analyze the propagation velocity of the boundary of this region below, in Section 9.

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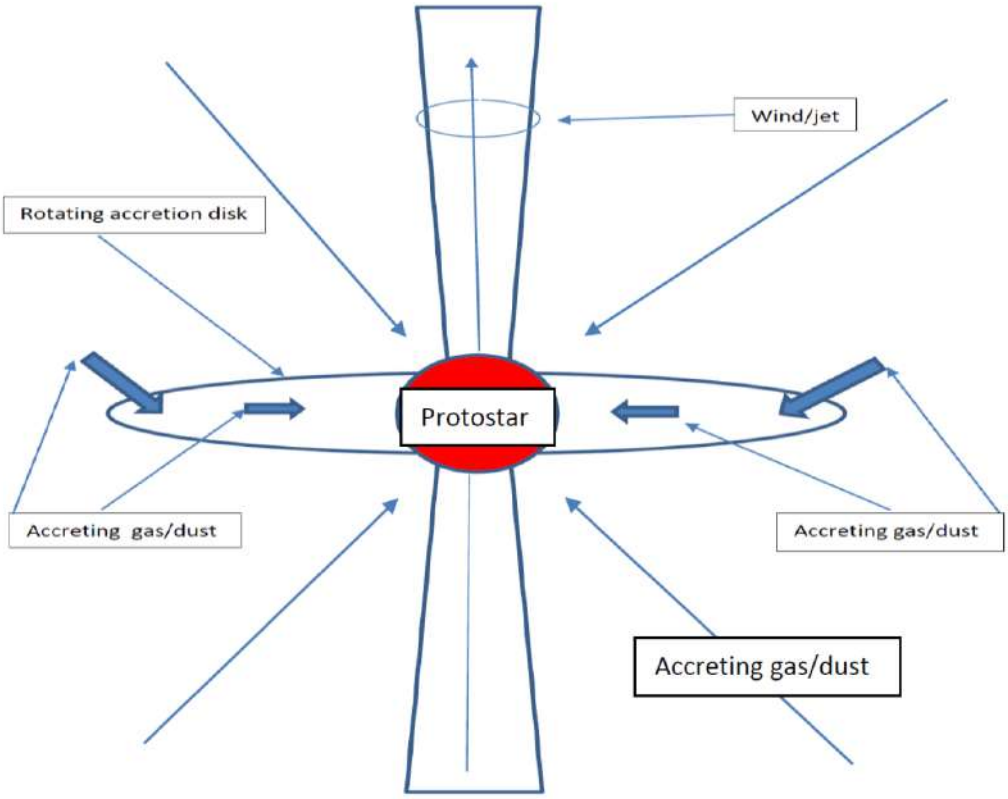

Figure 1. Scheme of the formation of a protostar.

4. On the formation of a disk rotating around a gravitating center

The rotation of the collapsing cloud, even with a low initial speed, due to the conservation of angular momentum, causes the resulting disk to rotate at a rather high speed. The point, obviously, is that the streamlines, together with the collapsing matter, deviate from the axis of rotation towards the equatorial plane (Cassen & Moosman (2019)). At the same time, since for the accretion of matter with a cessation, for example, on the surface of a star, the released energy (per unit mass) is estimated as (Frank et al. (2002))

$$v_{ff}^2/2 = GM/r, \tag{11}$$

and in the case of the formation of a disk with Keplerian orbital motion

$$E_{orb} = GM/2r, \tag{12}$$

and the rest can be converted into the internal energy of the gas, and/or, by means of radiation, leave the medium. Let us discuss some features of the resulting disk. Since the specific angular momentum of a homogeneous cloud rotating with a uniform angular velocity Ω

$$j = vr \tag{13}$$

is conserved under compression, and the ratio of the rotational to the absolute value of the gravitational energy is

$$\beta = E_{rot}/\mid E_{grav} \mid = r^2\Omega^2/2/(GM_*/r) = (r^3\Omega^2)/(2GM_*). \tag{14}$$

According to observations, β=0.1-0.01 (Bodenheimer (2011)). One can introduce the concept of the critical centrifugal radius, R_{ct} on which the centrifugal force is balanced by the gravitational one. According to the definitions

$$R_{ct} = (\Omega^2 r^4)/GM, \tag{15}$$

where it is necessary to put $r = R_*$ at the moment of equilibrium of these forces. From this radius value, all the infalling matter goes to the disk, since the conservation laws rule out another place of accretion, such as the surface of a star.

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So, part of the released energy of the accreted matter goes to increase the internal energy of the disk medium. In an adiabatic process, the internal energy is determined by the expression

$$e = P/(\gamma - 1)\rho, \tag{16}$$

where γ=5/3 and pressure is related to temperature by the usual equation of state,

$$P = R_{gas}\rho T/\mu, \tag{17}$$

$R_{gas} = 8.31 \cdot 10^7$ erg/(mole K) is the gas constant, and μ is the average molecular weight.

The disk temperature T is given by the energy balance of the disk, taking into account many processes, however, for orders of magnitude estimates, we can write, as shown above, $1/2(GM_*)/r = P/(\gamma-1)\rho = R_{gas}T/(\gamma-1)\mu$, and

$$T = 1/2(\gamma - 1)\frac{GM_*}{r}\frac{\mu}{R_{gas}}. \tag{18}$$

For a protostar with a solar mass, the disk temperature can be about 10^4 K during accretion to form a disk at a distance of 1 AU. Further, this energy can, for example, be radiated, respectively, the luminosity (of a black body) will be

$$L_* = \dot{M}GM_*/r = 4\pi r^2\sigma T^4. \tag{19}$$

5. On the gas-dynamic description of the accretion process

Let us now discuss the description of accretion in the absence of a magnetic field, more precisely, when the magnetic forces are insignificant compared to the gravitational ones. In the absence of sources (mass and momentum), the conservation laws dictate the following system of gas-dynamic equations (Frank et al. (2002))

$$\partial\rho/\partial t + \nabla\cdot(\rho\vec{v}) = -\nabla P + \vec{f}, \tag{20}$$

$$\rho\partial\vec{v}/\partial t + \rho\vec{v}\cdot\nabla\vec{v} = -\nabla P + \vec{f}, \tag{21}$$

$$\partial/\partial t(1/2\rho v^2 + \rho e) + \nabla\cdot[(1/2\rho v^2 + \rho e + P)\vec{v}] = \vec{f}\cdot\vec{v}. \tag{22}$$

Here

$$P = \rho kT/(\mu m_H), \tag{23}$$

and

$$\vec{f} = \rho\vec{g}, g = (GM_*)/r^2. \tag{24}$$

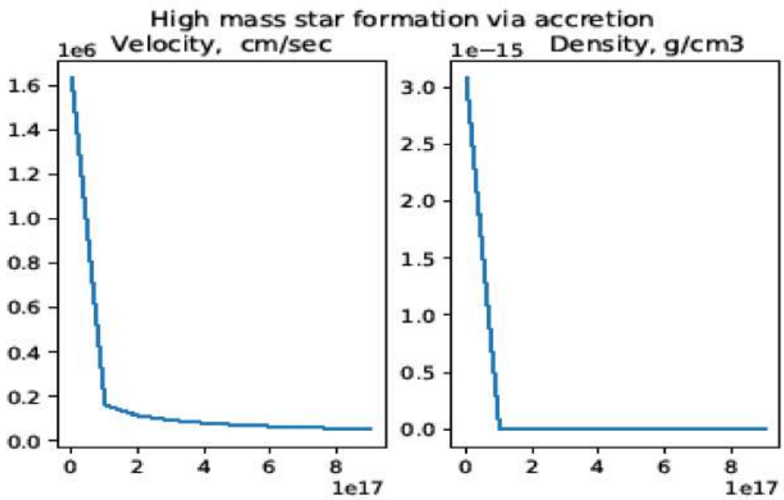

Figure 2. Distribution of velocity and density along the radius, according to free-fall dependencies for spherically symmetric accretion, $M_* = 10M_\odot, \dot{M} \approx 10^{22} g/s \approx 10^{-3} M_\odot/yr$

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First of all, we will discuss stationary spherically symmetric accretion. From the most general considerations, we have in this case:

$$\dot{M} = 4\pi r^2 \rho v = const., \tag{25}$$

and also $M_* \approx const.$. From conservation of mass in (20) it follows

$$\nabla \cdot (\rho \vec{v}) = 0, \tag{26}$$

or

$$d/dr(\rho v r^2) = 0. \tag{27}$$

Thus, in free fall mode, when $v = v_{ff} = \sqrt{2GM_*/r)}$, then

$$2/r + 1/\rho d\rho/dr + 1/v dv/dr = 0, \tag{28}$$

so $1/\rho d\rho/dr = -3/2r$, and $\rho(r) = const. \cdot r^{(-3/2)}$, in full agreement with (8), with spherically symmetric accretion onto a fixed gravitating center.

As noted above, for the accretion rate in this case we can write $\dot{M} \approx M_*/t_{ff}$, where, as is easy to see $t_{ff} \approx 1/\sqrt{G\rho}$. The exact value of the numerical coefficient is also easy to obtain (Dyson & Williams (1997), for the neutral HI cloud):

$$t_{ff} = \sqrt{(3\pi/32G\rho} = (4.3 \cdot 10^7/\sqrt{n_H} \quad years. \tag{29}$$

It is from the condition that the gravitational energy of the cloud exceeds the thermal one (in the absence of a magnetic field and turbulent vortices) that the Jeans condition of gravitational instability follows, when the cloud contracts into a protostar, and the critical value of the radius is easy to determine: $C \cdot GM_*^2/R_J \geq 3/2 \cdot R_{gas} T_* M_*/\mu$, and

$$R_J = 0.4GM_*/R_{gas}T. \tag{30}$$

It is easy to verify that the critical value of the Jeans radius is approximately equal to the time of free fall multiplied by the isothermal speed of sound a (the perturbation transfer rate):

$$R_J = \sqrt{(R_{gas}T/\mu)} \cdot 1/\sqrt{(G\rho)} \approx a \cdot t_{ff}. \tag{31}$$

Here $a = \sqrt{(R_{gas}T/\mu)}$, and, as noted above, also

$$\dot{M} \approx a^3/G, \tag{32}$$

with $L_* = G\dot{M} \cdot M_*/R_*$.

Let us give numerical estimates. For example, for a cold clump with $T = 10K, M_* \approx 0.14M_\odot, R_* \approx 10^{16}$ cm, the accretion rate is $\dot{M} \approx 10^{20}$ g/s, and the protostar luminosity is $L_* \approx 10^{34}$ erg/s (at $a = 0.187$ km/s), while for massive stars $M_* \approx 10M_\odot, R_* \approx 10^{18}$ cm, the accretion rate is $\dot{M} \approx 10^{22}$ g/s, while the luminosity of the protostar is $L_* \approx 10^{37}$ erg/s (at the same values of temperature and sound speed).

6. Bernoulli equation: inflow and outflow of flows in the vicinity of the center

Under the condition of stationarity, the Euler equation (21) can be written as (Frank et al. (2002)):

$$v\frac{dv}{dr} + \frac{1}{\rho} \cdot \frac{dP}{dr} + \frac{GM_*}{r^2} = 0 \tag{33}$$

direct integration of which will give,

$$\frac{v^2}{2} + \int \frac{dP}{\rho} - \frac{GM_*}{r} = const. \tag{34}$$

Taking into account (10) $dP = K\gamma\rho^{\gamma-1}d\rho$, so

$$\frac{v^2}{2} + \frac{K\gamma}{\gamma-1}\rho^{\gamma-1} - \frac{GM_*}{r} = const., \tag{35}$$

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where $\gamma \neq 1$. But $K\gamma\rho^{\gamma-1} = \gamma P/\rho = a^2$, where a is the isothermal sound speed , and we obtain the Bernoulli integral about conservation of (36) along the streamlines:

$$\frac{v^2}{2} + \frac{a^2}{\gamma - 1} - \frac{GM_*}{r} = const. \tag{36}$$

From it, as expected, at large distances from the protostar, the speed tends to the speed of sound, slightly exceeding it (and both together - to zero), and at small distances - to the speed of free fall.

Now we transform (33):

$$v\frac{dv}{dr} = \frac{\frac{2a^2}{r} - \frac{GM_*}{r^2}}{v^2 - a^2}, \tag{37}$$

and because $1/\rho \cdot dP/dr = a^2/\rho \cdot d\rho/dr$, so, from (27) $1/\rho \cdot d\rho/dr = -1/(vr^2) \cdot d/dr(vr^2)$. Then from (37) it follows that the flow velocity becomes equal to the speed of sound a at the coordinate of the "sound point" according to

$$r_s = (GM_*)/(2a^2), \tag{38}$$

moreover, from the definition of the "second cosmic velocity" at this point,

$$v_{esc} = \sqrt{2GM_*/r_s}, \tag{39}$$

it follows

$$v_{esc} = 2a. \tag{40}$$

Using de l'Hopital's rule at the sound point,

$$\frac{dv}{dr(r = r_s)} = \frac{\pm 2a^3}{GM_*} = \frac{\pm 2\dot{M}}{M_*}, \tag{41}$$

we obtain that both inflows and outflows of matter are possible during gas-dynamic spherically symmetric accretion, and we recall that the accretion (outflow) rate $\dot{M}$ is determined by the formula (32) $\dot{M} \approx a^3/G$.

7. On the expansion of supernova remnants after the explosion

Let us briefly discuss the process of interaction of expanding supernova remnants with the surrounding interstellar medium. As is known, during a supernova (type II) explosion, namely, during the collapse of the nucleus, it is possible to release energy of the order of $E_* \approx (GM^2)/R$, or up to 10^{53} erg during the collapse of the mass $M \approx M_\odot$ in a region with a size of $R \approx 10^6$ cm, that is, during the formation of a neutron star. The rest of the mass of the pre-supernova star is thrown out, the formed shell expands at a high speed of about 5000-10000 km/s (see Figures 3 and 4). At the same time, according to observational data (Dyson & Williams (1997)), about $\epsilon \approx 1\%$ is the kinetic energy of the remnant expansion ($\approx MV^2 \approx 10^{51}$ erg, at an expansion velocity of $\approx 10^4$ km/s with a mass of about $1M_\odot$). Let us mention, for completeness, that the excess energy of the explosion is carried away by neutrinos that practically do not interact with matter.

At the adiabatic stage, radiative energy losses are insignificant, and the total energy of the shell behind the shock wave front, thermal and kinetic (e_T and e_K, per unit mass), should be equal to the explosion energy, E_*. It can be shown (Dyson & Williams (1997)) that for a monatomic gas both types are equal,

$$e_T = e_K = 9/32 \cdot \dot{R}^2, \tag{42}$$

where R is the radius of the shell, and $\dot{R}$ is its velocity. Then the total energy of the shell, E_T, with a gas of density ρ_0 (and concentration n_0), is:

$$E_T = 4/3 \cdot \pi R^3 \cdot \rho_0 \cdot (e_T + e_K) = 3/4\pi n_0 m_H R^3 \dot{R}^2. \tag{43}$$

Because $E_T = E_*$ then

$$R^3 \cdot \dot{R}^2 = 4/3\pi E_*/\rho_0. \tag{44}$$

Since at $t \to 0, R \to 0$, then the solution to (44) is (the Sedov-Taylor formula - Dyson & Williams (1997))

$$R = (25/3\pi)^{1/5} \cdot (E_*/\rho_0)^{1/5} \cdot t^{2/5}, \tag{45}$$

and

$$\dot{R} = 2/5 \cdot (25/3\pi)^{1/5} \cdot (E_*/\rho_0)^{1/5} \cdot t^{-3/5}. \tag{46}$$

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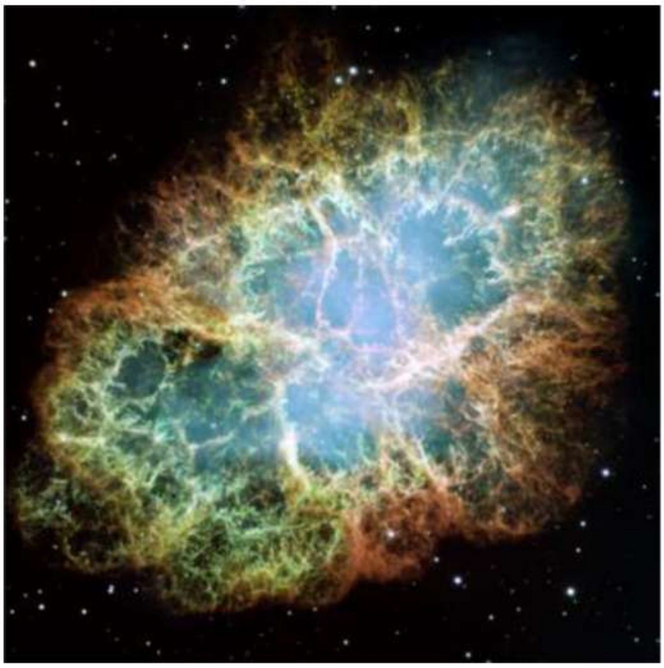

Figure 3. The Crab Nebula, SN 1054. (NASA, ESA, J.Hester and A.Loll). The Crab Nebula is the... remnant, of a massive star in our Milky Way that died 6,500 light-years away. Astronomers and careful observers saw the supernova in the year 1054. Image credit: NASA, ESA, J. Hester and A. Loll (Arizona State University)

8. On the use of dimension theory in the study of astrophysical flows associated with star formation processes

We will solve several problems concerning flows of interstellar matter (with one exception) using the methods of the theory of dimensions, including the problems discussed above. In some cases, the theory of dimensions of physical quantities is successfully used during the initial assessments of astrophysical processes (Dibay & Kaplan (1976)). Here the concepts of the defining parameters of a physical system in a quasi-stationary state are used. We will solve several problems on the application of the theory of dimension in astrophysics, from which the details of the use will become clear.

Problem 1. For example, for a planet rotating around the Sun, the determining parameters are the mass of the Sun (M), the period of rotation (P), the radius of the orbit (r), and, the gravitational interaction constant (G), which determines the process itself. Here, the number of defining parameters is one more than the number of basic dimensions, in this case, "cm, g, sec". Dimensions of defining parameters are given in a matrix of dimensions, compiled from a table of basic dimensions and defining dimensions. Let's make a matrix of dimensions.

Table 1.

defining parameters	**M**	**P**	**R**	**G**
basic dimensions	g	s	cm	$cm^3 \cdot g^{-1} \cdot s^{-2}$
g	1	0	0	-1
cm	0	0	1	3
s	0	1	0	-2

Multiplying the rows of the matrix with a vector row of exponents $\alpha, \beta, \gamma, \delta$, in accordance with the procedure for constructing a dimensionless complex $const. = M^\alpha \cdot P^\beta \cdot R^\gamma \cdot G^\delta$, we obtain the following system of linear algebraic equations (according to the theory, exponents can only be rational numbers, and arguments to functions other than rational fractions must be dimensionless):

$$1 \cdot \alpha + 0 \cdot \beta + 0 \cdot \gamma - 1 \cdot \delta = 0, \tag{47}$$

$$0 \cdot \alpha + 0 \cdot \beta + 1 \cdot \gamma + 3 \cdot \delta = 0, \tag{48}$$

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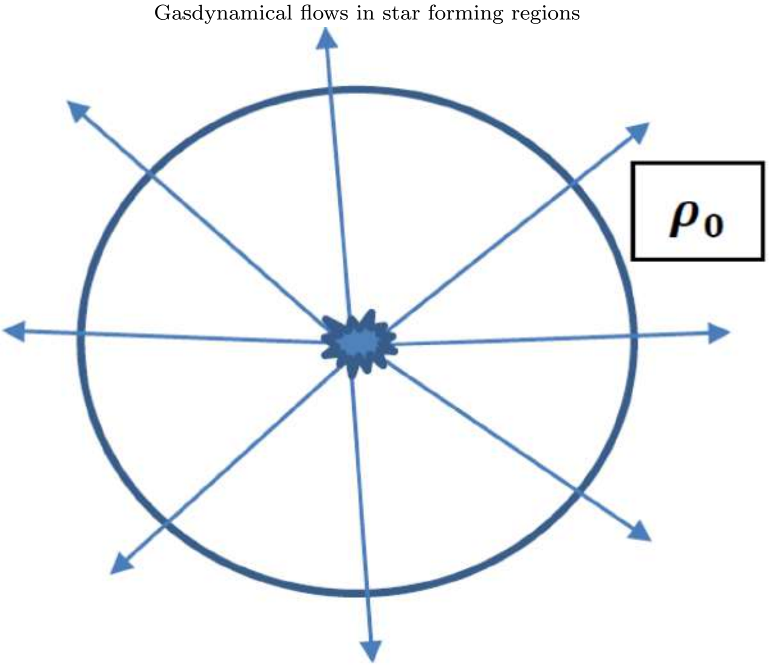

Figure 4. Supernova remnants inflowing into the ISM with density ρ_0.

$$0 \cdot \alpha + 1 \cdot \beta + 0 \cdot \gamma - 2 \cdot \delta = 0. \tag{49}$$

The number of variables is greater than the number of equations, therefore, without loss of generality (as is usually done when solving overdetermined systems), we can set $\alpha = 1$, then $\beta = 2, \gamma = -3, \delta = 1$, that is $MP^2R^{-3}G = const.$, or

$$\frac{P^2}{R^3} = const. \cdot \frac{1}{GM}. \tag{50}$$

Thus, from the most general considerations, Kepler's 3rd law was obtained. The numerical value of the constant is not determined, which is obvious, by the methods of the theory of dimensions, and can be obtained from a rigorous theory. But, since this constant is the same for all planets, we have for any two planets $P_1^2/R_1^3 = P_2^2/R_2^3$.

Problem 2. In the problem of a strong explosion, when describing the behavior of supernova remnants in the interstellar medium (cf. p.7), the defining parameters are E – explosion energy (erg), ρ – density of the interstellar medium (g/cm^3), R – radius (cm), and t is time (s). We make a matrix of dimensions.

Table 2.

defining parameters	**E**	ρ	**R**	**t**
basic dimensions	Erg or $g \cdot cm^2 \cdot s^{-2}$	$g \cdot cm^{-3}$	cm	s
g	2	-3	1	0
cm	1	1	0	0
s	-2	0	0	1

The dimensionless complex $const. = E^\alpha \rho^\beta R^\gamma t^\delta$ reduces to the system

$$2 \cdot -3 \cdot \beta + 1 \cdot \gamma + 0 \cdot \delta = 0 \tag{51}$$

$$1 \cdot \alpha + 1 \cdot \beta + 0 \cdot \gamma + 0 \cdot \delta = 0 \tag{52}$$

$$-2 \cdot \alpha + 0 \cdot \beta + 0 \cdot \gamma + 1 \cdot \delta = 0. \tag{53}$$

We choose $\gamma = 1$, and we get $\alpha = -1/5, \beta = 1/5, \delta = -2/5$. Eventually, $R = const. \cdot (E/\rho^{1/5} t^{2/5}$, and, $\dot{R} = const. \cdot 2/5(E/\rho)^{1/5} t^{-3/5}$. And this is the Sedov-Taylor formula in the problem of a strong explosion, which was successfully used to describe the explosion of a supernova and the expansion of its remnant (eq. 45-46). And again, the value of the numerical constant is not known, but it appears in the exact formula (45).

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Table 3.

Defining parameters	R	V	M	G
Basic dimensions	cm	cm/s	kg	$cm^3 \cdot s^{-2} \cdot g^{-1}$
g	1	1	0	3
cm	0	0	1	-1
s	0	-1	0	-2

Problem 3. Consider the problem of determining the velocity of a gas freely falling towards the center of gravitation. Expressions for the first (and second) cosmic velocities are defined similarly. The defining parameters here are R, V, M and G. We compose a matrix of dimensions,

The dimensionless complex $const. = R^{\alpha}V^{\beta}M^{\gamma}G^{\delta}$ is reduced to the system

$$1 \cdot \alpha + 1 \cdot \beta + 0 \cdot \gamma + 3 \cdot \delta = 0 \tag{54}$$

$$0 \cdot \alpha + 0 \cdot \beta + 1 \cdot \gamma - 1 \cdot \delta = 0 \tag{55}$$

$$0 \cdot \alpha - 1 \cdot \beta + 0 \cdot \gamma - 2 \cdot \delta = 0. \tag{56}$$

We get the solution: $\alpha = 1, \beta = 2, \gamma = -1, \delta = -1$, and write down the formula:

$$R = const. GM/V^2, \tag{57}$$

which coincides with formula (3) with an accuracy of a factor of 2.

Problem 4. Let's solve the problem of the spherically symmetric accretion rate of onto a fixed center. The defining parameters here are $\dot{M}, V, M$ and G. We compose a matrix of dimensions,

Table 4.

Defining parameters	$\dot{M}$	V	M	G
Basic dimensions	g/s	cm/s	g	$cm^3 \cdot s^{-2} \cdot g^{-1}$
g	1	1	0	3
cm	0	0	1	-1
s	0	-1	0	-2

The dimensionless complex $const. = \dot{M}^{\alpha}V^{\beta}M^{\gamma}G^{\delta}$ reduces to the system

$$1 \cdot \alpha + 1 \cdot \beta + 0 \cdot \gamma + 3 \cdot \delta = 0 \tag{58}$$

$$0 \cdot \alpha + 0 \cdot \beta + 1 \cdot \gamma - 1 \cdot \delta = 0 \tag{59}$$

$$0 \cdot \alpha - 1 \cdot \beta + 0 \cdot \gamma - 2 \cdot \delta = 0. \tag{60}$$

We get the solution: $\alpha = 1, \beta = 2, \gamma = -1, \delta = 1$, and write down the formula:

$$\dot{M} = const. \cdot (MV^3)/G, \tag{61}$$

which coincides with formula (32) with an accuracy of a numerical factor of order 1.

9. On mass-loaded flows during formation of massive stars

When a molecular cloud is contracted, followed by the formation of a star, flows with continuously added gas along the motion are possible. The source of this gas can be clumps - condensations, which are 3–4 orders of magnitude denser than the current density, and one should distinguish between the period of the onset of hard radiation, but without stellar wind, when UCHII is already formed. It is also possible that the influence of the stellar wind is insignificant, which is typical for not very massive stars. We should be interested in the period with an intense wind, on the order of $\approx 10^{-6} M_{\odot}/yr$, when UCHII is formed under the influence of both hard radiation and a sufficiently powerful wind. In this case, the wind flow itself can be considered isothermal, since the loaded mass is excited and immediately radiate, and the emission measure is proportional to the square of the density. As a result, a uniform temperature of the order of $\approx 10^4$ K

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is established in the region of intense HII emission. The ionization is responsible for the radiation of the protostar, where thermonuclear reactions are already taking place, the radiation energy from these reactions propagates to the outer layers and is radiated through the photosphere into the environment, in the first approximation, as an absolutely black body with an effective temperature inherent in massive O, B stars.

In the case of mass-loaded flows, taking into account stationarity and isothermality, equations (20-21) should be written as (Hartquist et al. (1986), Yeghikyan (1999)):

$$\frac{d}{dr} \cdot \rho v r^2 = S r^2, \tag{62}$$

$$\frac{1}{r^2}\frac{d}{dr}\rho v^2 r^2 = -S u_c - \frac{dP}{dr} - \frac{GM_*}{r^2}, \tag{63}$$

and also

$$P = \rho \cdot c^2, \tag{64}$$

where the first term on the right in (63) takes into account the momentum of gas portions flowing from moving clumps, and the minus sign is the direction towards the wind. Our goal is to show that a reasonable combination of the observed parameters of fast winds leads to the establishment of protostar parameters that do not contradict the observations. We emphasize that here we take into account one type of source, due to cloud clumps, with one type of elementary processes responsible for adding mass, namely, photoionization.

After some algebraic transformations, we bring the system to the form:

$$\frac{du}{dr}(u^2 - c^2) = -\frac{S}{\rho}(u^2 + c^2 + u u_c) + \frac{2uc^2}{r} - \frac{GM_* u}{r^2}, \tag{65}$$

$$\frac{d\rho}{dr} = -\frac{\rho}{u}\frac{du}{dr} - \frac{2\rho}{r} + \frac{S}{u}. \tag{66}$$

Let us first neglect the term on the right in (65), which takes into account gravity, and exclude ρ by means of (25), which in this case is written as

$$4\pi r^2 \cdot \rho u = \dot{M}_s + 4\pi \cdot \int_{r_0}^{r} S r'^2 dr', \tag{67}$$

where $\dot{M}_s$ is the mass loss rate of the central star,

$$\dot{M}_s = 4\pi r_0^2 \rho_0 v_\infty. \tag{68}$$

The values with the index "0" refer to the inner boundary of the cloud from the side of the star, and v_∞ is the limiting speed of the fast wind, and

$$4\pi \int_{r_0}^{r} S r'^2 dr' \equiv I(r). \tag{69}$$

After subsequent integration, we finally get:

$$u = -\frac{v_s I(r) + v_\infty \dot{M}_s}{\dot{M}_s + I(r)}, \tag{70}$$

and

$$\rho = \frac{[\dot{M}_s + I(r)]^2}{(4\pi r^2 [v_s I(r) + v_\infty \dot{M}_s])}. \tag{71}$$

Gravity is not taken into account here, but the momentum introduced during the loaded mass from condensations (clumps) is taken into account. For the first time in analytical theories of the formation of massive stars in this form, they were used in (Lizano et al. (1996)), and in models of planetary nebulae, in (Yeghikyan (1999)). Mention should also be made of (Johnson & Axford (1986)), where the same equations are used to describe the galactic wind. The distribution (70,71) for one set of parameters is shown in Fig. 5.

With a uniform distribution of mass loading centers, $S(r) = S_0 = const$, and $v_s = 20$ km/s=const, we have $I(r = r_m) = (4/3)S_0 \pi r_m^3$, where $r_m^3 = 10^{18}$ cm – is the maximum value of the radius from which accretion starts. With a power dependence of the form $S(r) = S_0(r_0/r)^\alpha, I(r) = 4\pi S_0(r^{3-\alpha} - r_0^{3-\alpha})/(3-\alpha)$.

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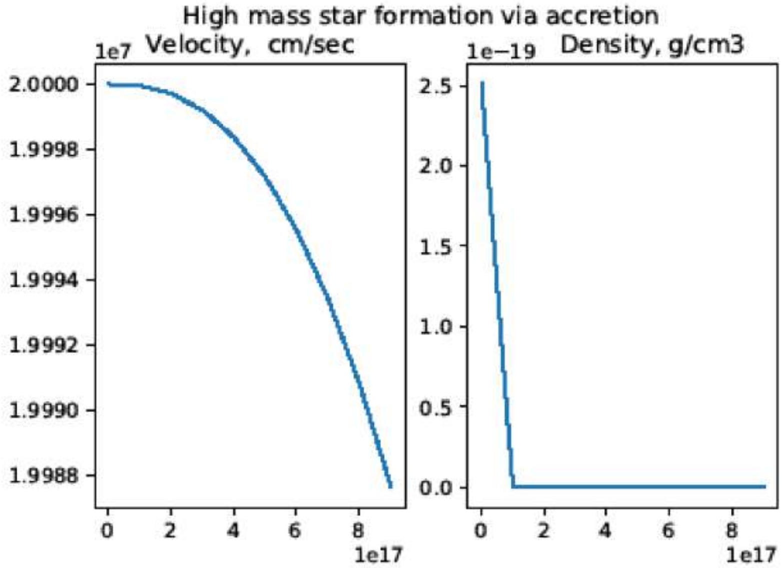

Figure 5. Velocity and density distribution along the radius, (70,71) for spherically symmetric stationary accretion with loaded mass, $M_* \approx 10 M_\odot, \dot{M} \approx 10^{19}$ g/s $(10^{-6} M_\odot), v_\infty = 2000$km/s, $v_s = 20$km/s, $\alpha = 0.5$
.

Finally, in the general case, one should separate the possibility of mass loading directly from cloud clumps (we denote it as (S_c-primary) and by means of the stellar wind (S_W-secondary), then the equations will be written in the form

$$\frac{d\rho}{dr} = -\frac{\rho}{u}\frac{du}{dr} - \frac{2\rho}{r} + \frac{(S_w + S_c)}{u}, \tag{72}$$

$$\frac{du}{dr} = \frac{u}{(\rho(u^2 - c^2))}[S_c(u_c - u) - S_w(u + u_c)] - \frac{(GM_* u)}{((u^2 - c^2)r^2)} - \frac{(c^2(S_w + S_c))}{\rho(u^2 - c^2)} + \frac{(2uc^2)}{(r(u^2 - c^2))}. \tag{73}$$

The solution of the system of differential equations (72,73) for one set of parameters is shown in Fig. 6.

The possibility of mass loading via two channels (photoionization and charge exchange reactions) is described in more detail in Yeghikyan et al. (2022), and a comparison with observations is also made there.

10. Propagation velocity of ionization of the HII region - the velocity of the photoionization front

As soon as a massive star reaches the MS and becomes a source of strong radiation that ionizes hydrogen, an HII region is formed in the molecular cloud, the boundary of which separates it from the HI region (Fig. 7), which, in turn, passes into the completely molecular H_2 region. The rate of progress of the boundary between the ionized and non-ionized parts of hydrogen, dr/dt, can be determined, under the condition of stationarity of the processes, by means of relations

$$J \cdot dt = n_0 \cdot dr, \tag{74}$$

where J is the flux of hydrogen-ionizing photons arriving at the interface at time t, and n_0 is the hydrogen concentration. Hence, $dr/dt = J/n_0$, and, since according to the definition of the Strömgren sphere at stationarity, the total number of L_c quanta emitted by the star must be equal to the total number of photons absorbed by hydrogen and photorecombination photons

$$S_*(L_c) = 4\pi r^2 J + 4/3\pi r^3 n_0^2 \beta_2, \tag{75}$$

where β_2 - is the total number of photorecombinations per level 2 of the hydrogen atom (Dyson & Williams (1997)). From (74,75) we find

$$\frac{dr}{dt} = \frac{S_*}{(4\pi r^2 n_0)} - 1/3 r n_0 \beta_2, \tag{76}$$

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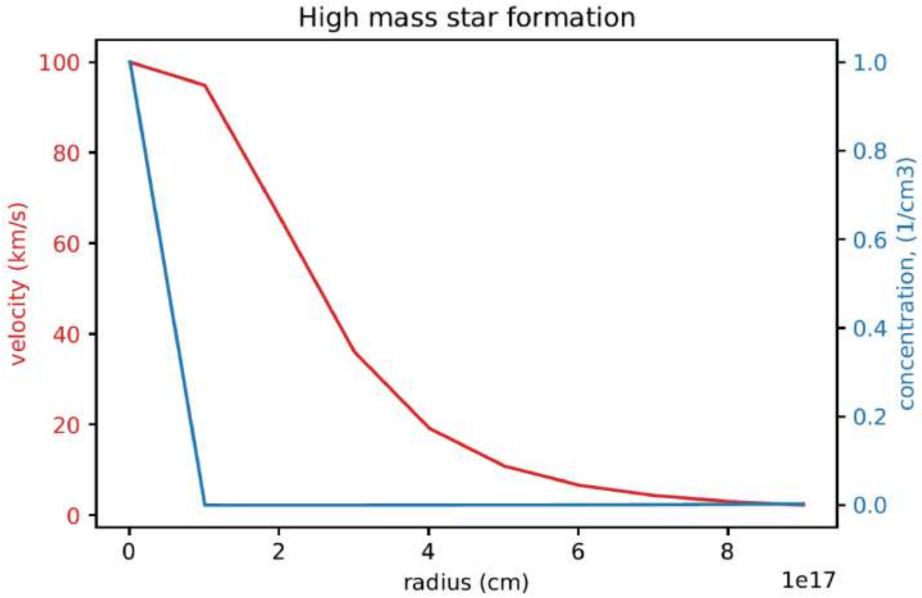

Figure 6. The distribution of velocity and density (concentration) along the radius, given by (72,73) for spherically symmetric accretion with loaded mass rate, primary ($10^{-3} M_\odot$/yr), and secondary $10^{-6} M_\odot$/yr), $M_* \approx 10 M_\odot, v_\infty = 2000$ km/s, $v_s = 20$ km/s. Initial conditions: $v = 100$ km/s, $n = 10^9$ cm^{-3} at $r_0 = 10^{15}$ cm.

.

where

$$R_s = \left(\frac{3S_*}{4\pi n_0^2 \beta_2}\right)^{1/3} \tag{77}$$

is the radius of the Strömgren sphere. As a result, introducing the notation for dimensionless quantities, $\lambda = r/R_s, \tau = t/t_r, V_r = R_s/t_r, \lambda = dr/dt/V_r$, and also $t_r \approx (n_0\beta_2)^(-1)$ – characteristic photorecombination time, we get equation (78)

$$\dot{\lambda} = 1/3((1-\lambda^3))/\lambda^2, \tag{78}$$

whose solution is

$$\lambda = (1 - e^{-\tau})^{1/3}, \tag{79}$$

with the initial condition, $\tau \to 0, \lambda \to 0$.

Table 5. The numerical example for $S_*(L_c) = 10^{49}$ s^{-1} and $n_0 = 100$ cm$^{-3}, \beta_2 = 2 \cdot 10^{-13}$ cm^3 s^{-1}, (Dyson & Williams (1997), Table 7.1).

τ	$\dot{\lambda}$	λ	**dr/dt**
0.1	1.42	0.46	2630
0.7	0.25	0.80	460
0.9	0.19	0.84	352
1.0	0.16	0.86	300
2.0	0.05	0.95	93
4.0	0.006	0.99	11

11. One example of a numerical solution of a system of gas-dynamic equations describing the gas flow of a molecular cloud around the Sun, passing through such a cloud

Every few hundred million years, the Sun, along with the Earth, passes through molecular clouds, while the heliosphere is compressed somewhere about to the Earth's orbit and the cloud material enters directly into the Earth's atmosphere (Yeghikyan & Fahr (2004a),Yeghikyan & Fahr (2004b), Yeghikyan & Fahr

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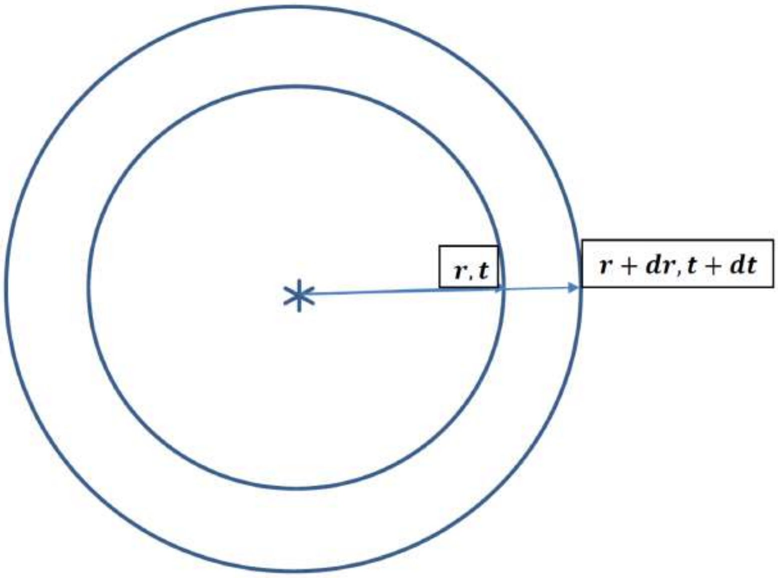

Figure 7. Velocity of the photoionization front separating the HII and HI regions around a massive star with a photon flux J (according to Dyson & Williams (1997)).

.

(2006)). The description of the flow of cloud matter around the heliosphere is reduced here to solving a system of gas-dynamic equations in the form

$$\frac{\partial \rho_j}{\partial t} + \nabla(\rho_j \mathbf{V_j}) = S_\rho, j, \tag{80}$$

$$\frac{\partial(\rho_j \mathbf{V_j})}{\partial t} + \nabla\mathbf{\Pi_j} = \rho_j(1-\mu_j)\nabla(GM/R) + S_V, j, \tag{81}$$

$$\frac{\partial E_j}{\partial t} + \nabla[\mathbf{V_j}(E_j + P_j)] = \rho_j(1-\mu_j)\mathbf{V_j}\nabla(GM/R) + S_E, j, \tag{82}$$

where ρ_j, V_j and P_j – are the density, velocity and (scalar) pressure of atoms (j=n) and protons (j=p) of the flow, respectively, $\mathbf{\Pi}_{ik,j} = \rho_j V_i V_k + P_j \delta_i k$ – is the hydrodynamic stress tensor, and $E_j = 1/2\rho_j V_j^2 + P_j/(\gamma-1)$ – is the energy density. The right-hand sides contain source terms due to gravity and photoionization, and mutual recharging of atoms and protons:

$$S_(\rho, n) = -\beta_{phi}\rho_n + (\rho_p^2\alpha)/m_p, \tag{83}$$

$$S_(\rho, p) = \beta_{phi}\rho_n - (\rho_p^2\alpha)/m_p, \tag{84}$$

$$S_(V, n) = -\beta_{phi}\rho_n\mathbf{V_n} - v_{ce} - \rho_n(\mathbf{V_n} - \mathbf{V_p}) + (\rho_p^2\alpha)/m_p, \tag{85}$$

$$S_(V, p) = \beta_{phi}\rho_n\mathbf{V_n} + v_{cen}\rho_n(\mathbf{V_n} - \mathbf{V_p}) - (\rho_p^2\alpha)/m_p, \tag{86}$$

$$S_(E, n) = -\beta_{phi}E_n + v_{ce} - (E_p - E_n), \tag{87}$$

$$S_(E, p) = \beta_{phi}E_n - v_{ce} + (E_p - E_n). \tag{88}$$

See (Yeghikyan & Fahr (2004a)) for more details. We choose the initial and boundary conditions as follows: the flow of initially neutral hydrogen around the undisturbed heliosphere from left to right, and, due to the symmetry of the flow with respect to the axis, the lower boundary is "reflecting", and the outflow of matter occurs through the upper and right boundaries. So, the system with right-hand sides (83-88), written in the standard form of a hyperbolic system of partial differential equations for hydrogen atoms and ions, was solved by the computer program CLAWPACK (Yeghikyan & Fahr (2003), Yeghikyan & Fahr (2004a),Yeghikyan & Fahr (2004b), Yeghikyan & Fahr (2006)) , separately for atoms and protons, by the method of successive approximations: usually, several iterations were sufficient for the relative error of the residual vector of the desired functions not to exceed 0.001. The calculation results are shown in Fig. 8. (Yeghikyan & Fahr (2004a)). Lines of equal concentration and axial components of hydrogen velocities are shown as functions of z and r.

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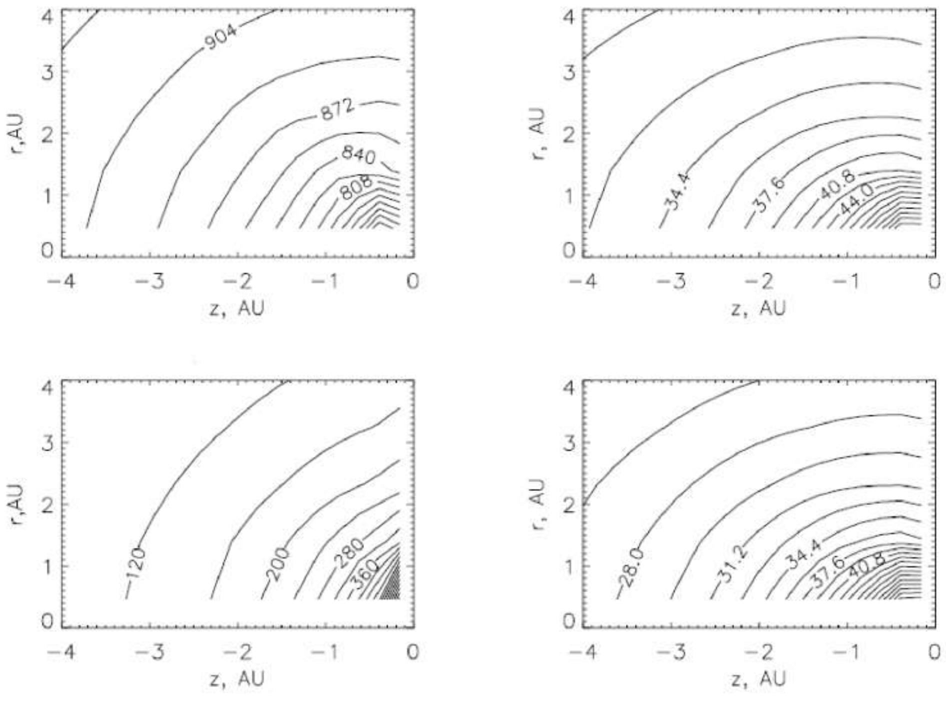

Figure 8. Lines of equal concentration and axial velocities of atoms (top) and protons (bottom) of the flow, on the plane (z, r). Contours of concentration values are marked with numbers in cm^{-3}, on the left, and velocities in km/s, on the right. Parameter values: $n_i = 1000$ $cm^{-3}, V_i = 26$ km/s, $T_i = 100$ K, $\beta_0 = \beta_{phi}, \mu = 0.1$.

12. Conclusion

A review of some problems of gas-dynamic astrophysical flows related to the formation of stars is given. The behavior of gas accreting onto a gravitating center is considered, some regularities in the formation of massive stars and the main characteristics of forming disks revolving around protostars, such as temperatures, are noted. When describing the accretion process in the spherically symmetric case, formulas for the dependence of velocity and density on the radius are given, as well as the Jeans criterion for the possibility of cloud collapse, which makes it possible to estimate the radius of the cloud from which a star is born, and the mass of the star itself. An analysis of the Bernoulli equation shows that the transition from supersonic to subsonic flow occurs at a point with a radius that depends only on the mass of the protostar and on the value of the speed of sound, which is equal to half the "second space velocity" in the system. It is also shown that both inflows and outflows of gas are possible, however, without taking into account the state of the disk. The Sedov-Taylor formula is obtained as applied to the expansion of the supernova remnant, on the power-law dependence of the radius and velocity on time. Examples of the use of dimension theory in the study of astrophysical objects associated with star formation processes are given. The gas dynamic equations of gas flow with the loaded mass are analyzed, taking into account the momentum introduced by the loaded mass. For the first time, analytical formulas for velocity and density (albeit in the absence of gravity) are given for known ratios for mass loading. Ordinary differential equations are also derived for the first time, which describe the velocity and density functions in the general case, when the mass loading is associated both with cloud condensations and can be caused by the stellar wind of O, B stars. Numerical solutions of these equations can be verified by observations. For completeness of description, an expression is given for the rate of propagation of the ionization of the HII region, that is, for the rate of the photoionization front around the emerging massive protostar. At the end, one example of a numerical solution of the complete system of gas-dynamic equations describing the gas flow of a molecular cloud matter around the Sun, passing through this cloud, with direct penetration of the neutral component into the Earth's atmosphere, is also given.

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