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Deformation of Special Relativity in ultra-high energy astrophysics

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Abstract

We review deformation of both postulates of Special Relativity (SR) theory, tested in experiments for ultra-high energy cosmic ray (UHECRs) and TeV-γ photons observed. To this aim, we utilize the theory of, so-called, *master space* (MS_p) induced supersymmetry (Ter-Kazarian, 2023, 2024), wherein the standard Lorentz code (SLC) is derived in a new perspective of double global MS_p-SUSY transformations in terms of Lorentz spinors $(\underline{\theta}, \bar{\underline{\theta}})$ referred to MS_p. This allows to introduce the physical finite *relative time* interval between two events as integer number of the *own atomic duration time* of double transition of a particle from M_4 to MS_p and back. While all the particles are living on M_4, their superpartners can be viewed as living on MS_p. This is a main ground for introducing MS_p, which is *unmanifested* individual companion to the particle of interest. Continuing along this line, in present communication we address the deformation of these spinors: $\underline{\theta} \to \tilde{\underline{\theta}} = \lambda^{1/2}\,\underline{\theta}$, etc., where λ appears as a scalar deformation function of the Lorentz invariance (LIDF). This yields both the DLE and DMAV, respectively, in the form $\tilde{ds} = \lambda ds$ and $\tilde{c} = \lambda c$, provided, the invariance of DLE, and the same value of DMAV in free space hold for all inertial systems. Thus, the LID-generalization of global MS_p-SUSY theory formulates the generalized relativity postulates in a way that preserve the relativity of inertial frames, in spite of the appearance of modified terms in the LID dispersion relations. We complement this conceptual investigation with testing of various LIDFs in the UHECR- and TeV-γ threshold anomalies by implications for several scenarios: the Coleman and Glashow-type perturbative extension of SLC, the LID extension of standard model, the LID in quantum gravity motivated space-time models, the LID in loop quantum gravity models, and the LIDF for the models preserving the relativity of inertial frames.

Keywords: *Deformation of Lorentz invariance–deformed geometry at LIV–cosmic rays–quantum spacetime–loop quantum gravity–spacetime noncommutativity*

1. Introduction

Numerous tests of Lorentz symmetry have been performed in recent years. The UHECR- and TeV-γ threshold anomalies found in high-energy experiments for UHECRs and TeV-γ photons observed provide a wealth of invaluable modern tests of the origin of LIV and has a strong potential in providing competitive constraints on suggested scenarios. The LIV phenomenology for UHECRs has been intensely studied in the last few decades, even the progress has been dramatic (for a comprehensive review see Batista & et al. (2019)). A propagation in intergalactic space through the cosmic microwave background (CMB) radiation necessarily should reduce energy of UHECRs below the Greisen-Zatsepin-Kuzmin (GZK) limit, 5×10^{19} eV. Stecker Stecker (1968) showed that for cosmic rays with energies above 1×10^{20} eV, an effective absorption mean-free-path should be less than 100 Mpc. More detailed approaches to the GZK cutoff feature have been made by e.g. Berezinsky & Grigorieva (1988), Scully & Stecker (2002). Hence, the GZK cutoff exists in the form of a suppression in the predicted flux of cosmic rays with energies above $\sim 8 \times 10^{19}$eV. The observations from the AGASA group found an abundant flux of incoming particles with energies above 1×10^{20}eV, violating the GZK cutoff Takeda & et al. (1998). Meantime, the observations from the HiRes Abbasi & et al. (2008) and Pierre Auger Observatory groups Abraham & et al. (2008), Abram & et al. (2010), Parizot (2007), with larger exposures, seem to be consistent with the presence of the GZK suppression effect. There are even some data Abram & et al. (2007, 2011), suggesting that some relatively close sources can be associated with observed cosmic rays with the highest energy. However, a reanalysis of the AGASA data

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has resulted in cutting their originally reported number of trans-GZK events by half Scully & Stecker (2009). The existence of such cutoff effect is uncertain owing to conflicting observational data and small number statistics. This is commonly referred as the GZK anomaly. Observed air showers in the KASCADE-Grande experiment show an excess of muons compared to predictions of standard hadronic interaction models above $\simeq 10^{16}$ eV, which indicates a longer-than-expected muon attenuation length Apel & et al. (2017). The Pierre Auger Observatory group also sees a muon excess by a factor $\simeq 1.5$ Aab & et al. (2016). For a solution of the GZK paradox it would be sufficient to push the threshold energy upwards by a factor of 6. If the muon excess cannot be explained by improved hadronic event generators with in the Standard Model, new physics could qualitatively play a role Batista & et al. (2019), Anchordoqui et al. (2017), Farrar & Allen (2013a,b), Olinto (2001). Comparatively large increases of muon number over a small primary energy range, which is possible in the hadronic channel in UHERCs, would likely be a hint for Lorentz symmetry violation at very high Lorentz factors.

The TeV-γ paradox is another one related to the transparency of the CMB. The HEGRA has detected high-energy photons with a spectrum ranging up to 24 TeV Aharonian & et al. (1999) from Mk 501, a BL Lac object at a redshift of 0.034 ($\sim$ 157 Mpc). Unlike the GZK paradox only a few solutions have been proposed for the TeV-γ paradox. The recent confirmation that at least some γ-ray bursts originate at cosmological distances Metzger & et al. (1997), van Paradis & et al. (1997) suggests that the radiation from these sources could be used to probe some of the fundamental laws of physics.

The quantum-gravitational effects may be playing a decisive role for LIV in the propagation of UHECR-particles. Most ideas in phenomenology reflect the expectation that the characteristic scale of quantum spacetime effects should be within a two or three orders of magnitude of the Planck scale, $E_P = M_P c^2$ ($\sim 1.22 \times 10^{19}$ GeV), at which quantum effects are expected to strongly affect the nature of spacetime. So that it should be possible to analyze quantum spacetime effects in terms of expansions in powers of Planck energy scale. The opportunity that the cosmic-ray threshold anomaly could be a signal of LIV had already been emphasized in e.g. Aloisio et al. (2000), Amelino-Camelias et al. (1998), Bertolami & Carvalho (2000), Coleman & Glashow (1999), Gonzalez-Mestres (1997), Jackiw & Kostelecky (1999), Sato (Sato), Stecker & Scully (2005). Enormous efforts have been made over the past decade to test LIV in various scenarious of quantum gravity Scully & Stecker (2009), Alfaro & Palma (2003), Amelino-Camelia & Piran (2001), Jacobson et al. (2003), Kifune (1999), Kluzniak (Kluzniak, 1999), Mattingly (2005), Protheroe & Meyer (2000), Stecker (2003). All these assumptions usually converged on a common way of solving and explaining the threshold UHECR and TeV-γ- anomalies with the help of efficient models for describing the propagation of astroparticles.

Since the theory of quantum gravity based on noncommutative geometry is at earlier stage of development Alfaro et al. (2000a), Amelino-Camelia & Majid (2000), Douglas & Nekrasov (2001), Gambini & Pullin (1999), Lukierski et al. (1992, 1995), Madore et al. (2000), Smolin (1999), and the possibility to preserve a Lorentz-invariant interpretation of noncommutative space-time is still not excluded Balachandran et al. (2008), Chaichian et al. (2004), Fiore & Wess (2007), there are claims in the literature that it is natural to tie the LIV to various alternatives for the yet nonexistent theory, see e.g. Aloisio et al. (2000), Alfaro & Palma (2003), Jacobson et al. (2003), Mattingly (2005), Stecker (2003), Alfaro & Palma (2002), Jacobson et al. (2004), Smolin (Smolin). In Alfaro & Palma (2002), Alfaro et al. (2000b, 2002a,b) the authors utilize the loop quantum gravity approach, whereas the discrete nature of space at short distances is expected to induce violations of Lorentz invariance and CPT. The loop quantum gravity provides the kinematics of the `Minkowski limit´ of the LIV may well be reachable.

However, even thanks to the fruitful interplay between phenomenological analysis and high energy astronomical experiments, the scientific situation remains, in fact, more inconsistent to day. Moreover, a systematic analysis of these properties happens to be surprisingly difficult by conventional theoretical methods. It should be noticed that currently no single scenario will be able to address explicit deformation of both postulates of the SLC. Literally speaking, when testing LID ansatze relied on a phenomenological approach to experiments with ultrahigh-energy astroparticles, as natural for phenomenology, we are allowed to determine only LIDF - λ_2 (for deformed maximum attainable velocity $\tilde{c} = \lambda_2 c$), but λ_1 (for deformed line element $\widetilde{ds} = \lambda_1\, ds$) is left unknown (see (24)). Therefore, other scenarios and possibilities should not be completely ruled out. The perturbative LIV expansions that are often needed for the analysis of these experiments might require the development of new techniques for description of deformed geometry at LIV.

In present paper we analyze the deformed geometry at LIV, for which we determine both LIDF - functions explicitly in terms of microscopic approach. We develop on MS$_p$-SUSY theory (Ter-Kazarian, 2023) (which is first paper of this series). Therewith, the SCL is derived in terms of Lorentz spinors $(\underline{\theta}, \underline{\bar{\theta}})$ referred to

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MS$_p$. On these premises, we derive the two postulates on which the theory of SR is based. This calls for a complete reconsideration of our ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property. To place the emphasis on the fundamental difference between the standard SUSY theories and some rather unusual properties of MS$_p$-SUSY theory, note that while the standard SUSY theory can be realized only as a spontaneously broken symmetry since the experiments do not show elementary particles to be accompanied by superpartners with different spin but identical mass, the MS$_p$-SUSY, in contrary, is realized as an *exact* SUSY, where all particles are living on 4D Minkowski space, but their superpartners can be viewed as living on MS$_p$. In MS$_p$-SUSY theory, obviously as in standard unbroken SUSY theory, the vacuum zero point energy problem, standing before any quantum field theory in M_4, is solved. The particles in M_4 themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4\oplus$ MS$_p$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2 \equiv \nu_b = \nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy.

Continuing along this line we now address the deformation of the Lorentz spinors: $\underline{\theta} \rightarrow \tilde{\underline{\theta}} = \lambda^{1/2}\,\underline{\theta}$, for given scalar LIDF, etc. This yields a deformed dispersion relation of a particle, which modifies the threshold condition for a reaction between a primary cosmic ray and a CMB radiation photon, leading to LIV effects at detectable levels. We relate LID to more general deformed smooth differential 4D-manifold $\mathcal{M}_4$. Since there are still issues of considerable interest, we finally complement this conceptual investigation with testing of various LIDFs in the UHECR- and TeV-γ- threshold anomalies with in several scenarios of the Coleman and Glashow-type perturbative extension of SLC developed in the context of conventional quantum field theory, the LID extension of standard model, the LID in quantum gravity motivated space-time models and the LID in loop quantum gravity models, and the LIDF for the models preserving the relativity of inertial frames.

Finally our main emphasis is on the important properties of the LID-generalization (see (27)) of global MS$_p$-SUSY theory. The quantum gravity theory implies that global Lorentz invariance is no more than an accidental symmetry of the ground state of the classical limit of the theory, therefore the corrections to Lorentz invariance appear as corrections to the laws of the relativity of inertial frames. The LID occurred at Planck scale energies leads to further predictions that there should be a preferred frame of reference in nature, or leads to prediction of an energy dependent speed of light. Such predictions are falsifiable in TeV energy gamma ray observations Amelino-Camelia (2002), Magueijo & Smolin (2003). In resolving this problems, Amelino-Camelia (2002) purports to shown that it is possible to formulate the relativity postulates in the new conceptual framework that does not lead to inconsistencies. The hypothesis that the Lorentz transformations may be modified at Planck scale is proposed by Magueijo & Smolin (2003), which preserve the relativity of inertial frames. In that respect, we stress that the proposed LID-generalization of global MS$_p$-SUSY theory strongly supports the major goal of works Amelino-Camelia (2002), Magueijo & Smolin (2003), because this theory formulates the LID-generalized relativity postulates in a way that preserve the relativity of inertial frames. (see subsect.5.5).

We will proceed according to the following structure. To start with, in Section 2 we necessarily recount some of the highlights behind of MS$_p$-SUSY. Section 3 deals with deformed geometry at LID in the framework of microscopic approach. In Section 4 we address possible deformation of two postulates of SR in the high-energy limit for the UHECRs and the TeV-γ photons observed. Various LIDFs allow us to test different LID ansatze yielding LIV in the UHECR- and TeV-γ- threshold anomalies with in several special scenarios: (Subsection 5.1) - The LIDF for Coleman and Glashow-type perturbative extension of SLC; (Subsection 5.2) - The LIDF for LIV extension of the standard model; (Subsection 5.3) - The LIDF for quantum gravity motivated space-time models; (Subsection 5.4) - The LIDF for the loop quantum gravity models; (Subsection 5.5) - The LIDF for the models preserving the relativity of inertial frames. A physical outlook and concluding remarks are presented in Section 6. For brevity, whenever possible undotted and dotted spinor indices often can be ruthlessly suppressed without ambiguity. Unless indicated otherwise, we take natural units, $h = c = 1$.

2. Probing SR behind the MS$_p$-SUSY, revisited

For a benefit of the reader, as a guiding principle to make the rest of paper understandable, in this section we necessarily recount some of the highlights behind of MS$_p$-SUSY (Ter-Kazarian, 2023, 2024), which are in use throughout the paper.

The flat MS$_p$ is the 2D composite space MS$_p \equiv \underline{M}_2 = \underline{R}^1_{(+)} \oplus \underline{R}^1_{(-)}$ with Lorentz metric. The ingredient

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1D-space $\underline{R}^1_m$ is spanned by the coordinates $\underline{\eta}^m$. The following notational conventions are used throughout this paper: all quantities related to the space $\underline{M}_2$ will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, ... = (\pm)$ denote the world indices related to $\underline{M}_2$. The MS$_p$ is smoothly (injective and continuous) embedded in 4D Minkowski space, $\underline{M}_2 \hookrightarrow M_4$. The elementary act of particle motion at each time step (t_i) through the infinitely small spatial interval $\triangle x_i = (x_{i+1} - x_i)$ in M_4 during the time interval $\triangle t_i = (t_{i+1} - t_i) = \varepsilon$ is probably the most fascinating challenge for physical research. Since this is beyond our perception, it appears legitimate to consider extension to the infinitesimal Schwinger transformation function, $F_{ext}(x_{i+1}, t_{i+1}; x_i, t_i)$, in fundamentally different aspect. We hypothesize that

in the limit $n \to \infty (\varepsilon \to 0)$, the elementary act of motion consists of an `annihilation´ of a particle at point $(x_i, t_i) \in M_4$, which can be thought of as the transition from initial state $|x_i, t_i>$ into unmanifested intermediate state, so-called, `motion´ state, $|\underline{x}_i, \underline{t}_i>$, and of subsequent `creation´ of a particle at infinitely close final point $(x_{i+1}, t_{i+1}) \in M_4$, which means the transition from `motion´ state, $|\underline{x}_i, \underline{t}_i>$, into final state, $|x_{i+1}, t_{i+1}>$. The motion state, $|\underline{x}_i, \underline{t}_i)>$, should be defined on unmanifested `master´ space, $\underline{M}_2$, which includes the points of all the atomic elements, $(\underline{x}_i, \underline{t}_i) \in \underline{M}_2$ $(i = 1, 2, ...)$.

This furnishes justification for an introduction of *unmanifested master space*, $\underline{M}_2$. If that is the case as above, a *creation* of a particle in $\underline{M}_2$ means its transition from initial state defined on M_4 into intermediate state defined on $\underline{M}_2$, while an *annihilation* of a particle in $\underline{M}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in M_4. All the fermionic and bosonic states taken together form a basis in the Hilbert space. The basis vectors in the Hilbert space composed of $H_B \otimes H_F$ is given by

$$\{|\underline{n}_b> \otimes |0>_f, \; |\underline{n}_b> \otimes f^\dagger\,|0>_f\},$$

or

$$\{|n_b> \otimes |\underline{0}>_{\underline{f}}, \; |n_b> \otimes \underline{f}^\dagger\,|\underline{0}>_{\underline{f}}\},$$

where we consider two pairs of creation and annihilation operators $(b^\dagger,\, b)$ and $(f^\dagger,\, f)$ for bosons and fermions, respectively, referred to the background space M_4, as well as $(\underline{b}^\dagger,\, \underline{b})$ and $(\underline{f}^\dagger,\, \underline{f})$ for bosons and fermions, respectively, as to background master space $\underline{M}_2$. Accordingly, we construct the quantum operators, $(q^\dagger,\, \underline{q}^\dagger)$ and $(q,\, \underline{q})$, which replace bosons by fermions and vice versa:

$$\begin{array}{l} q\,|\underline{n}_b,\, n_f >= q_0\sqrt{\underline{n}_b}\,|\underline{n}_b - 1,\, n_f + 1>, \\ q^\dagger\,|\underline{n}_b,\, n_f >= q_0\sqrt{\underline{n}_b + 1}\,|\underline{n}_b + 1,\, n_f - 1>, \end{array} \tag{1}$$

and that

$$\begin{array}{l} \underline{q}\,|n_b,\, \underline{n}_f >= q_0\sqrt{n_b}\,|n_b - 1,\, \underline{n}_f + 1>, \\ \underline{q}^\dagger\,|n_b,\, \underline{n}_f > q_0\sqrt{n_b + 1}\,|n_b + 1,\, \underline{n}_f - 1>. \end{array} \tag{2}$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators q and $\underline{q}$. So, we may refer the action of the supercharge operators q and $q^\dagger$ to the background space M_4, having applied in the chain transformations of fermion χ (accompanied with the auxiliary field F as it will be seen later on) to boson $\underline{A}$, defined on $\underline{M}_2$:

$$\longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow . \tag{3}$$

Respectively, we may refer the action of the supercharge operators $\underline{q}$ and $\underline{q}^\dagger$ to the $\underline{M}_2$, having applied in the chain transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field $\underline{F}$) to boson A, defined on the background space M_4:

$$\longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow . \tag{4}$$

The successive atomic double transitions of a particle $M_4 \rightleftharpoons \underline{M}_2$ is investigated within MS$_p$-SUSY, wherein all the particles are living on M_4, their superpartners can be viewed as living on MS$_p$. The underlying algebraic structure of MS$_p$-SUSY generators closes with the algebra of *translations* on the original space M_4 in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra those of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to M_4. Moreover, if there are several spinor generators $Q_\alpha{}^i$ with $i = 1, ..., N$ - theory with $N-$extended supersymmetry, can be written as a graded Lie algebra of SUSY field theories, with commuting and anticommuting generators:

$$\begin{array}{l} \{Q_\alpha{}^i,\, \bar{Q}^j{}_{\dot\alpha}\} = 2\delta^{ij}\,\sigma^{\hat m}_{\alpha\dot\alpha}\,p_{\hat m}; \\ \{Q_\alpha{}^i,\, Q_\beta{}^j\} = \{\bar{Q}^i{}_{\dot\alpha},\, \bar{Q}^j{}_{\dot\beta}\} = 0; \quad [p_{\hat m},\, Q_\alpha{}^i] = [p_{\hat m},\, \bar{Q}^j{}_{\dot\alpha}] = 0, \quad [p_{\hat m},\, p_{\hat n}] = 0. \end{array} \tag{5}$$

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The map from $SL(2,C)$ to the Lorentz group is established through the $\vec{\sigma}$-Pauli spin matrices, $\sigma^m = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \equiv (I_2, \vec{\sigma})$, $\bar{\sigma}^m \equiv (I_2, -\vec{\sigma})$, where I_2 is the identity two-by-two matrix. Both hermitian matrices P and P' or $\underline{P}$ and $\underline{P'}$ have expansions, respectively, in σ or $\underline{\sigma}$:

$$(\sigma^m p'_m) = M(\sigma^m p_m)M^\dagger, \quad (\sigma^{\underline{m}} p'_{\underline{m}}) = M(\sigma^{\underline{m}} p_{\underline{m}})M^\dagger, \tag{6}$$

where $M(M \in SL(2,C))$ is unimodular two-by-two matrix. The odd part of the supersymmetry algebra is composed entirely of the spin-1/2 operators $Q_\alpha{}^i$, $Q_\beta{}^j$. In order to trace a maximal resemblance in outward appearance to the standard SUSY theories, here we set one notation $\hat{m} = (m \quad \text{if} \quad Q = q$, or $\underline{m} \quad \text{if} \quad Q = \underline{q})$, and as before the indices α and $\dot{\alpha}$ run over 1 and 2.

The guiding principle of MS$_p$-SUSY resides in constructing the superspace which is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ (spanned by the 6D-coordinates $X^{\hat{m}} = (x^m, \eta^{\underline{m}})$ by the inclusion of additional 8D-fermionic coordinates $\Theta^\alpha = (\theta^\alpha, \underline{\theta}^\alpha)$ and $\bar{\Theta}_{\dot{\alpha}} = (\bar{\theta}_{\dot{\alpha}}, \underline{\bar{\theta}}_{\dot{\alpha}})$, as to $(q, \underline{q})$, respectively. Therewith thanks to the embedding $\underline{M}_2 \hookrightarrow M_4$, the spinors $(\underline{\theta}, \underline{\bar{\theta}})$, in turn, induce the spinors $\theta(\underline{\theta}, \underline{\bar{\theta}})$ and $\bar{\theta}(\underline{\theta}, \underline{\bar{\theta}})$, as to M_4. These spinors satisfy the following relations:

$$\begin{aligned} &\{\Theta^\alpha,\, \Theta^\beta\} = \{\bar{\Theta}_{\dot{\alpha}},\, \bar{\Theta}_{\dot{\beta}}\} = \{\Theta^\alpha,\, \bar{\Theta}_{\dot{\beta}}\} = 0, \\ &[x^m,\, \theta^\alpha] = [x^m,\, \bar{\theta}_{\dot{\alpha}}] = 0, \quad [\eta^{\underline{m}},\, \underline{\theta}^\alpha] = [\eta^{\underline{m}},\, \underline{\bar{\theta}}_{\dot{\alpha}}] = 0. \end{aligned} \tag{7}$$

and $\Theta^{\alpha *} = \bar{\Theta}^{\dot{\alpha}}$. Points in superspace are then identified by the generalized coordinates $z^{(M)} = (X^{\hat{m}},\, \Theta^\alpha,\, \bar{\Theta}_{\dot{\alpha}})$. We have then the one most commonly used `real´ or `symmetric´ superspace parametrized by

$$\Omega(X,\, \Theta,\, \bar{\Theta}) = e^{i(-X^{\hat{m}} p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})} = \Omega_q(x,\, \theta,\, \bar{\theta}) \times \Omega_{\underline{q}}(\underline{\eta},\, \underline{\theta},\, \underline{\bar{\theta}}), \tag{8}$$

where we now imply a summation over $\hat{m} = (m, \underline{m})$. To study the effect of supersymmetry transformations, we consider

$$g(0,\, \epsilon,\, \bar{\epsilon})\, \Omega(X,\, \Theta,\, \bar{\Theta}) = e^{i(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})}\, e^{i(-X^{\hat{m}} p_{\hat{m}} + \Theta^\alpha Q_\alpha + \bar{\Theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})}. \tag{9}$$

the transformation (9) induces the motion:

$$g(0,\, \epsilon,\, \bar{\epsilon})\, \Omega(X^{\hat{m}},\, \Theta,\, \bar{\Theta}) \;\rightarrow (X^{\hat{m}} + i\, \Theta\, \sigma^{\hat{m}}\, \bar{\epsilon} - i\, \epsilon\, \sigma^{\hat{m}}\, \bar{\Theta},\; \Theta + \epsilon,\; \bar{\Theta} + \bar{\epsilon}), \tag{10}$$

namely,

$$\begin{aligned} &g_q(0,\, \xi,\, \bar{\xi})\, \Omega_q(x,\, \theta,\, \bar{\theta}) \;\rightarrow (x^m + i\, \theta\, \sigma^m\, \bar{\xi} - i\, \xi\, \sigma^m\, \bar{\theta},\; \theta + \xi,\; \bar{\theta} + \bar{\xi}), \\ &g_{\underline{q}}(0,\, \underline{\xi},\, \underline{\bar{\xi}})\, \Omega_{\underline{q}}(\underline{\eta},\, \underline{\theta},\, \underline{\bar{\theta}}) \;\rightarrow (\underline{\eta}^{\underline{m}} + i\, \underline{\theta}\, \sigma^{\underline{m}}\, \underline{\bar{\xi}} - i\, \underline{\xi}\, \sigma^{\underline{m}}\, \underline{\bar{\theta}},\; \underline{\theta} + \underline{\xi},\; \underline{\bar{\theta}} + \underline{\bar{\xi}}). \end{aligned} \tag{11}$$

The spinors $\theta(\underline{\theta}, \underline{\bar{\theta}})$ and $\bar{\theta}(\underline{\theta}, \underline{\bar{\theta}})$ satisfy the embedding relations $\Delta\underline{x}^0 = \Delta x^0$ and $\Delta\underline{x}^2 = (\Delta\vec{x})^2$, so from (11) we obtain

$$\underline{\theta}\, \sigma^0\, \underline{\bar{\xi}} - \underline{\xi}\, \sigma^0\, \underline{\bar{\theta}} = \theta\, \sigma^0\, \bar{\xi} - \xi\, \sigma^0\, \bar{\theta}, \quad (\underline{\theta}\, \sigma^3\, \underline{\bar{\xi}} - \underline{\xi}\, \sigma^3\, \underline{\bar{\theta}})^2 = (\theta\, \vec{\sigma}\, \bar{\xi} - \xi\, \vec{\sigma}\, \bar{\theta})^2. \tag{12}$$

The *atomic displacement* caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ reads

$$\Delta\underline{\eta}_{(a)} = \underline{e}_{\underline{m}} \Delta\underline{\eta}^{\underline{m}}_{(a)} = \underline{u}\tau, \tag{13}$$

where the components $\Delta\underline{\eta}^{\underline{m}}_{(a)}$ are written

$$\Delta\underline{\eta}^{\underline{m}}_{(a)} = (\underline{\theta}\, \sigma^{\underline{m}}\, \underline{\bar{\theta}})\tau. \tag{14}$$

In Van der Warden notations for the Weyl two-component formalism $\underline{\bar{\theta}}_{\dot{\alpha}} = (\underline{\theta}_\alpha)^*$, the (13) can be recast into the form

$$\Delta\underline{\eta}^2_{(a)} = \tfrac{1}{2}\left[(\Delta\underline{x}^{\underline{0}}_{(a)} q)^2 - (\Delta\underline{x}^{\underline{1}}_{(a)})^2\right], \tag{15}$$

where $\Delta\underline{x}^{\underline{0}}_{(a)} = \underline{v}^{\underline{0}}\tau$, $\Delta\underline{x}^{\underline{1}}_{(a)} = \underline{v}^{\underline{1}}\tau$, and $\underline{v}^{(\pm)} = \frac{1}{\sqrt{2}}(\underline{v}^{\underline{0}} \pm \underline{v}^{\underline{1}})$. Hence the velocities of light in vacuum, $\underline{v}^{\underline{0}} = c$, and of a particle ,$\underline{\vec{v}}_{\underline{1}} = \underline{e}_{\underline{1}} \underline{v}^{\underline{1}} = \vec{n}|\vec{v}| = \vec{v}$ $(|\vec{v}| \le c)$, are

$$\begin{aligned} \underline{v}^{\underline{0}} &= \underline{\theta}\, \sigma^{\underline{0}}\, \underline{\bar{\theta}} = (\underline{\theta}_1\, \underline{\bar{\theta}}_1 + \underline{\theta}_2\, \underline{\bar{\theta}}_2) = \underline{\theta}\, \underline{\bar{\theta}}, \\ \underline{v}^{\underline{1}} &= \underline{\theta}\, \sigma^{\underline{1}}\, \underline{\bar{\theta}} = (\underline{\theta}_1\, \underline{\bar{\theta}}_1 - \underline{\theta}_2\, \underline{\bar{\theta}}_2), \end{aligned} \tag{16}$$

where

$$\begin{aligned} \theta_1(\underline{\theta},\, \underline{\bar{\theta}}) &= \tfrac{1}{2}\left[\left(\underline{v}^{\underline{0}} + \sqrt{\tfrac{2}{3}}\underline{v}^{\underline{1}}\right)^{1/2} + \left(\underline{v}^{\underline{0}} - \sqrt{\tfrac{2}{3}}\underline{v}^{\underline{1}}\right)^{1/2}\right], \\ \theta_2(\underline{\theta},\, \underline{\bar{\theta}}) &= \tfrac{1}{2}\left[\left(\underline{v}^{\underline{0}} + \sqrt{\tfrac{2}{3}}\underline{v}^{\underline{1}}\right)^{1/2} - \left(\underline{v}^{\underline{0}} - \sqrt{\tfrac{2}{3}}\underline{v}^{\underline{1}}\right)^{1/2}\right]. \end{aligned} \tag{17}$$

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Thus we derive the first founding property (i) that *the atomic displacement* $\Delta\underline{\eta}_{(a)}$, caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$, is an invariant:

$$\text{(i)} \quad \Delta\underline{\eta}_{(a)} = \Delta\underline{\eta}'_{(a)} = \cdots = inv. \tag{18}$$

The (16) gives the second (ii) founding property that the bilinear combination $\underline{\theta}\,\bar{\underline{\theta}}$ is a constant:

$$\text{(ii)} \quad c = \underline{\theta}\,\bar{\underline{\theta}} = \underline{\theta}'\,\bar{\underline{\theta}}' = \cdots = const. \tag{19}$$

The latter yields a second postulate of SR (Einstein's postulate) - *the velocity of light, c, in free space appears the same to all observers regardless the relative motion of the source of light and the observer*. The c is the maximum attainable velocity (16) for uniform motion of a particle in Minkowski background space, M_4. Equally noteworthy is the fact that (18) and (19) combined yield *invariance of the element of interval between two events* $\Delta x = k\Delta\underline{\eta}_{(a)}$ *(for given integer number k) with respect to the Lorentz transformation*:

$$\begin{aligned} &k^2\Delta\underline{\eta}^2_{(a)} = (c^2 - \underline{v}_1^2)\Delta t^2 = (c^2 - \vec{v}^{\,2})\Delta t^2 = (\Delta x^0)^2 - (\Delta\vec{x})^2 \equiv (\Delta s)^2 = (\Delta x'^0)^2 - \\ &(\Delta\vec{x})'^2 \equiv (\Delta s')^2 = \cdots = inv., \end{aligned} \tag{20}$$

where $x^0 = ct, \quad x^{0'} = ct', \ldots.$ We have here introduced a notion of physical relative finite *time intervals* between two events $\Delta t = k\tau/\sqrt{2}$, $\Delta t' = k\tau'/\sqrt{2}, \ldots.$

3. Deformation of both postulates of SR

Having SLC to be equipped with the MS$_p$-SUSY mechanism, the spinors $\underline{\theta}$ encode all of the information necessary for the two founding properties (18) and (19) of SR. In what follows, we will address violation of these properties, and that of both postulates of SR, and discuss the deformed geometry at LID. The LID is caused by two deformations

$$\Delta\underline{\widetilde{\eta}}_{(a)} = \lambda_1\,\Delta\underline{\eta}_{(a)} \Rightarrow (\widetilde{ds} = \lambda_1\,ds), \quad \tilde{c} = \lambda_2\,c, \tag{21}$$

where $\widetilde{ds}$ and $\tilde{c}$ are, respectively, deformed line element and deformed maximum attainable velocity of a particle. The deformed action of a free particle is written

$$\tilde{S} = -m_0\tilde{c}\int_a^b \widetilde{ds} = (\lambda_1\lambda_2)S, \tag{22}$$

where m_0 is the mass at rest, S is the undeformed action of a free particle. The (22) defines the energy-momentum eigenstates of the particle by means of deformed dispersion relation as follows:

$$\tilde{p}^2_\mu = (\lambda_1\lambda_2)^2 p^2_\mu = (\lambda_1\lambda_2)^2 m_0^2c^4, \tag{23}$$

which yields the following corrections to the energy-momentum on-shell relation or LID ansatz:

$$\begin{aligned} &\tilde{E}^2 - c^2\tilde{\vec{p}}^2 - m_0^2c^4 = f(\lambda_1, \lambda_2, \epsilon) \\ &\equiv m_0^2c^4\left[(\lambda_2^2 - 1)\epsilon^2 + (\lambda_1\lambda_2)^2 - 1\right], \end{aligned} \tag{24}$$

where $\tilde{E}$ and $\tilde{\vec{p}}$ denote respectively energy and (3-component) momentum of the particle, $\epsilon \equiv \tilde{E}/m_0c^2$, and we also set $c\tilde{\vec{p}} \simeq \tilde{E}$ at high-energy limit.

Remark: We emphasize that testing the $f(\lambda_1, \lambda_2, \epsilon)$ phenomenologically in the ultra-high energy astroparticle experiments, we are allowed to determine only λ_2 (for deformed maximum attainable velocity), but λ_1 (for deformed line element) is left unknown. To determine both coefficients, therefore, we consider a constitutive ansatz of simple, yet tentative, deformation of spinors $\underline{\theta}$ with given scalar LIDF, λ:

$$\underline{\theta} \to \tilde{\underline{\theta}} = \lambda^{1/2}\,\underline{\theta}. \tag{25}$$

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This, by virtue of (**??**), (16) and (25), leads to (21) for $\lambda = \lambda_1 = \lambda_2$. In this case

$$\begin{aligned} \widetilde{ds} &= \sqrt{2}(\underline{\tilde{\theta}}_1 \, \underline{\tilde{\tilde{\theta}}}_1 \, \underline{\tilde{\theta}}_2 \underline{\tilde{\tilde{\theta}}}_2)^{1/2} \, t = \lambda \, ds, \\ \tilde{c} &\equiv \underline{\tilde{\theta}} \, \underline{\tilde{\tilde{\theta}}} = \lambda c. \end{aligned} \tag{26}$$

To this end we note that the invariance of DLE, and the same value of DMAV in free space hold for all inertial frames

$$\begin{aligned} \widetilde{ds}^{\,2} &= \widetilde{ds'}^{\,2} = \widetilde{ds''}^{\,2} = \cdot = inv, \\ \tilde{c} &= \tilde{c'} = \tilde{c''} = \cdots . \end{aligned} \tag{27}$$

The relativity of inertial frames and validity of deformed Lorentz symmetry evidently is left unchanged, in spite of the appearance of modified terms in the LID dispersion relations. The LID ansatz (24) is reduced to

$$f(\lambda, \epsilon) = m_0^2 c^4 \left[(\lambda^2 - 1)\epsilon^2 + \lambda^4 - 1 \right]. \tag{28}$$

According to (24) and (28), in the relativistic limit, the group velocity to first order in $m_0^2 c^4$ and $f(\lambda, \epsilon)$ reads

$$\frac{v_{gr}^{\pm}}{c} = \frac{\partial E}{c \partial p} \simeq \frac{1}{2} - \frac{\lambda^4}{2\epsilon^2} + \frac{\lambda^2}{2} + \lambda(\epsilon + \frac{2\lambda^2}{\epsilon}) \frac{\partial \lambda}{\partial \epsilon}. \tag{29}$$

As we will see in next section, different choices of the LIDFs produce different LID ansatze yielding very different types of LIV behavior, which caused the UHECR- and TeV-γ threshold anomalies detected.

4. Deformed geometry at LID

A common conjecture for the behavior of spacetime in quantum gravity is that the algebra of spacetime coordinates is actually noncommutative (Douglas & Nekrasov, 2001, Mattingly, 2005), (Hinchliffe et al., 2004, Kowalski-Glikman & Nowak, 2003). The most familiar form of spacetime non-commutativity, "canonical" non-commutativity, where the spacetime coordinates acquire the non-commutativity relation, $[x_\alpha, x_\beta] = (i/\Lambda_{NC}^2)\Theta_{\alpha\beta}$, at the characteristic noncommutative energy scale Λ_{NC} (Mattingly, 2005). This scale is presumably near the Planck scale if the non-commutativity comes from quantum gravity. However, in large extra dimension scenarios Λ_{NC} could be as low as 1 TeV. The existence of Λ_{NC} manifestly breaks Lorentz invariance, the size of which is constrained by tests of Lorentz violation. All the approaches use an expansion in this scale to get some low energy effective field theory. For discussions of other types of non-commutativity, including those that preserve Lorentz invariance, see (Douglas & Nekrasov, 2001), (Hinchliffe et al., 2004, Kowalski-Glikman & Nowak, 2003).

In this section, therefore, we relate LIV to more general deformed smooth differential 4D-manifold $\mathcal{M}_4$. Meanwhile, there is a need to introduce the soldering tools, which are the linear frames and forms in tangent fiber-bundles to the external general smooth differential manifold, whose components are so-called tetrad (vierbein) fields. Consider a smooth deformation map (Ter-Kazarian, 2011)

$$\Omega : M_4 \rightarrow \mathcal{M}_4, \tag{30}$$

written in terms of the *world-deformation* tensor Ω, the flat Minkowski, M_4, and general, $\mathcal{M}_4$, smooth differential 4D-manifolds. Here we use Greek alphabet $(l, k, \rho, ... = 0, 1, 2, 3)$ to denote the holonomic world indices related to $\mathcal{M}_4$, and the second half of Latin alphabet $(l, m, k, ... = 0, 1, 2, 3)$ to stand for the world indices related to M_4. The M_4 has at each point a tangent space, $T_x M_4$, spanned by the holonomic orthonormal frame field, $\hat{e}_{(l)}$, as a shorthand for the collection of the 4-tuplet $(\hat{e}_{(0)}, \cdots, \hat{e}_{(3)})$. If the $\hat{e}_{(l)}$ are holonomic basis, their commutator will vanish $\left[\hat{e}_{(l)}, \hat{e}_{(k)}\right] = 0$, or converse, if the commutator vanishes one can find coordinates y^l such that $\hat{e}_{(l)} = \partial/\partial y^l$, which is known as Frobenius's Theorem (Schutz, 1982). Then any abstract vector $V \in M_4$ can be written as a linear combination of basis vectors, $V = V^l \hat{e}_{(l)}$. The real vector is an abstract geometrical entity, while the components are just the coefficients of the basis vectors in some convenient basis. Consider the tangent vector $V(u)$ with components $V^l(u) = dx^l/du$ to a parameterized curve $x^l(u)$, where u is the affine parameter. Under a Lorentz transformation the coordinates x^l change $x^{k\prime} = \Lambda_l^{k'} x^l$, while the parameterization u is unaltered. We can therefore deduce that the Lorentz transformation rule for basis vectors is $\hat{e}_{(k')} = \Lambda_{k'}^l \, \hat{e}_{(l)}$. The frame field, $\hat{e}$, then defines a dual vector, $\hat{\vartheta} \in {}^*T_x M_4$ (${}^*T_x M_4$ is the cotangent space), of differential forms, $\hat{\vartheta} = \begin{pmatrix} \hat{\vartheta}^{(0)} \\ \vdots \\ \hat{\vartheta}^{(3)} \end{pmatrix}$, as a shorthand for the

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collection of the $\hat{\vartheta}^{(k)} = e^k{}_l\, dx^l$, whose values at every point form the dual basis, such that $\hat{e}_{(l)} \rfloor \hat{\vartheta}^{(k)} = \delta^k_l$. Here $\rfloor$ denotes the interior product, namely, this is a C^∞-bilinear map $\rfloor : \Omega^1 \to \Omega^0$ with Ω^p denotes the C^∞-modulo of differential p-forms on M_4. In components $e_\sigma{}^k\, e^\sigma{}_l = \delta^k_l$.

Every dual vector $\omega \in M_4$ can be written in terms of its components labeled with lower indices: $\omega = \omega_l \hat{\vartheta}^{(l)}$. Whereas the dual space to the dual vector space is the original vector space itself: $V(\omega) = \omega(V) = \omega_l V^l$. The Lorentz transformation rule for dual basis vectors is $\hat{\vartheta}^{(k')} = \Lambda^{k'}_l\, \hat{\vartheta}^{(l)}$. To this end we could define the components of arbitrary tensor as $T^{l_1\cdots l_k}_{k_1\cdots k_l} = T(\hat{\vartheta}^{(l_1)}, \ldots, \hat{\vartheta}^{(l_k)}, \hat{e}_{(k_1)}, \ldots, \hat{e}_{(k_l)})$. The action of the tensors on a set of vectors and dual vectors is $T(\omega^{(1)}, \ldots, \omega^{(k)}, V^{(1)}, \ldots, V^{(l)}) = T^{l_1\cdots l_k}_{k_1\cdots k_l} \omega^{(1)}_{l_1}, \ldots, \omega^{(k)}_{l_k} V^{(1)k_1}, \ldots, V^{(l)k_l}$.

On the manifold, M_4, the tautological tensor field, id, of type (1,1) is defined which assigns to each tangent space the identity linear transformation. Thus for any point $x \in M_4$, and any vector $\xi \in T_x M_4$, one has $id(\xi) = \xi$. In terms of the frame field, the $\hat{\vartheta}^{(l)}$ give the expression for id as $id = \hat{e}\hat{\vartheta} = \hat{e}_{(0)} \otimes \hat{\vartheta}^{(0)} + \cdots \hat{e}_{(3)} \otimes \hat{\vartheta}^{(3)}$, in the sense that both sides yield ξ when applied to any tangent vector ξ in the domain of definition of the frame field. One can also consider general transformations of the linear group, $GL(4, R)$, taking any basis into any other set of four linearly independent fields. The notation, $\{\hat{e}_{(l)}, \hat{\vartheta}^{(k)}\}$, will be used below for general linear frames. The holonomic metric on the space M_4 can be recast in the form $\eta = \eta_{lk}\, \hat{\vartheta}^{(l)} \otimes \hat{\vartheta}^{(k)} = \eta(\hat{e}_{(l)}, \hat{e}_{(k)})\, \hat{\vartheta}^{(l)} \otimes \hat{\vartheta}^{(k)}$ with the components $\eta_{lk} = \eta(\hat{e}_{(l)}, \hat{e}_{(k)})$ in dual holonomic basis. We may define a p-form A to be closed if $dA = 0$, and exact if $A = dB$ for some $(p-1)$-form B. The pth de Rham cohomology vector space $H^p(M) = Z^p(M)/B^p(M)$, defined on the arbitrary manifold M, depends only on the topology of the manifold M, where $Z^p(M)$ is the vector space of closed p-forms. Minkowski space M_4 is topologically equivalent to $\mathbf{R^4}$, so that all of the $H^p(M)$ vanish for $p > 0$; for $p = 0$ we have $H^0(M) = \mathbf{R}$. Therefore in Minkowski space all closed forms are exact except for zero-forms; zero-forms can't be exact since there are no -1-forms for them to be the exterior derivative.

In turn, the general manifold $\mathcal{M}_4$ in (30) has at each point a tangent space, $T_x\mathcal{M}_4$, spanned by the anholonomic orthonormal frame field, e, as a shorthand for the collection of the 4-tuplet $(e_0, \cdots, e_3)$, where $e_a = e_a{}^\mu\, \partial_\mu$. We use the first half of Latin alphabet $(a, b, c, ... = 0, 1, 2, 3)$ to denote the anholonomic indices related to the tangent space. The frame field, e, then defines a dual vector, $\vartheta \in {}^*T_x\mathcal{M}_4$, of differential forms, $\vartheta = \begin{pmatrix} \vartheta^0 \\ \vdots \\ \vartheta^3 \end{pmatrix}$, as a shorthand for the collection of the $\vartheta^b = e^b{}_\mu\, dx^\mu$, whose values at every point form the dual basis, such that $e_a \rfloor \vartheta^b = \delta^b_a$. In components $e_a{}^\mu\, e^b{}_\mu = \delta^b_a$.

Constructing a smooth deformation map (30), let introduce so-called the *first deformation matrices*, $\pi_a{}^l$ and $\pi^a{}_k\, (\in GL(4, \mathcal{M})\, \forall\, x^\mu)$, which yield local tetrad deformations

$$e_a = \pi_a{}^l\, \hat{e}_{(l)}, \quad \vartheta^a = \pi^a{}_k\, \hat{\vartheta}^{(k)}, \tag{31}$$

provided,

$$\pi_a{}^l = \lambda^{1/2} e_a{}^l, \quad \pi^a{}_k = \lambda^{1/2} e^a{}_k, \tag{32}$$

so that

$$e_\mu = \pi_\mu{}^l\, \hat{e}_{(l)}, \quad \vartheta^\mu = \pi^\mu{}_k\, \hat{\vartheta}^{(k)}, \quad \pi_\mu{}^l = e_\mu{}^a \pi_a{}^l, \quad \pi^\mu{}_k = e^\mu{}_a \pi^a{}_k. \tag{33}$$

With this provision, we build up a *world-deformation* tensor,

$$\Omega^k{}_l = \pi^k{}_\mu\, \pi^\mu{}_l = \pi^k{}_a\, \pi^a{}_l = e^a{}_l e_b{}^l \Omega^b{}_a = \lambda\, (e^k{}_\mu\, e^\mu{}_l) = \lambda\, (e^k{}_a\, e^a{}_l) = \lambda\, \delta^k_l. \tag{34}$$

Thus, according to (26), the tautological tensor field $\tilde{id} \in \mathcal{M}_4$ of type (1,1), assigned to each tangent space $T_x\mathcal{M}_4$ the identity linear transformation:

$$\tilde{ds} = \tilde{id} = e_a \otimes \vartheta^a = \lambda\, \hat{e}_{(l)} \otimes \hat{\vartheta}^{(l)} = \lambda\, id = \lambda\, ds, \tag{35}$$

where $ds = id \in M_4$. The *first deformation matrices* π, in general, give rise to the right cosets of the Lorentz group, i.e. they are the elements of the quotient group $GL(4, \mathcal{M})/SO(3, 1)$. If we deform the co-tetrad according to (31), we have two choices to recast metric as follows: either writing the deformation of the metric in the space of tetrads or deforming the tetrad field:

$$g = o_{ab}\, \vartheta^a \otimes \vartheta^b = o_{ab}\, \pi^a{}_l\, \pi^b{}_k\;\, \hat{\vartheta}^{(l)} \otimes\, \hat{\vartheta}^{(k)} = \gamma_{lk}\, \hat{\vartheta}^{(l)} \otimes\, \hat{\vartheta}^{(k)}, \tag{36}$$

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where γ_{lk} is, so-called, *second deformation matrix*:

$$\gamma_{lk} = o_{ab}\, \pi^a{}_l\, \pi^b{}_k = \lambda\, o_{ab}\, e^a{}_l\, e^b{}_k. \tag{37}$$

The deformed metric splits as

$$g_{\mu\nu} = \Upsilon^2\, \eta_{\mu\nu} + \gamma_{\mu\nu}, \tag{38}$$

provided,

$$\gamma_{\mu\nu} = (\gamma_{al} - \Upsilon^2\, o_{al})\, e^a{}_\mu\, \pi^l{}_\nu = (\gamma_{ks} - \Upsilon^2\, \eta_{ks})\, \pi^k{}_\mu\, \pi^s{}_\nu, \tag{39}$$

where $\Upsilon = \pi^a{}_a = \pi^k{}_k$, and $\gamma_{al} = \gamma_{kl}\pi_a{}^k$. The anholonomic orthonormal frame field, e, relates g to the tangent space metric, $o_{ab} = diag(+---)$, by $o_{ab} = g(e_a, e_b) = g_{\mu\nu}\, e_a{}^\mu\, e_b{}^\nu$, which has the converse $g_{\mu\nu} = o_{ab}\, e^a{}_\mu\, e^b{}_\nu$ because $e_a{}^\mu\, e^a{}_\nu = \delta^\mu_\nu$. The γ_{lm} can be decomposed in terms of symmetric, $\pi_{(al)}$, and antisymmetric, $\pi_{[al]}$, parts of the matrix $\pi_{al} = o_{ac}\pi^c{}_l$ (or respectively in terms of $\pi_{(kl)}$ and $\pi_{[kl]}$, where $\pi_{kl} = \eta_{ks}\, \pi^s{}_l$) as

$$\gamma_{al} = \Upsilon^2\, o_{al} + 2\Upsilon\, \Theta_{al} + o_{cd}\, \Theta^c{}_a\, \Theta^d{}_l + o_{cd}\, (\Theta^c{}_a\, \varphi^d{}_l + \varphi^c{}_a\, \Theta^d{}_l) + o_{cd}\, \varphi^c{}_a\, \varphi^d{}_l, \tag{40}$$

where

$$\pi_{al} = \Upsilon o_{al} + \Theta_{al} + \varphi_{al}, \tag{41}$$

provided Θ_{al} is the traceless symmetric part and φ_{al} is the skew symmetric part of the first deformation matrix. The anholonomy objects defined on the tangent space, $T_x\mathcal{M}_4$, read

$$C^a : \quad = d\,\vartheta^a = \tfrac{1}{2}\, C^a{}_{bc}\, \vartheta^b \wedge \vartheta^c, \tag{42}$$

where the anholonomy coefficients, $C^a{}_{bc}$, which represent the curls of the basis members, are

$$\begin{aligned} &C^c{}_{ab} = -\vartheta^c([e_a, e_b]) = e_a{}^\mu e_b{}^\nu(\partial_\mu e^c{}_\nu - \partial_\nu e^c{}_\mu) - e^c{}_\mu[e_a(e_b{}^\mu) - e_b(e_a{}^\mu)] \\ &= 2\, \pi^c{}_l\, \pi^\mu_m \left(\pi^{-1m}{}_{[a}\partial_\mu\, \pi^{-1l}{}_{b]}\right). \end{aligned} \tag{43}$$

In particular case of constant metric in the tetradic space, the deformed connection can be written as

$$\Gamma^a{}_{bc} = \tfrac{1}{2}\left(C^a{}_{bc} - o^{aa'}\, o_{bb'}\, C^{b'}{}_{a'c} - o^{aa'}\, o_{cc'}\, C^{c'}{}_{a'b}\right). \tag{44}$$

A deformed general spin connection defined on the tangent space $T_x\mathcal{M}_4$ then is

$$\omega^a{}_{b\mu} = \pi_c{}^a \hat{\omega}^c{}_{d\mu} \pi^d{}_b + \pi_c{}^a\, \partial_\mu\, \pi^c{}_b = \pi_c{}^a\, \partial_\mu\, \pi^c{}_b, \tag{45}$$

because the spin connection, $\hat{\omega}^c{}_{d\mu}$, defined on the tangent space T_xM_4 is zero. As far as

$$\pi_c{}^a = \pi_\mu{}^a e_c{}^\mu = \lambda^{1/2}\, e_\mu{}^a e_c{}^\mu = \lambda^{1/2}\, \delta^a_c, \tag{46}$$

the spin connection (45) becomes

$$\omega^a{}_{b\mu} = \tfrac{1}{2}\delta^a_b\, \partial_\mu\, \lambda. \tag{47}$$

To have a complete picture of deformed geometry, there is a need for testing different LIDFs in several scenarios of LIV.

5. Testing LIDFs in the UHECR- and TeV-γ threshold anomalies

Having given some examples of the ways in which LIV arises, in this section let us turn to the complementary type of issues that are in focus when one studies the UHECRs and TeV-γ photons observed. We give a brief sketch of testing different LIDFs in several scenarios of the Coleman and Glashow-type perturbative extension of SLC, the LID extension of standard model, the LID in quantum gravity motivated space-time models and the LID in loop quantum gravity models.

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5.1. The LIDF for Coleman and Glashow-type perturbative extension of SLC

Let us consider a particular LIDF

$$\lambda_a = 1 + \delta_a, \quad 0 < |\delta_a| \ll 1, \tag{48}$$

with constant δ_a. Then a particle of specie (a) has, in addition to its own mass m_a, its own maximum attainable velocity $c_a = c\lambda_a$ (its own velocity of light c_a) as measured in the preferred frame:

$$\tilde{E}_a^2 = c_a^2 \tilde{\vec{p}}_a^2 + m_a^2 c_a^4, \tag{49}$$

where $\tilde{E}_a$ and $\tilde{\vec{p}}_a$ denote the energy and the (3-component) momentum of a-th particle. In this case, according to (28), the LID ansatz is

$$f_a = m_a^2 c^4 \left[\epsilon_a^2 \left(1 - \lambda_a^2\right) + 1 - \lambda_a^4\right], \tag{50}$$

where $\epsilon_a = \tilde{E}_a / m_a c^2$. This ansatz yields the CG Coleman & Glashow (1999)-type perturbative extension of SLC. Coleman and Glashow have developed a perturbative framework of LIV, by introducing noninvariant terms in the context of conventional quantum field theory. These terms assumed to be renormalizable in dimension no greater than four, gauge invariant under $SU(3)\otimes SU(2)\otimes U(1)$ local group, and the Lagrangian is rotationally and translationally invariant in a preferred frame which is presumed to be the rest frame of the CMB. The threshold condition for a reaction to take place can be substantially modified if the difference $\lambda_a - \lambda_b$ is nonzero (a and b are two particles involved in the reaction). This leads to new effects and predictions such as an abundant flux of cosmic rays well beyond the GZK cutoff energy. The reaction $(p + \gamma \to p + \pi)$ happens through several channels, for example, the baryonic $\triangle$ and N and mesonic ρ and ω resonance channels, and is the main reason for the appearance of the GZK cutoff. Following Stecker & Scully (2005), the CBR photopion production $(p + \gamma \to N + \pi's)$, on CBR photons of energy E' and temperature $T_{CBR} = 2.73$ K, is turned off if in terms of LIDF $\lambda_\pi - \lambda_p > 5 \times 10^{-24}(E'/2.73K)^2/c$. In this case, the description of the UHECR threshold anomaly requires together with conditions on $\lambda_\triangle - \lambda_\pi$ that $\lambda_\pi - \lambda_p > 10^{-24}/c$, where λ_π and λ_p are the λ_a for $a \equiv$ pions and $a \equiv$ protons, respectively. A resolution of the TeV-γ threshold anomaly with in this scheme Amelino-Camelia & Piran (2001) requires the additional condition $\lambda_e - \lambda_\gamma > 5\times 10^{-16}/c$. This combines with the absence of vacuum Cherenkov radiation by electrons with energies up to 500 GeV in such a way that the allowed values for $\lambda_e - \lambda_\gamma$ lying in a relatively narrow range of $5 \times 10^{-16}/c < \lambda_e - \lambda_\gamma < 5 \times 10^{-13}/c$.

5.2. The LIDF for LIV extension of the standard model

The development of air showers can be influenced both by Lorentz invariant new physics acting on primaries or secondaries with energy $E \gtrsim 2\times 10^{17}$eV, and also by LIV involving large Lorentz boosts. Therefore we now address LIDF for LIV extension of the minimal $SU(3) \otimes SU(2) \otimes U(1)$ standard model, including CPT-even and CPT-odd terms. This can be viewed as the low-energy limit of a physically relevant fundamental theory with Lorentz-covariant dynamics in which spontaneous Lorentz violation occurs Colladay & Kostelecky (1997, 1998), Kostelecky & Mewes (2008). The LIV is induced by non-renormalizable operators that conserve gauge invariance but break parts of the Poincaré group. One possible effect is vacuum birefringence, which could be bounded from cosmological observations. The LIDF for left- and right-handed photons or fermions can be written

$$\lambda_\pm(\epsilon/\epsilon_P, \eta_\pm, n) = \frac{\epsilon}{\sqrt{2}} \left\{ -1 + \sqrt{1 + \frac{4}{\epsilon^4}\left[1 + \epsilon^2 (1 + f_\pm(\epsilon/\epsilon_P, \eta_\pm, n))\right]} \right\}^{1/2}, \tag{51}$$

provided the LID ansatz, following Amelino-Camelia et al. (2005), Christian (2005), Diaz (2014), is

$$f_\pm(\epsilon/\epsilon_P, \eta_\pm, n) = \eta_\pm \epsilon^2 \left(\frac{\epsilon}{\epsilon_P}\right)^n, \tag{52}$$

where dimensionless numbers $\eta_\pm$ refer to positive and negative helicity states, and $n = d - 4$ for a d-dimensional operator. On the right-hand side we highlighted the fact that this correction would come in suppressed with respect to the standard leading term by a factor of given by the degree of ratio ϵ/ϵ_P of the

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energy and Planck energy scale. In general, in effective field theory one has $\eta_+ = (-1)^n \eta_-$. In (51), $d \leq 4$ for renormalizable LIV terms, and n is negative. A tiny deformation

$$\underline{\tilde{\theta}} \simeq \left[1 + \tfrac{\eta_\pm}{2}(n+1)\left(\tfrac{\epsilon}{\epsilon_P}\right)^n\right]^{1/2} \underline{\theta}, \tag{53}$$

where the corresponding parameter $\eta_\pm$ should be of order 1, yields variation in photon speed, which, when accumulated over cosmological light-travel times, may be revealed by observing LIV effects. For the LID ansatz (52), the group velocity (29) can be recast into the form

$$\tfrac{v^\pm_{gr}}{c} \simeq 1 - \tfrac{1}{2\epsilon^2} + \tfrac{\eta_\pm}{2}(n+1)\left(\tfrac{\epsilon}{\epsilon_P}\right)^n. \tag{54}$$

This leads to energy-dependent delays in the propagation time from the sources to Earth. However, the interplay between LIV and violations of CPT symmetry is still the subject of a lively debate, see e.g. Chaichian et al. (2011).

5.3. The LIDF for quantum gravity motivated space-time models

In models for quantum gravity involving large extra dimensions, the energy scale at which gravity becomes strong can occur at a scale, $E_{QG} << E_P$, even approaching a TeV. In the most commonly considered case, the phenomenological approach is motivated by the role that the deformed dispersion relation might have in quantum gravity, the energy scale E_{QG} of which is expected to be somewhere between $E_{GUT} < E_{QG} < E_P$, where E_{GUT} is the energy scale of grand unified theory (GUT). The quantum-gravity effects inducing some level of nonlocality or noncommutativity would affect even the most basic flat-space continuous symmetries, such as Lorentz invariance. This intuition would be seen as a way to develop the LIV. The expectations of the quantum gravity motivated space-time models known to support such violations of Lorentz invariance: an integer value of the dimensionless parameter and a characteristic energy scale constrained to a narrow interval in the neighborhood of the Planck scale Amelino-Camelia & Piran (2001), Amelino-Camelias et al. (1998). In this case, the LIDF should be the scalar function, $\lambda(\epsilon,\, \epsilon_P)$ of the energy and Planck energy scale, which yields $\tilde{c}(\epsilon,\, \epsilon_P) = \underline{\tilde{\theta}}(\epsilon,\, \epsilon_P)\, \underline{\tilde{\tilde{\theta}}}(\epsilon,\, \epsilon_P)$ - a deformed velocity. Within the phenomenological approach, the UHECR and TeV-γ threshold anomalies are due to a class of deformed dispersion relations, which are invariant under the subgroup of rotations $SO(3)$, but is not locally Lorentz invariant. In this scenario, the LIDF can be written as

$$\lambda(\epsilon,\, \epsilon_P, \eta, \alpha) = \tfrac{\epsilon}{\sqrt{2}}\left\{-1 + \sqrt{1 + \tfrac{4}{\epsilon^4}\left[1 + \epsilon^2(1 + f(\epsilon,\, \epsilon_P, \eta, \alpha))\right]}\right\}^{1/2}, \tag{55}$$

where the LID ansatz, following Amelino-Camelia & Piran (2001), Amelino-Camelias et al. (1998) for $\alpha = 1$, and Amelino-Camelia (1997, 2000, 2013) for a general α, takes the form

$$f(\epsilon,\, \epsilon_P, \eta, \alpha) \equiv \epsilon^2 \eta \left(\tfrac{\epsilon}{\epsilon_P}\right)^\alpha. \tag{56}$$

The free parameters α and η are characterizing the deviation from standard Lorentz invariance. The η is a dimensionless parameter of order 1. The α specifies how strongly the magnitude of the violation is suppressed by E_P. Using the UHECR and TeV-γ data, as well as upper bounds on time-of-flight differences between photons of different energies, the LID parameter space is constrain. Concerning the consistency of the interpretation of the threshold anomalies as manifestations of LID it is also important to observe that the modified dispersion relation (56), in spite of affecting so significantly the GZK and TeV-γ thresholds, does not affect significantly the processes used for the detection of the relevant high-energy particles. The quantum-gravity intuition would then be seen as a way to develop a theoretical prejudice for plausible values of η and α . In particular, corrections going as ϵ/ϵ_P a typically emerge in quantum gravity as leading order pieces of some more complicated analytic structures Amelino-Camelias et al. (1998), Amelino-Camelia & Majid (2000), Lukierski et al. (1992, 1995), Amelino-Camelia (1997). This provides, of course, a special motivation for the study of the cases $\alpha = 1$ and $\alpha = 2$ $[f(\epsilon/\epsilon_P) \approx 1 + \alpha_1(\epsilon/\epsilon_P)^{n_1} + \cdots]$. Moreover, the fact that $E_{GUT} < E_{QG} < E_P$ corresponds to the expectation that η should not be far from the range $1 \leq \eta \leq 10^{3\alpha}$. At $f(\epsilon,\, \epsilon_P, \eta, \alpha) \ll 1$, in the first order over f, (55) gives a deformation

$$\underline{\tilde{\theta}} \simeq \left[1 + \epsilon^2 \eta \left(\tfrac{\epsilon}{\epsilon_P}\right)^{\alpha(\sim 1,2)}\right]^{1/2} \underline{\theta}, \tag{57}$$

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which will cause a significant upward shift above the GZK cutoff determined by the threshold equation Amelino-Camelia & Piran (2001)

$$E_{p,th} = \frac{c^4[(m_p+m_\pi)^2-m_p^2]}{4E_\gamma} + \eta\frac{E^{2+\alpha}_{p,th}}{4E_\gamma E_P^\alpha}\left(\frac{m_p^{1+\alpha}+m_\pi^{1+\alpha}}{(m_p+m_\pi)^{1+\alpha}}-1\right), \tag{58}$$

where $E_{p,th}$ is the threshold energy of the proton and E_γ is the energy of the photon, in the head-on collision between a soft photon and a high-energy proton of photoproduction process $p+\gamma \to p+\pi$.

5.4. The LIDF for the loop quantum gravity models

Alternatively, loop quantum gravity is a canonical approach to the problem of gravity quantization. It is based on the construction of a spin network basis, labeled by graphs embedded in a three-dimensional insertion Σ in spacetime. In Alfaro & Palma (2002, 2003), Alfaro et al. (2000b, 2002a,b) the authors present some techniques to establish and analyze new constraints on the loop quantum gravity parameters. The effects of the loop structure of space at the Planck level are treated semiclassically through a coarse-grained approximation. An interesting feature of these methods is the explicit appearance of the two length scales: Planck scale $\ell_P \approx 1.62\times 10^{-33}$cm and, so-called , `weave´ scale $\mathcal{L} \gg \ell_P$. It is possible, however, to introduce a loop state which approximates a flat three-metric on Σ at length scales greater than the length scale $\mathcal{L} \gg \ell_P$. There with for distances $d \ll \mathcal{L}$ the quantum loop structure of space is manifest, while for distances $d \geq \mathcal{L}$ the continuous flat geometry is regained. Such effective theories introduce LID's to the dispersion relations. The dispersion relation for fermions can be obtained through the development of a Klein-Gordon-like equation. The lower contributions in both scales ℓ_P and $\mathcal{L}$ yield the following LIDF:

$$\lambda_\pm(\epsilon,\ell_P,\mathcal{L}) = \frac{\epsilon}{\sqrt{2}}\left\{-1+\sqrt{1+\frac{4}{\epsilon^4}\left[1+\epsilon^2(1+f_\pm(\epsilon,\ell_P,\mathcal{L}))\right]}\right\}^{1/2}. \tag{59}$$

Provided the LID ansatz is given Alfaro & Palma (2002, 2003)

$$f_\pm(\epsilon,\ell_P,\mathcal{L}) = 2\alpha\epsilon^2+\eta\epsilon^4\pm 2\lambda\epsilon, \tag{60}$$

where the $\pm$ signs correspond to the helicity state of the described particle, and the parameters (α,η,λ) are functions of these two scales as

$$\alpha = k_\alpha(\ell_P/\mathcal{L})^2, \quad \eta = k_\eta \ell_P^2, \quad \lambda = k_\lambda \ell_P/2\mathcal{L}^2, \tag{61}$$

and the dimensionless parameters $k_\alpha, k_\eta, k_\lambda$ are of order 1. For the electromagnetic sector of the theory, the LIDF for the photon is written

$$\lambda_\pm(\epsilon,\ell_P,\mathcal{L}) = \frac{\epsilon}{\sqrt{2}}\left\{-1+\sqrt{1+\frac{4}{\epsilon^4}\left[1+\epsilon^2(1+2\alpha_\gamma k^2\pm 2\theta_\gamma\ell_P k^3\right]}\right\}^{1/2}. \tag{62}$$

where $\alpha_\gamma = k_\gamma$. There with the condition for significantly increasing the threshold for electron-positron pair production interactions $(p+\gamma\to p+e^++e^-)$ is obtained $\lambda_e-\lambda_p > (m_p+m_e)m_pc^3/E_f^2$, which is $O(10^{-22}/c)$ for a fiducial energy $E_f = 100$ EeV. Thus, given even a very small amount of deformation,

$$\underset{\sim}{\tilde{\theta}} \simeq \left[1+2\alpha\epsilon^2+\eta\epsilon^4\pm 2\lambda\epsilon\right]^{1/2}\underset{\sim}{\theta}, \tag{63}$$

both photopion and pair-production interactions of UHECR with the CBR can be turned off, and it could explain the UHECR spectrum which shows an excess of UHECRs at energies above GZK limit. Since a detailed knowledge of the deviation parameters figured in (63) is absent, the different corrections are studied independently in Alfaro & Palma (2003), with hope that there will always exist the possibility of having an adequate combination of these parameter values that could affect the threshold conditions simultaneously. This threshold analysis gives the following constraints for a significant increase or decrease in the threshold energy for pair production: $|\alpha_p-\alpha_e| < 9.8\times 10^{-22}$; $\quad 2.6\times 10^{-18}\text{eV}^{-1} \lesssim \mathcal{L} \lesssim 1.6\times 10^{-17}\text{eV}^{-1}$. If no GZK anomaly is confirmed in future experimental observations, then one should state a stronger bound $|\alpha_\pi-\alpha_p| < 2.3\times 10^{-23}$, or $\lesssim \mathcal{L} \gtrsim 1.7\times 10^{-17}\text{eV}^{-1}$. For η- and λ-corrections are respectively obtained: $|\eta| < 1.6\times 10^{-60}\text{eV}^{-2}$ and $|\lambda_e| < 1.6\times 10^{-5}$eV. However, the present stage of UHECR observations requires that one proceeds with caution, as far as each one of these parameters will be significant at different energy ranges.

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5.5. The LIDF for the models preserving the relativity of inertial frames

Einstein's Relativity postulates provide an example in which the Relativity Principle coexists with an observer-independent (velocity) scale. In Amelino-Camelia (2002) it is assumed that Relativity can also be "doubly special", in the sense that the Relativity Principle can coexist with observer-independent scales of both velocity and length. As a corollary, the Relativity Principle can be formulated in a way that does not lead to inconsistencies in the case of space-times whose short-distance structure is governed by an observer-independent length scale. In the proposed a new conceptual framework, it is shown by Amelino-Camelia (2002) that the relativistic treatment of the known in literature dispersion relation, $E^2 = c^4m^2 + c^2p^2 - \widetilde{\ell}_P cp^2 E$, leads to threshold anomalies. This, by virtue of (28), yields following LIDF:

$$\lambda_\pm(\epsilon, \ell_P, \mathcal{L}) = \frac{\epsilon}{\sqrt{2}}\Big\{-1+ \sqrt{1+\frac{4}{\epsilon^4}\left[1+\epsilon^2(1+\widetilde{\ell}_P cp^2E)\right]}\Big\}^{1/2}. \tag{64}$$

The interesting hypothesis that the Lorentz transformations may be modified at Planck scale is proposed by Magueijo & Smolin (2003), which preserve the relativity of inertial frames with a non-linear action of the Lorentz transformations on momentum space.In contrast to other definitions, this leads to a commutative spacetime geometry. But the commutation relations between position and momentum become energy dependent, leading to a new energy dependent modification of the uncertainty relations. The most general invariant associated with the new group action is $||p||^2 \equiv \eta^{ab}U(p_a)U(p_b)$, where the modified boost generators can be written in the form $K^i = U^{-1}[p_0]L^i_0 U[p_0]$. This, incorporating with (28), yield

$$\lambda_\pm(\epsilon, \ell_P, \mathcal{L}) = \frac{\epsilon}{\sqrt{2}}\Big\{-1+ \sqrt{1+\frac{4}{\epsilon^4}\left[1+\epsilon^2(\frac{\eta^{ab}U(p_a)U(p_b)}{m^2c^2})\right]}\Big\}^{1/2}, \tag{65}$$

where L_{ab} are the standard Lorentz generators. For the particular boosts, one has $U[p_0] \equiv \exp(\varsigma p_0 D)$ (ς may have either sign, and that it is expected to be proportional to plus or minus the Planck length), where $D = p_a\frac{\partial}{\partial p_a}$ is a dilatation, or more specifically: $U[p_0](p_a) = p_a/(1-\varsigma p_0)$ (for a more detailed analysis see Magueijo & Smolin (2003)). In that respect, we stress that the proposed LID-generalization of global MS_p-SUSY theory strongly supports the major goal of works Amelino-Camelia (2002) and Magueijo & Smolin (2003), because this theory formulates the LID-generalized relativity postulates in a way that preserve the relativity of inertial frames.

6. The physical outlook and concluding remarks

In this paper we have presented a general overview of the effects of deviations from exact Lorentz invariance in the context of double space- or MS_p-SUSY.

Deformation of the spinors: $\underline{\theta} \to \underline{\tilde{\theta}} = \lambda^{1/2}\,\underline{\theta}$, etc., where λ is figured as the scalar LIDF, yields both the DLE and DMAV, respectively, in the form $\tilde{ds} = \lambda ds$ and $\tilde{c} = \lambda c$, provided, the invariance of DLE, and the same value of DMAV in free space hold for all inertial systems. This evidently maintains the relativity of inertial frames and validity of deformed Lorentz symmetry, in spite of the appearance of modified terms in the LID dispersion relations.

Deformed geometry at LIV. We relate LID to more general deformed smooth differential 4D-manifold $\mathcal{M}_4$. Whereas, a smooth deformation map $\Omega : M_4 \to \mathcal{M}_4$, is written in terms of the *world-deformation* tensor Ω, the flat Minkowski, M_4, and general, $\mathcal{M}_4$, smooth differential 4D-manifolds. The general manifold $\mathcal{M}_4$ in (30) has at each point a tangent space, $T_x\mathcal{M}_4$, spanned by the anholonomic orthonormal frame field. Building up the *world-dqeformation* tensor, we introduce the *first deformation matrices*, which, in general, give rise to the right cosets of the Lorentz group, i.e. they are the elements of the quotient group $GL(4, \mathcal{M})/SO(3,1)$. If we deform the co-tetrad, we have two choices to recast metric either writing the deformation of the metric in the space of tetrads or deforming the tetrad field.

We complement this investigation with testing of various LIDFs in the UHECR- and TeV-γ- threshold anomalies with in several scenarios of the Coleman and Glashow-type perturbative extension of SLC developed in the context of conventional quantum field theory, the LID extension of standard model, the LID in quantum gravity motivated space-time models and the LID in loop quantum gravity models.

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LIDF in CG-type LIV. In the case of CG-type perturbative extension of SLC ($\lambda_a = 1+\delta_a, \quad 0 < |\delta_a| \ll 1$), the CBR photopion production ($p + \gamma \to N + \pi's$), on CBR photons of energy E' and temperature $T_{CBR} = 2.73$ K, is turned off if in terms of LIDF $\lambda_\pi - \lambda_p > 5 \times 10^{-24}(E'/2.73K)^2/c$. The description of the UHECR threshold anomaly requires together with conditions on $\lambda_\triangle - \lambda_\pi$ that $\lambda_\pi - \lambda_p > 10^{-24}/c$. A resolution of the TeV-γ threshold anomaly with in this scheme requires the additional condition $\lambda_e - \lambda_\gamma > 5 \times 10^{-16}/c$. This combines with the absence of vacuum Cherenkov radiation by electrons with energies up to 500 GeV in such a way that the allowed values for $\lambda_e - \lambda_\gamma$ lying in a relatively narrow range of $5 \times 10^{-16}/c < \lambda_e - \lambda_\gamma < 5 \times 10^{-13}/c$.

LIDF in LIV extension of standard model. The LIV extension of the minimal standard model, including CPT-even and CPT-odd terms, shows that a tiny deformation $\underline{\tilde{\theta}} \simeq \left[1 + \frac{\eta_\pm}{2}(n+1)\left(\frac{\epsilon}{\epsilon_P}\right)^n\right]^{1/2} \underline{\theta}$, where the corresponding parameter $\eta_\pm$ should be of order 1, yields variation in photon speed, which, when accumulated over cosmological light-travel times, may be revealed by observing LIV effects.

LIDF in quantum gravity models. The corrections going as E/E_P in quantum gravity motivate a deformation $\underline{\tilde{\theta}} = \left[E^2\eta\left(\frac{E}{E_P}\right)^{\alpha(\sim 1,2)}\right]^{1/2} \underline{\theta}$, which causes a significant upward shift above the GZK cutoff determined by the threshold equation.

LIDF in loop quantum gravity models. In the coarse-grained approximation of the loop quantum gravity models, the condition for significantly increasing the threshold for electron-positron pair production interactions ($p+\gamma \to p+e^+ + e^-$) is obtained $\lambda_e - \lambda_p > (m_p + m_e)m_p c^3/E_f^2$, which is $O(10^{-22}/c)$ for a fiducial energy $E_f = 100$ EeV. Thus, for a very small amount of deformation, $\underline{\tilde{\theta}} \simeq \left[1 + 2\alpha\epsilon^2 + \eta\epsilon^4 \pm 2\lambda\epsilon\right]^{1/2} \underline{\theta}$, both photopion and pair-production interactions of UHECR with the CBR can be turned off, and it could explain the UHECR spectrum which shows an excess of UHECRs at energies above GZK limit.

LIDF for the models preserving the relativity of inertial frames. We stress that the proposed LID-generalization of global MS$_p$-SUSY theory strongly supports the major goal of works Amelino-Camelia (2002), Magueijo & Smolin (2003), because this theory formulates the LID-generalized relativity postulates in a way that preserve the relativity of inertial frames.

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