Inertia II: The local $\widetilde{MS}_p$-SUSY induced inertia effects

G.Ter-Kazarian*

Byurakan Astrophysical Observatory, Byurakan, Aragatsotn Province, Armenia

Abstract

In the framework of *local* $\widetilde{MS}_p$-SUSY theory, which is extension of *global*, so called, *master space* (MS$_p$)-SUSY theory (Ter-Kazarian, 2023, 2024), we address the *accelerated motion and inertia effects*. The superspace is a direct sum of curved background double spaces $\widetilde{M}_4 \oplus \widetilde{MS}_p$, with an inclusion of additional fermionic coordinates $(\Theta, \bar{\Theta})$ induced by the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, which refer to $\widetilde{MS}_p$. We take the Lorentz group as our structure group in order to recover rigid superspace as a limiting solution to our dynamical theory. The local $\widetilde{MS}_p$-SUSY is conceived as a quantum field theory whose action includes the *fictitious gravitation field term*, where the graviton coexists with a fermionic field of, so-called, gravitino (sparticle) described by the Rarita-Scwinger kinetic term. A significant difference between standard theories of supergravity and the local $\widetilde{MS}_p$-SUSY theory is that a coupling of supergravity with matter superfields no longer holds. We argue that a deformation/(distortion of local internal properties) of MS$_p$, is the origin of the *absolute acceleration* ($\vec{a}_{abs} \neq 0$) and inertia effects (*fictitious graviton*). These gravitational fields had no sources and were generated by coordinate transformations. A curvature of $\widetilde{MS}_p$ arises entirely due to the inertial properties of the Lorentz-rotated frame of interest. This refers to the particle of interest itself, without relation to other matter fields, so that this can be globally removed by appropriate coordinate transformations. The supervielbein $E^A(z)$, being an alogue of Cartan's local frame, is the dynamical variable of superspace formulation, which identifies the tetrad field $e_{\hat{m}}{}^{\hat{a}}(X)$ and the Rarita-Schwinger fields. The connection is the second dynamical variable in this theory. The field $e_{\hat{m}}{}^{\hat{a}}(X)$ plays the role of a gauge field associated with local transformations (*fictitious graviton*). The *fictitious gravitino* is the gauge field related to local supersymmetry. The two fields differ in their spin: 2 for the graviton, 3/2 for the gravitino. These two particles are the two bosonic and fermionic states of a gauge particle in the curved background spaces $\widetilde{M}_4$ and $\widetilde{MS}_p$, respectively, or vice versa. Following (Ter-Kazarian, 2012), in the framework of classical physics, we discuss the inertia effects by going beyond the hypothesis of locality, and derive the explicit form of the vierbien $e_{\hat{m}}{}^{\hat{a}}(\varrho) \equiv (e_m{}^a(\varrho), e_{\underline{m}}{}^{\underline{a}}(\varrho))$. This theory furnishes justification for the introduction of the *weak principle of equivalence* (WPE). We derive a general expression of the relativistic inertial force exerted on the extended spinning body moving in the Rieman-Cartan space.

Keywords: *Supersymmetry–Supergravity–Inertia effects*

1. Introduction

The phenomenon of inertia may be the most profound mystery in physics. Its solution could shed light on or be central to unraveling other important puzzles. In (Ter-Kazarian, 2023, 2024) we have studied the first part of phenomenon of inertia, which governs the *inertial uniform motion* of a particle in 4D flat Minkowski space, M_4. This is probably the most fascinating challenge for physical research. We have developed the theory of global, so-called, \`double space´- or *master space* (MS$_p$)- supersymmetry, subject to certain rules, wherein the superspace is a 14D-extension of a direct sum of background spaces $M_4 \oplus$ MS$_p$ by the inclusion of additional 8D fermionic coordinates. The latter is induced by the spinors $\underline{\theta}$ and $\bar{\underline{\theta}}$ referred to MS$_p$. While all the particles are living on M_4, their superpartners can be viewed as living on MS$_p$. This is a main ground for introducing MS$_p$, which is *unmanifested* individual companion to the particle of interest. Supersymmetry transformation is defined as a translation in superspace, specified by the group element with corresponding anticommuting parameters. The multiplication of two successive transformations induce the motion. As a corollary, we have derived SLC in a new perspective of global double MS$_p$-SUSY transformations in terms of Lorentz spinors $(\underline{\theta}, \bar{\underline{\theta}})$. This calls for a complete reconsideration of our ideas of Lorentz motion code,

*gago_50@yahoo.com

doi: https://doi.org/10.52526/25792776-23.70.2-212

to be now referred to as the *individual code of a particle*, defined as its intrinsic property. In MS_p-SUSY theory, obviously as in standard unbroken SUSY theory, the vacuum zero point energy problem, standing before any quantum field theory in M_4, is solved. The particles in M_4 themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4\oplus$ MS_p, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2 \equiv \nu_b = \nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy. On these premises, we have derived the two postulates on which the theory of Special Relativity (SR) is based.

The second part of phenomenon of inertia describes how the *inertial uniform motion* of a particle is affected by applied forces (*the accelerated motion and inertia effects*). A puzzling underlying reality of inertia effects stood open more than four centuries, and that this physics is still an unknown exciting problem to be challenged and allows various attempts. The beginning of the study of inertia effects can be traced back to the works developed by Galileo (Drake, 1978) and Newton (Newton, 1687). Certainly, more than four centuries passed since the famous far-reaching discovery of Galileo (in 1602-1604) that all bodies fall at the same rate (Drake, 1978) - an apparent enigmatic equality of inertial and passive gravitational mass, which led to an early empirical version of the suggestion that the gravity and inertia may somehow result from a single mechanism. Besides describing these early gravitational experiments, Newton in *Principia Mathematica* (Newton, 1687) has proposed a comprehensive approach to studying the relation between the gravitational and inertial masses of a body. Newton describes his precise pendulum experiments with "gold, silver, lead, glass, sand, common salt, wood, water and wheat" testing WPE - equality of inertial and passive gravitational mass.

Einstein's biographer Leopold Infeld wrote in 1950 (Infeld, 1950). "No one in our century, with the exception of Einstein, wondered about this law any longer", These efforts led to Einstein's extension in 1907 of WPE to all of physics (Beck & Havas, 1989): "...We consider two systems Σ_1 and Σ_2 in motion. Let Σ_1 be accelerated in the direction of its X-axis, and let γ be the (temporally constant) magnitude of that acceleration. Σ_2 shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration -γ in the direction of the X-axis". And then he formulate his principle of equivalence (EPE): "At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system". This involved *relative* acceleration in an attempt to introduce ideas of Ernst Mach into his theory. These ideas gave the theory its name. This did not prove to be a happy idea since this notion makes mathematical sense only for bodies having the same 4-velocity and from a physical point of view accelerations are *absolute*.

Ever since, there is an ongoing quest to understand the reason for the universality of the gravity and inertia, attributing to the WPE, which establishes the independence of free-fall trajectories of the internal composition and structure of bodies. At present, the variety of consequences of the precision experiments from astrophysical observations makes it possible to probe this fundamental issue more deeply by imposing the constraints of various analyzes. Currently, the observations performed in the Earth-Moon-Sun system (Everitt & et al., 2009, 2011, Faller & et al., 1990, Gabriel & Haugan, 1990, Gillies, 1997, Haugan & Kauffmann, 1995, Hayasaka & Takeuchi, 1989, Heifetz & et al., 2009, Imanishi & et al., 1991, Keiser & et al., 2009, Luo & et al., 2002, Muhlfelder & et al., 2009, Ni, 2011, Nitschke & Wilmarth, 1990, Quinn & Picard, 1990, Silbergleit & et al., 2009, Turyshev, 2008, Will, 2006, Zhou & et al., 2002), or at galactic and cosmological scales (Haugan & Lämmerzahl, 2001, Lämmerzahl & Bordé, 2001, Ni, 2005a,b,c, 2008), probe more deeply both WPE and strong EPE. The intensive efforts have been made, for example, to clear up whether the rotation state would affect the trajectory of test particle. Shortly after the development of the work by Hayasaka & Takeuchi (1989), in which is reported that, in weighing gyros, it would be a violation of WPE, by Faller & et al. (1990), Imanishi & et al. (1991), Nitschke & Wilmarth (1990), Quinn & Picard (1990) performed careful weighing experiments on gyros with improved precision, but found only null results which are in disagreement with the report of (Hayasaka & Takeuchi, 1989). The interferometric free-fall experiments by Luo & et al. (2002) and Zhou & et al. (2002) again found null results in disagreement with (Hayasaka & Takeuchi, 1989). For rotating bodies, the ultraprecise Gravity Probe B experiment (Everitt & et al., 2009, 2011, Heifetz & et al., 2009, Keiser & et al., 2009, Muhlfelder & et al., 2009, Silbergleit & et al., 2009), which measured the frame-dragging effect and geodetic precession on four quartz gyros, has the best accuracy. GP-B serves as a starting point for the measurement of the gyrogravitational factor of particles. Whereas, the gravitomagnetic field, which is locally equivalent to a *Coriolis field* and generated

doi: https://doi.org/10.52526/25792776-23.70.2-212

by the absolute rotation of a body, has been measured too. This, with its superb accuracy, verifies WPE for unpolarized bodies to an ultimate precision - a four-order improvement on the noninfluence of rotation on the trajectory, and ultraprecision on the rotational equivalence (Ni, 2011). Moreover, the theoretical models may indicate cosmic polarization rotations which are being looked for and tested in the CMB experiments (Ni, 2008). To look into the future, measurement of the gyrogravitational ratio of particle would be a further step, see (Ni, 2005c) and references therein, towards probing the microscopic origin of gravity. Also, the inertia effects in fact are of vital interest for the phenomenological aspects of the problem of neutrino oscillations, see e.g. (Atwood & et al., 1984, Bonse & Wroblewski, 1983, Capozziello & Lambiase, 2000, Cardall & Fuller, 1997, Colella et al., 1975, Gasperini, 1988, Halprin & Leung, 1991, Pantaleone et al., 1993, Piniz et al., 1997, de Sabbata & Gasperini, 1981). All these have evoked the study of the inertial effects in an accelerated and rotated frame. In doing this, it is a long-established practice in physics to use the hypothesis of locality for extension of the Lorentz invariance to accelerated observers in Minkowski spacetime (Misner et al., 1973, Synge, 1960). This in effect replaces the accelerated observer by a continuous infinity of hypothetical momentarily comoving inertial observers along its wordline. This assumption, as well as its restricted version, so-called, clock hypothesis, which is a hypothesis of locality only concerned about the measurement of time, are reasonable only if the curvature of the wordline could be ignored. As long as all relevant length scales in feasible experiments are very small in relation to the huge acceleration lengths of the tiny accelerations we usually experience, the curvature of the wordline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small. In this line, in 1990, Hehl and Ni proposed a framework to study the relativistic inertial effects of a Dirac particle (Hehl & Ni, 1990), in agreement with (Li & Ni, 1979, Ni, 1977, Ni & Zimmermann, 1978). Ever since this question has become a major preoccupation of physicists, see e.g. (Bakke & Furtado, 2010, Bini et al., 2004, Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Marzlin, 1996, Mashhoon, 2002, 2011, Pan & Ren, 2011, Silenko & Teryaev, 2007). Even this works out, still, it seems quite clear that such an approach is a work in progress, and that it will have to be extended to describe physics for arbitrary accelerated observers. Beyond the WPE, there is nothing convincing in the basic postulates of physics for the origin and nature of inertia to decide on the issue. Despite our best efforts, all attempts to obtain a true knowledge of the geometry related to the noninertial reference frames of an arbitrary observer seem doomed, unless we find *a physical principle the inertia might refer to*, and that a working alternative *relativistic theory of inertia* is formulated. Otherwise one wanders in a darkness.

In particular, the concept of a uniformly accelerated, gravitation-free reference system without the use of special relativity is the problem (Schucking, 2009, Schucking & Surowitz, 2012). In Newton's theory the acceleration, γ, could be a vector in the x-direction constant in space and time independent of the velocity of a body moving in the x-direction. Not so in Minkowski spacetime. There the acceleration vector has to be orthogonal to the 4-velocity and it would appear that homogeneity of the acceleration field in spacetime could no longer be achieved. That is, the notion of relative acceleration exists only for particles whose four-velocities agree. Interpreting the gravitational acceleration of a falling object as minus the acceleration of the reference system had to be restricted to objects at rest. It was a letter from Max Planck that had alerted Einstein to this fact, which was acknowledged by Einstein in the Erratum (1908, p. 317) to Ref. (Beck & Havas, 1989): "A letter by Mr. Planck induced me to add the following supplementary remark so as to prevent a misunderstanding that could arise easily. ...A reference system at rest situated in a temporally constant, homogeneous gravitational field is treated as physically equivalent to a uniformly accelerated, gravitation-free reference system. The concept 'uniformly accelerated' needs further clarification". Einstein then pointed out that "the equivalence was to be restricted to a body with zero velocity in the accelerated system. In a linear approximation, he concluded, this was sufficient because only linear terms had to be taken into account". Einstein's retreat raises the question whether it is impossible to find a homogeneous uniformly accelerated reference system, or, assuming exact validity of his principle of equivalence, a homogeneous gravitational field. But, as it is pointed out by Schucking & Surowitz (2012), a homogeneous gravitational field in Minkowski spacetime can be constructed if the reference frames in gravitational theory are understood as spaces with a flat connection and torsion defined through teleparallelism. In 1907 Einstein did not show the equivalence of acceleration and gravitation described by spacetime curvature. He did not show either the equivalence of geodesics and non-geodesics or the equivalence of rotating and non-rotating systems. What he did, we now can see more clearly, was the introduction of accelerated reference systems exhibiting torsion through distant parallelism. This kind of torsion was first introduced by Einstein in 1928. There were physical consequences that needed to be checked for these systems, like the constancy of the speed of light independent of acceleration, no influence

doi: https://doi.org/10.52526/25792776-23.70.2-212

of acceleration on the rate of clocks and the length of standards. As far as these assumptions have been tested, they appear to be in order.

Various attempts at the resolution of difficulties that are encountered in linking Mach's principle with Einstein's theory of gravitation have led to many interesting investigations. For example, by Mashhoon & Wesson (2012) is shown that the GR can be locally embedded in a Ricci-flat 5D manifold such that every solution of GR in 4D can be locally embedded in a Ricci-flat 5D manifold, and that the resulting inertial mass of a test particle varies in space-time.

However, the inertial forces are not of gravitational origin as it was proposed by EPE, because there are many controversies to question the validity of such a description. Synge confesses in his monograph (Synge (1960), in introduction) ". . . I have never been able to understand this Principle. ...Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's worldline. Space-time is either flat or curved, and in several places of the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's worldline (in most ordinary cases the latter predominate)..."

The difficulty is brought into sharper focus by considering the laws of inertia, including their quantitative aspects. That is, Mach principle and its modifications do not provide a quantitative means for computing the inertial forces. Brans's thorough analysis (Brans, 1977) has shown that no extra inertia is induced in a body as a result of the presence of other bodies. If the matter distribution is not isotropic, and if the inertia is due to gravitational interactions, then one of the consequences of Mach's principle could be conceivable that the concentration of matter near the centre of our galaxy may result in an anisotropy of inertia at the earth which could be detected experimentally. Testing this question, the experiments by Cocconi & Salpeter (1960), Drever (1961), Hughes et al. (1960) do not found such anisotropy of inertial mass. The experiment suggested by Cocconi and Salpeter was to observe the Zeeman splitting in the excited nuclear state of Fe^{57} by use of the Mössbauer effect. The effect of some anisotropy of inertia is then very similar to that for the atomic Zeeman effect, except that one is now dealing with the motion of the nucleon in the Fe^{57} responsible for the γ-ray transition (at least on a shell-model picture) instead of the motion of an electron. They found that the variation Δm of mass with direction, if it exists, should satisfy $\frac{\Delta m}{m} \leq 10^{-9}$. The most sensitive test is obtained in (Hughes et al., 1960) from a nuclear magnetic resonance experiment with a Li^7 nucleus of spin $I = 3/2$. This method gives a sensitivity some factor of 10^6 greater than could be achieved in the experiment suggested by Cocconi and Salpeter using the Mössbauer effect. The magnetic field was of about 4700 gauss was stabilized against the proton resonance frequency with the Atomichron as a frequency standard. Only a single line was observed. The increase in sensitivity over that which one could obtain from the Mössbauer effect is due to the far narrower line width obtainable for a transition with a nucleus in its ground state as compared with a nucleus in an excited state. The south direction in the horizontal plane points within 22 degrees towards the center of our galaxy, and 12 hour later this same direction along the earth's horizontal plane points 104 degrees away from the galactic center. If the nuclear structure of Li^7 is treated as a single $P_{3/2}$ proton in a central nuclear potential, the variation Δm of mass with direction, if it exists, was found to satisfy $\frac{\Delta m}{m} \leq 10^{-20}$. This is by now very strong evidence that there is no anisotropy of mass which is due to the effects of mass in our galaxy. Hence it seems that within the framework of the Mach theory as discussed by Cocconi and Salpeter one should conclude that there is no anisotropy of mass of the type which varies as $P_2(cos\theta)$ associated with effects of mass in our galaxy. Here $P_2(cos\theta)$ is the Legendre polynomial of order 2, and θ is the angle between the direction of acceleration of the particle (determined by the direction of an external magnetic field H and by the magnetic quantum state) and the direction to the galactic center. In the same time, Dicke (1961) and Ni (1983) have pointed out that when anisotropic effects on both kinetic and potential energies are considered, the null results are to be expected provided that the anisotropy couples in the same way to both forms of energy. It is concluded that the extremely accurate null result of the experiment of (Hughes et al., 1960) seemed does not cast doubt upon the validity of Mach's principle. They stated that, on the contrary, this important experiment shows, with great precision, that inertial anisotropy effects are universal, the same for all particles. These experiments can thus be regarded as a test of the universal coupling of gravity to all forms of mass-energy. Anyway, the subsequent experimental test along this line (Prestage & et al., 1985), using nuclear-spin-polarized $^9Be^+$ ions, also gives null result on spatial anisotropy and thus supporting local Lorentz invariance. In this experiment the frequency of a nuclear spin-flip transition in $^9Be^+$ has been compared to the frequency of a hydrogen maser transition to

doi: https://doi.org/10.52526/25792776-23.70.2-212

see if the relative frequencies depend on the orientation of the ${}^9Be^+$ ions in space. Obtained null result represents a decrease in the limits set by (Hughes et al., 1960) and (Drever, 1961) on a spatial anisotropy by a factor of about 300.

Our idea is that the *universality* of gravitation and inertia attribute to the single mechanism of origin from geometry but having a different nature. We have ascribed, therefore, the inertia effects (second part of inertia) to the geometry itself but as having a nature other than 4D Riemannian space (Ter-Kazarian, 2012). The key to our construction procedure of the toy model is an assignment to each and every particle individually a new fundamental constituent of hypothetical 2D, so-called, *master-space* (MS_p), subject to certain rules. The MS_p, embedded in the background 4D-space, is an *unmanifested* indispensable individual companion to the particle of interest. Within this scheme, the MS_p was presumably allowed to govern the motion of individual particle in the background 4D-space. The particle has to live with MS_p-companion as an intrinsic property devoid of any external influence. This together with the idea that the inertia effects arise as a deformation/(distortion of local internal properties) of MS_p, are the highlights of the alternative *relativistic theory of inertia* (RTI) (Ter-Kazarian, 2012). The crucial point is to observe that, in spite of totally different and independent physical sources of gravity and inertia, the RTI furnishes justification for the introduction of the PE. However, this investigation obviously is incomplete unless it has conceptual problems for further motivation and justification of introducing the fundamental concept of MS_p. The way we assigned such a property to the MS_p is completely *ad hoc* and there are some obscure aspects of this hypothesis. Moreover, this theory will certainly be incomplete without revealing the physical processes that underly the *inertial uniform motion* of a particle in flat space.

Since in (Ter-Kazarian, 2023), we already have solved these questions in the framework of *global* MS_p-SUSY theory, then on these premises, in the present paper we develop the *local* $\widetilde{MS}_p$-SUSY theory in order to address the *accelerated motion and inertia effects*. A curvature of $\widetilde{MS}_p$ arises entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a *fictitious gravitation*, which can be globally removed by appropriate coordinate transformations. Whereas, in order to become on the same footing with the distorted space $\widetilde{MS}_p$, the space $\widetilde{M}_4$ refers only to the accelerated proper reference frame of a particle.

With this perspective in sight, we will proceed according to the following structure. To start with, in Section 2 we revisit the global \`double space´- or MS_p-SUSY as a guiding principle to make the rest of paper understandable. We give a glance at MS_p, and outline the key points of the proposed symmetry. More about the accelerated motion is said in Section 3, but this time in the framework of local $\widetilde{MS}_p$-SUSY. In Section 4, we turn to non-trivial linear representation of the $\widetilde{MS}_p$-SUSY algebra. In Section 5 we briefly discuss the inertia effects. In Section 6, we turn to model building in the 4D background Minkowski space-time On these premises, we discuss the theory beyond the hypothesis of locality in Section 7. In this, we compute the improved metric and other relevant geometrical structures in noninertial system of arbitrary accelerating and rotating observer in Minkowski space-time. The case of semi-Riemann background space V_4 is studied in Section 8, whereas we give justification for the introduction of the *weak* principle of equivalence (WPE) on the theoretical basis, which establishes the independence of free-fall trajectories of the internal composition and structure of bodies. The implications of the inertial effects for the more general post-Riemannian geometry are briefly discussed in Section 9. Concluding remarks are presented in Section 10. For brevity, whenever possible undotted and dotted spinor indices often can be ruthlessly suppressed without ambiguity. Unless indicated otherwise, we take natural units, $h = c = 1$.

2. Probing SR behind the MS_p-SUSY, revisited

For a benefit of the reader, as a guiding principle to make the rest of paper understandable, in this section we necessarily recount some of the highlights behind of *global* MS_p-SUSY (Ter-Kazarian, 2023, 2024), on which the *local* $\widetilde{MS}_p$-SUSY is based. The latter is the only framework in use throughout the paper.

The flat MS_p is the 2D composite space

$$MS_p \equiv \underline{M}_2 = \underline{R}^1_{(+)} \oplus \underline{R}^1_{(-)}, \tag{1}$$

with Lorentz metric. The ingredient 1D-space $\underline{R}^1_{\underline{m}}$ is spanned by the coordinates $\underline{\eta}^{\underline{m}}$. The following notational conventions are used throughout this paper: all quantities related to the space $\underline{M}_2$ will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, ... = (\pm)$ denote the world indices related to $\underline{M}_2$.

Suppose the position of the particle is specified by the coordinates $x^m(s)$ $(x^0 = t)$ in the basis e_m (m=0,1,2,3) at given point in the background M_4 space. Consider a smooth (injective and continuous)

doi: https://doi.org/10.52526/25792776-23.70.2-212

embedding $\underline{M}_2 \hookrightarrow M_4$. That is, a smooth map $f: \underline{M}_2 \longrightarrow M_4$ is defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{e}_0 = e_0, \quad \underline{x}^0 = x^0, \quad \underline{e}_1 = \vec{n}, \quad \underline{x}^1 = |\vec{x}|, \tag{2}$$

where $\vec{x} = e_i x^i = \vec{n}|\vec{x}|$ $(i = 1, 2, 3)$. Given the inertial frames $S_{(4)}, S'_{(4)}, S''_{(4)}, ...$ in unaccelerated uniform motion in M_4, we may define the corresponding inertial frames $\underline{S}_{(2)}$, $\underline{S}'_{(2)}$, $\underline{S}''_{(2)}$,... in $\underline{M}_2$, which are used by the non-accelerated observers for the positions $\underline{x}^r, \underline{x}'^r, \underline{x}''^r, ...$ of a free particle in flat $\underline{M}_2$. According to (2), the time axes of the two systems $\underline{S}_{(2)}$ and $S_{(4)}$ coincide in direction, and the time coordinates are taken the same. For the case at hand,

$$\underline{v}^{(\pm)} = \frac{d\eta^{(\pm)}}{d\underline{x}^0} = \frac{1}{\sqrt{2}}(\underline{v}^0 \pm \underline{v}^1), \quad \underline{v}^1 = \frac{d\underline{x}^1}{d\underline{x}^0} = |\vec{v}| = |\frac{d\vec{x}}{dx^0}|, \tag{3}$$

and that

$$\underline{u} = \underline{e}_m \underline{v}^m = (\underline{\vec{v}}_0, \underline{\vec{v}}_1), \quad \underline{\vec{v}}_0 = \underline{e}_0 \underline{v}^0, \quad \underline{\vec{v}}_1 = \underline{e}_1 \underline{v}^1 = \vec{n}|\vec{v}| = \vec{v}, \tag{4}$$

therefore, $\underline{u} = u = (e_0, \vec{v})$. To explain why MS$_p$ is two dimensional, we note that only 2D real *null vectors* are allowed as the basis at given point in MS$_p$, which is embedded in M_4. Literally speaking, the $\underline{M}_2$ can be viewed as 2D space living on the 4D world sheet.

The elementary act of particle motion at each time step (t_i) through the infinitely small spatial interval $\triangle x_i = (x_{i+1} - x_i)$ in M_4 during the time interval $\triangle t_i = (t_{i+1} - t_i) = \varepsilon$ is probably the most fascinating challenge for physical research. Since this is beyond our perception, it appears legitimate to consider extension to the infinitesimal Schwinger transformation function, $F_{ext}(x_{i+1}, t_{i+1}; x_i, t_i)$, in fundamentally different aspect. We hypothesize that

in the limit $n \to \infty (\varepsilon \to 0)$, the elementary act of motion consists of an ´annihilation´ of a particle at point $(x_i, t_i) \in M_4$, which can be thought of as the transition from initial state $|x_i, t_i >$ into unmanifested intermediate state, so-called, ´motion´ state, $|\underline{x}_i, \underline{t}_i >$, and of subsequent ´creation´ of a particle at infinitely close final point $(x_{i+1}, t_{i+1}) \in M_4$, which means the transition from ´motion´ state, $|\underline{x}_i, \underline{t}_i >$, into final state, $|x_{i+1}, t_{i+1} >$. The motion state, $|\underline{x}_i, \underline{t}_i) >$, should be defined on unmanifested ´master´ space, $\underline{M}_2$, which includes the points of all the atomic elements, $(\underline{x}_i, \underline{t}_i) \in \underline{M}_2$ $(i = 1, 2, ...)$.

This furnishes justification for an introduction of *unmanifested master space*, $\underline{M}_2$.

The fields of spin-zero $(\vec{S} = \vec{K} = 0)$ scalar field $A(x)$ and spin-one $A^n(x)$, corresponding to the $(1/2, 1/2)$ representation, transform under a general Lorentz transformation as follows:

$$\begin{array}{ll} \underline{A}(\underline{\eta}) \equiv A(x), & (\text{spin} \quad 0); \\ \underline{A}^m(\underline{\eta}) = \Lambda^m{}_n \, A^n(x), & (\text{spin} \quad 1). \end{array} \tag{5}$$

The map from $SL(2, C)$ to the Lorentz group is established through the $\vec{\sigma}$-Pauli spin matrices, $\sigma^m = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \equiv (I_2, \vec{\sigma})$, $\bar{\sigma}^m \equiv (I_2, -\vec{\sigma})$, where I_2 is the identity two-by-two matrix.

According to embedding map (2), the $\underline{\sigma}$-matrices are

$$\sigma^{\underline{m}} = \sigma^{(\pm)} = \frac{1}{\sqrt{2}}(\sigma^{\underline{0}} \pm \sigma^{\underline{1}}) = \frac{1}{\sqrt{2}}(\sigma^0 \pm \sigma^3). \tag{6}$$

The matrices $\sigma^{\underline{m}}$ form a basis for two-by-two complex matrices $\underline{P}$:

$$\underline{P} = (p_{\underline{m}} \sigma^{\underline{m}}) = (p_{(\pm)} \sigma^{(\pm)}) = (p_{\underline{0}} \sigma^{\underline{0}} + p_{\underline{1}} \sigma^{\underline{1}}), \tag{7}$$

provided $p_{(\pm)} = i\partial_{\underline{\eta}^{(\pm)}}$, $p_{\underline{0}} = i\partial_{\underline{x}^0}$ and $p_{\underline{1}} = i\partial_{\underline{x}^1}$. The real coefficients $p'_{\underline{m}}$ and $p_{\underline{m}}$, like p'_m and p_m, are related by a Lorentz transformation $p'_{\underline{m}} = \Lambda^{\underline{n}}_{\underline{m}} p_{\underline{n}}$, because the relations $det(\sigma^{\underline{m}} p_{\underline{m}}) = p_{\underline{0}}^2 - p_{\underline{1}}^2$ and $det\, M = 1$ yield $p'^2_{\underline{0}} - p'^2_{\underline{1}} = p_{\underline{0}}^2 - p_{\underline{1}}^2$. Correspondence of $p_{\underline{m}}$ and $\underline{P}$ is uniquely: $p_{\underline{m}} = \frac{1}{2} Tr(\sigma^{\underline{m}} \underline{P})$, which combined with (9) yields

$$\Lambda^{\underline{m}}_{\underline{n}}(M) = \frac{1}{2}\, Tr\left(\sigma^{\underline{m}} M \sigma^{\underline{n}} M^\dagger\right). \tag{8}$$

Thus, both hermitian matrices P and P' or $\underline{P}$ and $\underline{P}'$ have expansions, respectively, in σ or $\underline{\sigma}$:

$$(\sigma^m p'_m) = M(\sigma^m p_m)M^\dagger, \quad (\sigma^{\underline{m}} p'_{\underline{m}}) = M(\sigma^{\underline{m}} p_{\underline{m}})M^\dagger, \tag{9}$$

where $M (M \in SL(2, C))$ is unimodular two-by-two matrix. Meanwhile $(\underline{\chi}\, \sigma^{\underline{m}}\, \bar{\underline{\zeta}})\, \underline{A}_{\underline{m}}$ is a Lorentz scalar if the following condition is satisfied:

$$\Lambda^{\underline{m}}_{\underline{n}}(M) \sigma^{\underline{n}}_{\alpha\dot{\alpha}} = (M^{-1})_\alpha{}^\beta \sigma^{\underline{m}}_{\beta\dot{\beta}} (M^{-1})^{\dagger\dot{\beta}}{}_{\dot{\alpha}}. \tag{10}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

A two-component $(1/2,0)$ Weyl fermion, $\chi_\beta(x)$, therefore, transforms under Lorentz transformation to yield $\underline{\chi}_\alpha(\underline{\eta})$:

$$\chi_\beta(x) \longrightarrow \underline{\chi}_\alpha(\underline{\eta}) = (M_R)_\alpha{}^\beta \chi_\beta(x), \quad \alpha,\beta = 1,2, \tag{11}$$

where the orthochronous Lorentz transformation, corresponding to a rotation by the angles ϑ_3 and ϑ_2 about, respectively, the axes n_3 and n_2, is given by rotation matrix

$$M_R = e^{i\frac{1}{2}\sigma_2\vartheta_2} e^{i\frac{1}{2}\sigma_3\vartheta_3}. \tag{12}$$

There with the rotation of an hermitian matrix P is

$$p_{\underline{m}}\sigma^{\underline{m}} = M_R\, p_m\sigma^m\, M_R^\dagger, \tag{13}$$

where p_m and $p_{\underline{m}}$ denote the momenta $p_m \equiv m(ch\beta,\, sh\beta\sin\vartheta_2\cos\vartheta_3,\, sh\beta\sin\vartheta_2\sin\vartheta_3,$ $sh\beta\cos\vartheta_2)$, and $p_{\underline{m}} \equiv m(ch\underline{\beta},\, 0,\, 0,\, sh\underline{\beta})$.

A two-component $(0,1/2)$ Weyl spinor field is denoted by $\bar{\chi}^{\dot\beta}(x)$, and transforms as

$$\bar{\chi}^{\dot\beta}(x) \longrightarrow \underline{\bar{\chi}}^{\dot\alpha}(\underline{\eta}) = (M_R^{-1})^{\dagger\dot\alpha}{}_{\dot\beta}\, \bar{\chi}^{\dot\beta}(x), \quad \dot\alpha,\dot\beta = 1,2. \tag{14}$$

The so-called `dotted´ indices have been introduced to distinguish the $(0,1/2)$ representation from the $(1/2,0)$ representation. The `bar´ over the spinor is a convention that this is the $(0,1/2)$-representation. We used the Van der Waerden notations for the Weyl two-component formalism: $(\underline{\bar\chi}_{\dot\alpha})^* = \underline{\chi}_\alpha$ and $\underline{\bar\chi}_{\dot\alpha} = (\underline{\chi}_\alpha)^*$.

The odd part of the supersymmetry algebra is composed entirely of the spin-1/2 operators $Q_\alpha{}^i$, $Q_{\dot\beta}{}^j$. In order to trace a maximal resemblance in outward appearance to the standard SUSY theories, here we set one notation $\hat{m} = (m \quad \text{if} \quad Q = q, \text{ or } \underline{m} \quad \text{if} \quad Q = \underline{q})$, and as before the indices α and $\dot\alpha$ run over 1 and 2.

If that is the case as above, a *creation* of a particle in $\underline{M}_2$ means its transition from initial state defined on M_4 into intermediate state defined on $\underline{M}_2$, while an *annihilation* of a particle in $\underline{M}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in M_4. All the fermionic and bosonic states taken together form a basis in the Hilbert space. The basis vectors in the Hilbert space composed of $H_B \otimes H_F$ is given by

$$\{|\underline{n}_b > \otimes|0>_f,\, |\underline{n}_b > \otimes f^\dagger\,|0>_f\},$$

or

$$\{|n_b > \otimes|\underline{0}>_f,\, |n_b > \otimes \underline{f}^\dagger\,|\underline{0}>_f\},$$

where we consider two pairs of creation and annihilation operators $(b^\dagger,\, b)$ and $(f^\dagger,\, f)$ for bosons and fermions, respectively, referred to the background space M_4, as well as $(\underline{b}^\dagger,\, \underline{b})$ and $(\underline{f}^\dagger,\, \underline{f})$ for bosons and fermions, respectively, as to background master space $\underline{M}_2$. Accordingly, we construct the quantum operators, $(q^\dagger,\, \underline{q}^\dagger)$ and $(q,\, \underline{q})$, which replace bosons by fermions and vice versa:

$$\begin{aligned} q\,|\underline{n}_b,\, n_f >&= q_0\sqrt{\underline{n}_b}\,|\underline{n}_b - 1,\, n_f + 1>, \\ q^\dagger\,|\underline{n}_b,\, n_f >&= q_0\sqrt{\underline{n}_b + 1}\,|\underline{n}_b + 1,\, n_f - 1>, \end{aligned} \tag{15}$$

and that

$$\begin{aligned} \underline{q}\,|n_b,\, \underline{n}_f >&= q_0\sqrt{n_b}\,|n_b - 1,\, \underline{n}_f + 1>, \\ \underline{q}^\dagger\,|n_b,\, \underline{n}_f >& q_0\sqrt{n_b + 1}\,|n_b + 1,\, \underline{n}_f - 1> . \end{aligned} \tag{16}$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators q and $\underline{q}$. So, we may refer the action of the supercharge operators q and $q^\dagger$ to the background space M_4, having applied in the chain transformations of fermion χ (accompanied with the auxiliary field F as it will be seen later on) to boson $\underline{A}$, defined on $\underline{M}_2$:

$$\longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow . \tag{17}$$

Respectively, we may refer the action of the supercharge operators $\underline{q}$ and $\underline{q}^\dagger$ to the $\underline{M}_2$, having applied in the chain transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field $\underline{F}$) to boson A, defined on the background space M_4:

$$\longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow . \tag{18}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

The successive atomic double transitions of a particle $M_4 \rightleftharpoons \underline{M}_2$ is investigated within MS$_p$-SUSY, wherein all the particles are living on M_4, their superpartners can be viewed as living on MS$_p$. The underlying algebraic structure of MS$_p$-SUSY generators closes with the algebra of *translations* on the original space M_4 in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra those of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to M_4. Moreover, if there are several spinor generators $Q_\alpha{}^i$ with $i = 1, ..., N$ - theory with $N-$extended supersymmetry, can be written as a graded Lie algebra of SUSY field theories, with commuting and anticommuting generators:

$$\begin{array}{l} \{Q_\alpha{}^i,\, \bar{Q}^j{}_{\dot\alpha}\} = 2\delta^{ij}\, \sigma^{\hat m}_{\alpha\dot\alpha}\, p_{\hat m}; \\ \{Q_\alpha{}^i,\, Q_\beta{}^j\} = \{\bar{Q}^i{}_{\dot\alpha},\, \bar{Q}^j{}_{\dot\beta}\} = 0; \quad [p_{\hat m},\, Q_\alpha{}^i] = [p_{\hat m},\, \bar{Q}^j{}_{\dot\alpha}] = 0, \quad [p_{\hat m},\, p_{\hat n}] = 0. \end{array} \tag{19}$$

The anticommuting (Grassmann) parameters $\epsilon^\alpha(\xi^\alpha,\, \underline{\xi}^\alpha)$ and $\bar\epsilon^\alpha(\bar\xi^\alpha,\, \underline{\bar\xi}^\alpha)$:

$$\{\epsilon^\alpha,\, \epsilon^\beta\} = \{\bar\epsilon^\alpha,\, \bar\epsilon^\beta\} = \{\epsilon^\alpha,\, \bar\epsilon^\beta\} = 0, \quad \{\epsilon^\alpha,\, Q_\beta\} = \cdots = [p_{\hat m},\, \epsilon^\alpha] = 0, \tag{20}$$

allow us to write the algebra (19) for $(N = 1)$ entirely in terms of commutators:

$$[\epsilon Q,\, \bar{Q}\bar\epsilon] = 2\epsilon\sigma^{\hat m}\bar\epsilon p_{\hat m}, \quad [\epsilon Q,\, \epsilon Q] = [\bar Q\bar\epsilon,\, \bar Q\bar\epsilon] = [p^{\hat m},\, \epsilon Q] = [p^{\hat m},\, \bar Q\bar\epsilon] = 0. \tag{21}$$

For brevity, here the indices $\epsilon Q = \epsilon^\alpha Q_\alpha$ and $\bar\epsilon\bar Q = \bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha}$ will be suppressed unless indicated otherwise. This supersymmetry transformation maps tensor fields $\mathcal{A}(A,\, \underline{A})$ into spinor fields $\psi(\chi,\, \underline{\chi})$ and vice versa. From the algebra (21) we see that Q has mass dimension $1/2$. Therefore, as usual, fields of dimension ℓ transform into fields of dimension $\ell + 1/2$ or into derivatives of fields of lower dimension. It can be checked that the supersymmetry transformations close supersymmetry algebra:

$$(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})\underline{A} = -2i(\xi_1\sigma^m\bar\xi_2 - \xi_2\sigma^m\bar\xi_1)(\delta^0_m\underline{\partial}_0 + \tfrac{1}{|\vec x|}x^i\delta_{im}\underline{\partial}_1)\underline{A}. \tag{22}$$

The guiding principle of MS$_p$-SUSY resides in constructing the superspace which is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ (spanned by the 6D-coordinates $X^{\hat m} = (x^m,\, \underline{\eta}^{\underline{m}})$ by the inclusion of additional 8D-fermionic coordinates $\Theta^\alpha = (\theta^\alpha,\, \underline{\theta}^\alpha)$ and $\bar\Theta_{\dot\alpha} = (\bar\theta_{\dot\alpha},\, \underline{\bar\theta}_{\dot\alpha})$, as to $(q,\, \underline{q})$, respectively. Therewith thanks to the embedding $\underline{M}_2 \hookrightarrow M_4$, the spinors $(\underline{\theta}, \underline{\bar\theta})$, in turn, induce the spinors $\theta(\underline{\theta}, \underline{\bar\theta})$ and $\bar\theta(\underline{\theta}, \underline{\bar\theta})$, as to M_4. These spinors satisfy the following relations:

$$\begin{array}{l} \{\Theta^\alpha,\, \Theta^\beta\} = \{\bar\Theta_{\dot\alpha},\, \bar\Theta_{\dot\beta}\} = \{\Theta^\alpha,\, \bar\Theta_{\dot\beta}\} = 0, \\ [x^m,\, \theta^\alpha] = [x^m,\, \bar\theta_{\dot\alpha}] = 0, \quad [\underline{\eta}^{\underline{m}},\, \underline{\theta}^\alpha] = [\underline{\eta}^{\underline{m}},\, \underline{\bar\theta}_{\dot\alpha}] = 0. \end{array} \tag{23}$$

and $\Theta^{\alpha *} = \bar\Theta^{\dot\alpha}$. Points in superspace are identified by the generalized coordinates

$$z^{(M)} = (X^{\hat m},\, \Theta^\alpha,\, \bar\Theta_{\dot\alpha}) = (x^m,\, \theta^\alpha,\, \bar\theta_{\dot\alpha}) \oplus (\underline{\eta}^{\underline{m}}, \underline{\theta}^\alpha,\, \underline{\bar\theta}_{\dot\alpha}).$$

We have then the one most commonly used `real´ or `symmetric´ superspace parametrized by

$$\Omega(X,\, \Theta,\, \bar\Theta) = e^{i(-X^{\hat m}p_{\hat m} + \Theta^\alpha Q_\alpha + \bar\Theta_{\dot\alpha}\bar Q^{\dot\alpha})} = \Omega_q(x,\, \theta,\, \bar\theta) \times \Omega_{\underline{q}}(\underline{\eta},\, \underline{\theta},\, \underline{\bar\theta}), \tag{24}$$

where we now imply a summation over $\hat m = (m, \underline{m})$. To study the effect of supersymmetry transformations, we consider

$$g(0,\, \epsilon,\, \bar\epsilon)\, \Omega(X,\, \Theta,\, \bar\Theta) = e^{i(\epsilon^\alpha Q_\alpha + \bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha})}\, e^{i(-X^{\hat m}p_{\hat m} + \Theta^\alpha Q_\alpha + \bar\Theta_{\dot\alpha}\bar Q^{\dot\alpha})}. \tag{25}$$

the transformation (25) induces the motion:

$$g(0,\, \epsilon,\, \bar\epsilon)\, \Omega(X^{\hat m},\, \Theta,\, \bar\Theta) \;\to (X^{\hat m} + i\,\Theta\,\sigma^{\hat m}\,\bar\epsilon - i\,\epsilon\,\sigma^{\hat m}\,\bar\Theta,\; \Theta + \epsilon,\, \bar\Theta + \bar\epsilon), \tag{26}$$

namely,

$$\begin{array}{l} g_q(0,\, \xi,\, \bar\xi)\, \Omega_q(x,\, \theta,\, \bar\theta) \;\to (x^m + i\,\theta\,\sigma^m\,\bar\xi - i\,\xi\,\sigma^m\,\bar\theta,\; \theta + \xi,\, \bar\theta + \bar\xi), \\ g_{\underline{q}}(0,\, \underline{\xi},\, \underline{\bar\xi})\, \Omega_{\underline{q}}(\underline{\eta},\, \underline{\theta},\, \underline{\bar\theta}) \;\to (\underline{\eta}^{\underline{m}} + i\,\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar\xi} - i\,\underline{\xi}\,\sigma^{\underline{m}}\,\underline{\bar\theta},\; \underline{\theta} + \underline{\xi},\, \underline{\bar\theta} + \underline{\bar\xi}). \end{array} \tag{27}$$

The spinors $\theta(\underline{\theta}, \underline{\bar\theta})$ and $\bar\theta(\underline{\theta}, \underline{\bar\theta})$ satisfy the embedding relations $\Delta\underline{x}^0 = \Delta x^0$ and $\Delta\underline{x}^2 = (\Delta\vec x)^2$, so from (27) we obtain

$$\underline{\theta}\,\sigma^0\,\underline{\bar\xi} - \underline{\xi}\,\sigma^0\,\underline{\bar\theta} = \theta\,\sigma^0\,\bar\xi - \xi\,\sigma^0\,\bar\theta, \quad (\underline{\theta}\,\sigma^3\,\underline{\bar\xi} - \underline{\xi}\,\sigma^3\,\underline{\bar\theta})^2 = (\theta\,\vec\sigma\,\bar\xi - \xi\,\vec\sigma\,\bar\theta)^2. \tag{28}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

The *atomic displacement* caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$ reads

$$\Delta\underline{\eta}_{(a)} = \underline{e}_m \Delta\underline{\eta}^m_{(a)} = \underline{u}\tau, \tag{29}$$

where the components $\Delta\underline{\eta}^m_{(a)}$ are written

$$\Delta\underline{\eta}^m_{(a)} = (\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar\theta})\tau. \tag{30}$$

In Van der Warden notations for the Weyl two-component formalism $\underline{\bar\theta}_{\dot\alpha} = (\underline{\theta}_\alpha)^*$, the (29) can be recast into the form

$$\Delta\underline{\eta}^2_{(a)} = \tfrac{1}{2}\left[(\Delta\underline{x}^0_{(a)}q)^2 - (\Delta\underline{x}^1_{(a)})^2\right], \tag{31}$$

where $\Delta\underline{x}^0_{(a)} = \underline{v}^0\tau$, $\Delta\underline{x}^1_{(a)} = \underline{v}^1\tau$, and $\underline{v}^{(\pm)} = \frac{1}{\sqrt{2}}(\underline{v}^0 \pm \underline{v}^1)$. Hence the velocities of light in vacuum, $\underline{v}^0 = c$, and of a particle ,$\vec{\underline{v}}_1 = \underline{e}_1\underline{v}^1 = \vec{n}|\vec{v}| = \vec{v}$ $(|\vec{v}| \leq c)$, are

$$\begin{aligned}\underline{v}^0 &= \underline{\theta}\,\sigma^0\,\underline{\bar\theta} = (\underline{\theta}_1\,\underline{\bar\theta}_1 + \underline{\theta}_2\,\underline{\bar\theta}_2) = \underline{\theta}\,\underline{\bar\theta},\\ \underline{v}^1 &= \underline{\theta}\,\sigma^1\,\underline{\bar\theta} = (\underline{\theta}_1\,\underline{\bar\theta}_1 - \underline{\theta}_2\,\underline{\bar\theta}_2),\end{aligned} \tag{32}$$

where

$$\begin{aligned}\theta_1(\underline{\theta},\,\underline{\bar\theta}) &= \tfrac{1}{2}\left[\left(\underline{v}^0 + \sqrt{\tfrac{2}{3}}\underline{v}^1\right)^{1/2} + \left(\underline{v}^0 - \sqrt{\tfrac{2}{3}}\underline{v}^1\right)^{1/2}\right],\\ \theta_2(\underline{\theta},\,\underline{\bar\theta}) &= \tfrac{1}{2}\left[\left(\underline{v}^0 + \sqrt{\tfrac{2}{3}}\underline{v}^1\right)^{1/2} - \left(\underline{v}^0 - \sqrt{\tfrac{2}{3}}\underline{v}^1\right)^{1/2}\right].\end{aligned} \tag{33}$$

Thus we derive the first founding property (i) that *the atomic displacement* $\Delta\underline{\eta}_{(a)}$, caused by double transition of a particle $M_4 \rightleftharpoons \underline{M}_2$, is an invariant:

$$\text{(i)}\quad \Delta\underline{\eta}_{(a)} = \Delta\underline{\eta}'_{(a)} = \cdots = inv. \tag{34}$$

The (32) gives the second (ii) founding property that the bilinear combination $\underline{\theta}\,\underline{\bar\theta}$ is a constant:

$$\text{(ii)}\quad c = \underline{\theta}\,\underline{\bar\theta} = \underline{\theta}'\,\underline{\bar\theta}' = \cdots = const. \tag{35}$$

The latter yields a second postulate of SR (Einstein's postulate) - *the velocity of light, c, in free space appears the same to all observers regardless the relative motion of the source of light and the observer.* The c is the maximum attainable velocity (32) for uniform motion of a particle in Minkowski background space, M_4. Equally noteworthy is the fact that (34) and (35) combined yield *invariance of the element of interval between two events* $\Delta x = k\Delta\underline{\eta}_{(a)}$ *(for given integer number k) with respect to the Lorentz transformation*:

$$\begin{aligned}&k^2\Delta\underline{\eta}^2_{(a)} = (c^2 - \underline{v}^2_1)\Delta t^2 = (c^2 - \vec{v}^{\,2})\Delta t^2 = (\Delta x^0)^2 - (\Delta\vec{x})^2 \equiv (\Delta s)^2 = (\Delta x'^0)^2 -\\ &(\Delta\vec{x})'^2 \equiv (\Delta s')^2 = \cdots = inv.,\end{aligned} \tag{36}$$

where $x^0 = ct,\quad x^{0'} = ct', \ldots$. We have here introduced a notion of physical relative finite *time intervals* between two events $\Delta t = k\tau/\sqrt{2}$, $\Delta t' = k\tau'/\sqrt{2}, \ldots$.

3. Accelerated motion and local $\widetilde{MS}_p$-SUSY

On the premises of Section 2, in what follows, we address the *accelerated motion and inertia effects* in the framework of *local* $\widetilde{MS}_p$-SUSY.

In case of an accelerated ($a = |\vec{a}| \neq 0$) motion of a particle in M_4, according to (27), we have then

$$\begin{aligned}&\tfrac{i}{\sqrt{2}}\left(\underline{\theta}\,\sigma^3\frac{d^2\bar\xi}{dt^2} - \frac{d^2\xi}{dt^2}\sigma^3\underline{\bar\theta}\right) = \frac{d^2q}{dt^2} = a = \tfrac{1}{\sqrt{2}}\left(\frac{d^2\eta^{(+)}}{dt^2} - \frac{d^2\eta^{(-)}}{dt^2}\right) =\\ &\tfrac{1}{\sqrt{2}}\left(a^{(+)} - a^{(-)}\right),\quad a^{(\pm)} = \frac{dv^{(\pm)}}{dt}.\end{aligned} \tag{37}$$

So, we may relax the condition $\partial_{\hat m}\epsilon = 0$ and promote this symmetry to a local supersymmetry in which the parameter $\epsilon = \epsilon(X^{\hat m})$ depends explicitly on $X^{\hat m} = (\widetilde{x}^m,\, \widetilde{\underline{\eta}}^{\underline{m}})$, where $\widetilde{x}^m \in \widetilde{M}_4$ and $\widetilde{\underline{\eta}}^{\underline{m}} \in \widetilde{\underline{M}}_2$. The

doi: https://doi.org/10.52526/25792776-23.70.2-212

mathematical structure of the local $\widetilde{MS}_p$-SUSY theory has much in common with those used in the geometrical framework of standard supergravity theories. Such a local SUSY can already be read off from the algebra (21) in the form

$$[\epsilon(X)Q,\, \bar{Q}\bar{\epsilon}(X)] = 2\epsilon(X)\sigma^{\hat{m}}\bar{\epsilon}(X)\widetilde{p}_{\hat{m}}, \tag{38}$$

which says that the product of two supersymmetry transformations corresponds to a translation in space-time of which the four momentum $\widetilde{p}_{\hat{m}}$ is the generator. In accord, the transformation (22) is expected to be somewhat of the form

$$[\delta_{\epsilon_1(X)},\, \delta_{\epsilon_2(X)}]V \sim \epsilon_1(X)\sigma^{\hat{m}}\bar{\epsilon}_2(X)\,\partial_{\hat{m}}V, \tag{39}$$

that differ from point to point, namely this is the notion of a general coordinate transformation. Whereupon we see that for the local $\widetilde{MS}_p$-SUSY to exist it requires the background spaces $(\widetilde{M}_4,\, \widetilde{\underline{M}}_2)$ to be curved. Thereby, the space $\widetilde{M}_4$, in order to become on the same footing with the distorted space $\widetilde{\underline{M}}_2$, refers to the accelerated proper reference frame of a particle, without relation to other matter fields. A useful guide in the construction of local superspace is that it should admit rigid superspace as a limit. The reverse is also expected, since if one starts with a constant parameter ϵ (20) and performs a local Lorentz transformation, then this parameter will in general become space-time dependent as a result of this Lorentz transformation.

Supergravity theories have been successfully formulated in terms of differential forms in superspace. We may introduce supergravity in a way which is manifestly covariant under such coordinate transformations. This leads us to extend the concept of differential forms to superspace. Coordinates of curved superspace are denoted

$$z^M = (X^{\hat{m}},\, \Theta,\, \bar{\Theta}) = z^{(\widetilde{M}_4)} \oplus z^{(\widetilde{\underline{M}}_2)} = (\widetilde{x}^m,\, \theta,\, \bar{\theta}) \oplus (\widetilde{\eta}^{\underline{m}},\, \underline{\theta},\, \bar{\underline{\theta}}), \tag{40}$$

and differential elements

$$dz^M = (dX^{\hat{m}},\, d\Theta,\, d\bar{\Theta}) = dz^{(\widetilde{M}_4)} \oplus dz^{(\widetilde{\underline{M}}_2)} = (d\widetilde{x}^m,\, d\theta,\, d\bar{\theta}) \oplus (d\widetilde{\eta}^{\underline{m}},\, d\underline{\theta},\, d\bar{\underline{\theta}}), \tag{41}$$

where $M \equiv (\hat{m}, \hat{\mu}, \hat{\dot{\mu}})$, $\hat{m} \equiv (m, \underline{m})$, $\Theta^{\hat{\mu}} \equiv (\theta^{\mu}, \underline{\theta}^{\underline{\mu}})$ and $\bar{\Theta}_{\hat{\dot{\mu}}} \equiv (\bar{\theta}_{\dot{\mu}}, \bar{\underline{\theta}}_{\underline{\dot{\mu}}})$. The $\hat{\mu}$ are all upper indices, while $\hat{\dot{\mu}}$ is a lower index. Elements of superspace obey the following multiplication law:

$$z^M z^N = (-1)^{nm} z^N z^M. \tag{42}$$

Here n is a function of N and m is a function of M. These functions take the values zero or one, depending on whether N and M are vector or spinor indices. Exterior products in superspace are defined in complete analogy to ordinary space:

$$\begin{aligned} dz^M \wedge z^N &= -(-1)^{nm} dz^N \wedge dz^M, \\ dz^M z^N &= (-1)^{nm} dz^N \wedge z^M. \end{aligned} \tag{43}$$

With this definition, differential forms have a standard extension to superspace. The differentials are written to the left of the coefficient function and the indices are labeled in such a way that there is always an even number of indices between those being summed. We shall, as usual, drop the symbol $\wedge$ for exterior multiplication. Functions of the superspace variable z are called zero-forms. Having defined superspace forms, we must also introduce exterior derivatives. Exterior derivatives map zero-forms into one-forms. Equations written in terms of differential forms and exterior derivatives are covariant under coordinate changes. Objects which transform linearly under a representation of the structure group are called tensors. Note that exterior derivatives do not map tensors into tensors. Connections are Lie algebra valued one-forms. The curvature tensor is a Lie algebra valued two-form. The curvature form and the covariant derivative of a tensor are, in general, the only tensorial quantities which may be constructed by taking derivatives. Higher derivatives lead to identities (and not to new tensors) because of the fact that $dd = 0$. These identities are called Bianchi identities. Bianchi identities of the first type are found from the covariant derivative. Bianchi identities of the second type are found from the curvature form. Here we shall forbear to write the details out as the standard theory is so well known. Together with other details of the theory, they can be seen in the textbooks, e.g. (Wess & Bagger, 1983, West, 1987).

Similar to (26), the multiplication of two local successive supersymmetric transformations induces the motion

$$\begin{aligned} &g(0,\, \epsilon(X),\, \bar{\epsilon}(X))\,\Omega(X^{\hat{m}},\, \Theta,\, \bar{\Theta}) \longrightarrow \\ &(X^{\hat{m}} + i\,\Theta\,\sigma^{\hat{m}}\,\bar{\epsilon}(X) - i\,\epsilon(X)\,\sigma^{\hat{m}}\,\bar{\Theta},\, \Theta + \epsilon(X),\, \bar{\Theta} + \bar{\epsilon}(X)). \end{aligned} \tag{44}$$

In its simplest version, supergravity was conceived as a quantum field theory whose action included the gravitation field term, where the graviton coexists with a fermionic field called gravitino, described by the

doi: https://doi.org/10.52526/25792776-23.70.2-212

Rarita-Scwinger kinetic term. The two fields differ in their spin: 2 for the graviton, 3/2 for the gravitino. The different 4D $N = 1$ supergravity multiplets all contain the graviton and the gravitino, but differ by their systems of auxiliary fields. For a detailed discussion we refer to the papers by (Binetruy et al., 2001, Fayet & Ferrara, 1977, Jacob, 1987, Nilles, 1984, Wess & Bagger, 1983, West, 1987, van Nieuwenhuizen, 1981, van Nieuwenhuizen et al., 1976). These fields would transform into each other under local supersymmetry. We may use the usual language which is almost identical to the vierbien formulation of GR with some additional input. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The motion (44) generates certain coordinate transformations:

$$z^M \longrightarrow z'^M = z^M - \zeta^M(z), \tag{45}$$

where $\zeta^M(z)$ arc arbitrary functions of z. The dynamical variables of superspace formulation are the frame field $E^A(z)$ and connection Ω. Using the analogue of Cartan's local frame, the superspace $(z^M, \Theta, \bar{\Theta})$ has at each point a tangent superspace spanned by the frame field defined as a 1-form over superspace

$$E^A(z) = dz^M E_M^A(z), \tag{46}$$

with coefficient superfields, generalizing the usual frame, namely supervierbien $E_M^A(z)$. Here, we use the first half of capital Latin alphabet $A, B, \ldots$ to denote the anholonomic indices related to the tangent superspace structure group, which is taken to be just the Lorentz group. The inverse vielbein $E_A{}^M(z)$ is defined by the relations

$$E_M^A(z)E_A{}^N(z) = \delta_M^N, \quad E_A{}^M(z)E_M^B(z) = \delta_A{}^B, \tag{47}$$

where

$$\delta_M^N = \begin{pmatrix} \delta_{\hat{m}}^{\hat{n}} & 0 & 0 \\ 0 & \delta_{\hat{\mu}}{}^{\hat{\nu}} & 0 \\ 0 & 0 & \delta_{\hat{\dot{\mu}}}{}^{\hat{\dot{\nu}}} \end{pmatrix}, \tag{48}$$

The formulation of supergravity in superspace provides a unified description of the vierbein and the Rarita-Schwinger fields, which are identified in a common geometric object, the local frame $E^A(z)$ of superspace. They are manifestly coordinate independent. The upper index A is reserved for the structure group, for which we take the Lorentz group. This is because we would like to recover supersymmetric flat space as a solution to our dynamical theory. With this choice, the reference frame defined by the vielbein is locally Lorentz covariant.

$$\delta E^A = E^B L_B^A(z), \quad \delta E_M^A = E_M^B L_B^A(z). \tag{49}$$

The indices transforming under the structure group will be called Lorentz indices. The Lorentz generators $L_B^A(z)$ have three irreducible components: $L_{\hat{b}}{}^{\hat{a}}, L_\beta{}^\alpha$ and $L^{\dot{\beta}}{}_{\dot{\alpha}}$. The vielbein forms $E^{\hat{a}} = dz^M E_M^{\hat{a}}$, $E^{\hat{\alpha}} = dz^M E_M^{\hat{\alpha}}$, and $E_{\hat{\dot{\alpha}}} = dz^M E_{M\hat{\dot{\alpha}}}$ are coordinate-independent irreducible Lorentz tensors.

To formulate covariant derivatives one must introduce a connection form

$$\phi = dz^M \phi_M, \quad \phi_M = \phi_{MA}^{B}, \tag{50}$$

transforming as follows under the structure group:

$$\delta\phi = \phi L - L\phi - dL. \tag{51}$$

Connections are Lie algebra valued one-forms

$$\phi = dz^M \phi_M^r(z) i T^r, \tag{52}$$

with the following transformation law:

$$\phi' = X^{-1}\phi X - X^{-1}dX, \tag{53}$$

where r runs over the dimension of the algebra. The connection is the second dynamical variable in this theory. The ϕ_{MA}^{B} is Lie algebra valued in its two Lorentz indices:

$$\phi_{MAB} = -(-1)^{ab}\phi_{MBA}. \tag{54}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

The covariant derivative of the vielbein is called torsion:

$$T^A = dE^A + E^B \phi_B{}^A. \tag{55}$$

In flat space it is possible to transform the vielbein into the global reference frame: $E^A = e^A$. It is defined up to rigid Lorentz transformations. In this frame the connection vanishes: $\phi = 0$. The torsion, however, is non-zero because of the following non-zero components:

$$T_{\hat{\alpha}\hat{\dot{\beta}}}{}^{\hat{c}} = T_{\hat{\dot{\beta}}\hat{\alpha}}{}^{\hat{c}} = 2i\sigma_{\hat{\alpha}\hat{\dot{\beta}}}{}^{\hat{c}}. \tag{56}$$

The curvature tensor is defined in terms of the connection:

$$R = d\phi + \phi\phi. \tag{57}$$

It is a Lie algebra valued two-form:

$$R_A{}^B = \tfrac{1}{2} dz^M dz^N R_{NMA}{}^B. \tag{58}$$

Covariant derivatives with respect to local Lorentz transformations are constructed by means of the spin connection Ω, which is a 1-form in superspace. Supergauge transformations are constructed from the general coordinate and structure group transformations of superspace:

$$L_B{}^A = -\zeta^C \phi_{CB}{}^A. \tag{59}$$

They amount to a convenient reparametrization of these transformations. Supergauge transformations map Lorentz tensors into Lorentz tensors and reduce to supersymmetry transformations in the limit of flat space. The parameter ζ characterizes infinitesimal changes in coordinates. Whereas, either ζ^A or ζ^M may be chosen as the field-independent transformation parameter. Its companion then depends on the fields through the vielbein. Since we would like Lorentz tensors to transform into Lorentz tensors, we shall choose ζ^A to be field-independent. Supergauge transformations consist of a general coordinate transformation with field-independent parameter ζ^A followed by a structure group Lorentz transformation with field-dependent parameter (68). It is among this restricted class of transformations that we shall find the gauged supersymmetry transformations.

The super-vielbein $E_M{}^A$ and spin-connection Ω contain many degrees of freedom. Although some of these are removed by the tangent space and supergeneral coordinate transformations, there still remain many degrees of freedom. There is no general prescription for deducing necessary covariant constraints which if imposed upon the superfields of super-vielbein and spin-connection will eliminate the component fields. However, some usual constraints can be found using tangent space and supergeneral coordinate transformations of the torsion and curvature covariant tensors, given in appropriate super-gauge. The transformation parameters ζ^A and $L_{\hat{a}\hat{b}}$ are functions of superspace. Their lowest components characterize general coordinate transformations in six-dimensional X-space $[\zeta^{\hat{a}}(X^{\hat{m}})]$, gauged supersymmetry transformations $[\zeta^{\hat{\alpha}}(X)], \zeta_{\hat{\dot{\alpha}}}(X)]$, and local Lorentz transformations $L_{\hat{a}\hat{b}}(X)$. We will use their higher components to transform away certain $\Theta = \bar{\Theta} = 0$ components of the vielbein and the connection. Let us consider the vielbein. Its transformation law may be written as a super-gauge transformation together with an additional Lorentz transformation $L_B{}^A$:

$$\delta_\zeta E_M{}^A = -\mathcal{D}_M \zeta^A - \zeta^B T_{BM}{}^A + E_M{}^B L_B{}^A. \tag{60}$$

The lowest component of this equation gives the transformation property of $E_M{}^A|_{\Theta=\bar{\Theta}=0}$. Higher components of ζ^A enter $\delta E_M{}^A|$ through the covariant derivatives $\mathcal{D}_{\hat{\mu}}\zeta^A$ and $\bar{\mathcal{D}}^{\hat{\dot{\mu}}}\zeta^A$. One may use these higher components to transform super-vielbein to the final form, see e.g. (Wess & Bagger, 1983), where the minimum number of independent component fields are the graviton, $e_{\hat{m}}{}^{\hat{a}}(X)$, and the gravitino, $\psi_{\hat{m}}{}^{\hat{\alpha}}(X), \bar{\psi}_{\hat{m}\hat{\dot{\alpha}}}(X)$. Since

$$E_A{}^M(z)\big|_{\Theta=\bar{\Theta}=0} = E_{\underline{a}}{}^{\underline{m}}(z^{(\widetilde{\underline{M}}_2)})\Big|_{\underline{\theta}=\underline{\bar{\theta}}=0} \oplus E_a{}^m(z^{(\widetilde{M}_4)})\Big|_{\theta=\bar{\theta}=0}, \tag{61}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

accordingly, we find

$$
\begin{aligned}
E_{\underline{a}}{}^{\underline{m}}(z^{(\underline{\widetilde{M}}_2)})\Big|_{\underline{\theta}=\underline{\bar{\theta}}=0} &= \begin{pmatrix} e_{\underline{m}}{}^{\underline{a}}(\widetilde{\eta}) & \frac{1}{2}\psi_{\underline{m}}{}^{\underline{\alpha}}(\widetilde{\eta}) & \frac{1}{2}\bar{\psi}_{\underline{m}\,\underline{\dot{\alpha}}}(\widetilde{\eta}) \\ 0 & \delta_{\underline{\mu}}{}^{\underline{\alpha}} & 0 \\ 0 & 0 & \delta^{\underline{\dot{\mu}}}{}_{\underline{\dot{\alpha}}} \end{pmatrix}, \\
E_a{}^m(z^{(\widetilde{M}_4)})\Big|_{\theta=\bar{\theta}=0} &= \begin{pmatrix} e_m{}^a(x) & \frac{1}{2}\psi_m{}^\alpha(x) & \frac{1}{2}\bar{\psi}_{m\dot{\alpha}}(x) \\ 0 & \delta_\mu{}^\alpha & 0 \\ 0 & 0 & \delta^{\dot{\mu}}{}_{\dot{\alpha}} \end{pmatrix}.
\end{aligned} \tag{62}
$$

The fields of graviton and gravitino cannot be gauged away. Provided, we have

$$
e_{\hat{a}}{}^{\hat{m}} e_{\hat{m}}{}^{\hat{b}} = \delta_{\hat{a}}^{\hat{b}}, \quad \psi_{\hat{a}}{}^{\hat{\mu}} = e_{\hat{a}}{}^{\hat{m}} \psi_{\hat{m}}{}^{\hat{\alpha}} \delta_{\hat{\alpha}}^{\hat{\mu}}, \quad \bar{\psi}_{\hat{a}\hat{\dot{\mu}}} = e_{\hat{a}}{}^{\hat{m}} \bar{\psi}_{\hat{m}\hat{\dot{\alpha}}} \delta^{\hat{\dot{\mu}}}{}_{\hat{\dot{\alpha}}}. \tag{63}
$$

The tetrad field

$$
e_{\hat{m}}^{\hat{a}}(X) = (e_{\underline{m}}{}^{\underline{a}}(\widetilde{\eta}), e_m{}^a(x))
$$

plays the role of a gauge field associated with local transformations. The Majorana type field

$$
\frac{1}{2}\psi_{\hat{m}}{}^{\hat{\alpha}} = \frac{1}{2}(\psi_{\underline{m}}{}^{\underline{\alpha}}(\widetilde{\eta}), \psi_m{}^\alpha(x))
$$

is the gauge field related to local supersymmetry. These two fields belong to the same supergravity multiplet which also accommodates auxiliary fields so that the local supersymmetry algebra closes. Under infinitesimal transformations of local supersymmetry, they transformed as

$$
\begin{aligned}
&\delta e_{\hat{m}}^{\hat{a}} = i(\psi_{\hat{m}}\sigma^{\hat{a}}\zeta - \zeta\sigma^{\hat{a}}\bar{\psi}_{\hat{m}}), \\
&\delta\psi_{\hat{m}} = -2\mathcal{D}_{\hat{m}}\zeta^{\hat{\alpha}} + ie_{\hat{m}}^{\hat{c}}\{\tfrac{1}{3}M(\varepsilon\sigma_{\hat{c}}\bar{\zeta})^{\hat{\alpha}} + b_{\hat{c}}\zeta^{\hat{\alpha}} + \tfrac{1}{3}b^{\hat{d}}(\zeta\sigma_{\hat{d}}\bar{\sigma}_{\hat{c}})\},
\end{aligned} \tag{64}
$$

etc., where $M(X) = -6R(z)|_{\Theta=\bar{\Theta}=0}$ and $b_{\hat{a}}(X) = -3G(z)|_{\Theta=\bar{\Theta}=0}$ are the auxiliary fields, and

$$
\begin{aligned}
&\zeta^{\hat{\alpha}}(z) = \zeta^{\hat{\alpha}}(X), \quad \bar{\zeta}^{\hat{\alpha}}(z) = \bar{\zeta}^{\hat{\alpha}}(X), \\
&\zeta^{\bar{a}}(z) = 2i[\Theta\sigma^{\hat{a}}\bar{\zeta}(X) - \zeta(X)\sigma^{\hat{a}}\bar{\Theta}].
\end{aligned} \tag{65}
$$

These auxiliary fields are not restricted by any differential equations in X-space.

4. Non-trivial linear representation of the $\widetilde{MS}_p$-SUSY algebra

With these guidelines to follow, we start by considering the simplest example of a supersymmetric theory in six dimensional background curved spaces $\widetilde{M}_4 \oplus \underline{\widetilde{M}}_2$ as the $\widetilde{MS}_p$-generalization of flat space MS_p-SUSY model. The chiral superfields are defined as $\bar{\mathcal{D}}_{\hat{\dot{\alpha}}}\Phi = 0$, which reduces to $\bar{D}_{\hat{\dot{\alpha}}}\Phi = 0$ in flat space. To obtain a feeling for this model we may consider first example of non-trivial linear representation $(\widehat{\chi}, \mathcal{A}, \mathcal{F})$, of the $\widetilde{MS}_p$-SUSY algebra. This has $N = 1$ and $s_0 = 0$, and contains two Weyl spinor states of a massive Majorana spinor $\widehat{\chi}(\chi,\, \underline{\chi})$, two complex scalar fields $\mathcal{A}(A,\, \underline{A})$, and two more real scalar degrees of freedom in the complex auxiliary fields $\mathcal{F}(F,\, \underline{F})$, which provide in supersymmetry theory the fermionic and bosonic degrees of freedom to be equal off-shell as well as on-shell, and are eliminated when one goes on-shell. The component multiplets, $(\widehat{\chi}, \mathcal{A}, \mathcal{F})$, are called the chiral or scalar multiplets. We could define the component fields as the coefficient functions of a power series expansion in Θ and $\bar{\Theta}$. This decomposition, however, is coordinate-dependent. It is, therefore, more convenient to define them as

$$
\mathcal{A} = \Phi|_{\Theta=\bar{\Theta}=0}\,, \quad \widehat{\chi}_{\hat{\alpha}} = \tfrac{1}{\sqrt{2}}\mathcal{D}_{\hat{\alpha}}\,\Phi|_{\Theta=\bar{\Theta}=0}\,, \quad \mathcal{F} = -\tfrac{1}{4}\mathcal{D}^{\hat{\alpha}}\mathcal{D}_{\hat{\alpha}}\,\Phi|_{\Theta=\bar{\Theta}=0}\,, \tag{66}
$$

which carry Lorentz indices. They are related to the Θ and $\bar{\Theta}$ expansion coefficients through a transformation which depends on the supergravity multiplet. The transformation laws of the component fields are found from the transformation law of the superfield Φ:

$$
\delta\Phi = -\zeta^A\mathcal{D}_A\Phi, \tag{67}
$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

provided, the parameters ζ^A are specified as (65). Under infinitesimal transformations of local supersymmetry, the transformation law of the chiral multiplet (see e.g. Wess & Bagger (1983),), incorporating with embedding map $\underline{\widetilde{M}}_2 \hookrightarrow \widetilde{M}_4$, which is the map (2) rewritten for curved spaces $\underline{\widetilde{M}}_2$ and $\widetilde{M}_4$, and the transformation law $\underline{A}(\widetilde{\eta}) = A(\widetilde{x}$ (5) for spin-zero scalar field, give

$$\begin{aligned}
&\delta\underline{A} = -\sqrt{2}\zeta^\alpha\chi_\alpha, \\
&\delta\chi_\alpha = -\sqrt{2}\zeta_\alpha F - i\sqrt{2}\sigma_{\alpha\dot{\beta}}{}^{a}\bar{\zeta}^{\dot{\beta}}\widehat{D}_a\underline{A}, \\
&\delta F = -\frac{\sqrt{2}}{3}M^*\zeta^\alpha\chi_\alpha + \bar{\zeta}^{\dot{\alpha}}\left(\frac{1}{6}\sqrt{2}b_{\alpha\dot{\alpha}}\chi^\alpha - i\sqrt{2}\widehat{D}_{\alpha\dot{\alpha}}\chi^\alpha\right),
\end{aligned} \tag{68}$$

where

$$\begin{aligned}
&\widehat{D}_a\underline{A} \equiv e_a{}^m\left[\left(\frac{\partial\widetilde{x}^0}{\partial\underline{\widetilde{x}}^m}\right)\widetilde{\partial}_{\underline{0}}\,\underline{A} + \left(\frac{\partial|\vec{\widetilde{x}}|}{\partial\underline{\widetilde{x}}^m}\right)\widetilde{\partial}_{\underline{1}}\,\underline{A} - \frac{1}{\sqrt{2}}\psi_m{}^\mu\chi_\mu\right], \\
&\widehat{D}_a\chi_\alpha = e_a{}^m\left(\mathcal{D}_m\chi_\alpha - \frac{1}{\sqrt{2}}\psi_{m\alpha}F - \frac{i}{\sqrt{2}}\bar{\psi}_m{}^{\dot{\beta}}\widehat{D}_{\alpha\dot{\beta}}\underline{A}\right).
\end{aligned} \tag{69}$$

In the same way, we should define the spinor $\underline{\chi}$ as the field into which $A(x)$ transforms. In this case, the infinitesimal supersymmetry transformations for $Q = \underline{q}$ read

$$\begin{aligned}
&\delta A = -\sqrt{2}\underline{\zeta}^{\underline{\alpha}}\underline{\chi}_{\underline{\alpha}}, \\
&\delta\underline{\chi}_{\underline{\alpha}} = -\sqrt{2}\underline{\zeta}_{\underline{\alpha}}\underline{F} - i\sqrt{2}\sigma_{\underline{\alpha}\underline{\dot{\beta}}}{}^{\underline{a}}\bar{\underline{\zeta}}^{\underline{\dot{\beta}}}\widehat{D}_{\underline{a}}A, \\
&\delta\underline{F} = -\frac{\sqrt{2}}{3}\underline{M}^*\underline{\zeta}^{\underline{\alpha}}\underline{\chi}_{\underline{\alpha}} + \bar{\underline{\zeta}}^{\underline{\dot{\alpha}}}\left(\frac{1}{6}\sqrt{2}\underline{b}_{\underline{\alpha}\underline{\dot{\alpha}}}\underline{\chi}^{\underline{\alpha}} - i\sqrt{2}\widehat{D}_{\underline{\alpha}\underline{\dot{\alpha}}}\underline{\chi}^{\underline{\alpha}}\right),
\end{aligned} \tag{70}$$

where

$$\begin{aligned}
&\widehat{D}_{\underline{a}}A \equiv \underline{e}_{\underline{a}}{}^{\underline{m}}\left[\left(\frac{\partial\widetilde{x}^0}{\partial\underline{\widetilde{x}}^{\underline{m}}}\right)\widetilde{\partial}_0\,A + \left(\frac{\partial\widetilde{x}^i}{\partial\underline{\widetilde{x}}^{\underline{m}}}\right)\widetilde{\partial}_i\,A - \frac{1}{\sqrt{2}}\underline{\psi}_{\underline{m}}{}^{\underline{\mu}}\underline{\chi}_{\underline{\mu}}\right], \\
&\widehat{D}_{\underline{a}}\chi_{\underline{\alpha}} = \underline{e}_{\underline{a}}{}^{\underline{m}}\left(\mathcal{D}_{\underline{m}}\underline{\chi}_{\underline{\alpha}} - \frac{1}{\sqrt{2}}\underline{\psi}_{\underline{m}\,\underline{\alpha}}\underline{F} - \frac{i}{\sqrt{2}}\bar{\underline{\psi}}_{\underline{m}}{}^{\underline{\dot{\beta}}}\widehat{D}_{\underline{\alpha}\underline{\dot{\beta}}}A\right).
\end{aligned} \tag{71}$$

The graviton and the gravitino form thus the basic multiplet of local $\widetilde{MS}_p$-SUSY, and one expects the simplest locally supersymmetric model to contain just this multiplet. The spin 3/2 contact term in total Lagrangian arises from equations of motion for the torsion tensor, and that the original Lagrangian itself takes the simpler interpretation of a minimally coupled spin (2, 3/2) theory.

5. Inertial effects

We would like to place the emphasis on the essential difference arisen between the standard supergravity theories and some rather unusual properties of local $\widetilde{MS}_p$-SUSY theory. In the framework of the standard supergravity theories, as in GR, a curvature of the space-time acts on all the matter fields. The source of graviton is the energy-momentum tensor of matter fields, while the source of gravitino is the spin-vector current of supergravity. In the local $\widetilde{MS}_p$-SUSY theory, unlike the supergravity, instead we argue that a deformation/(distortion of local internal properties) of MS_p, is the origin of the *absolute acceleration* ($\vec{a}_{abs} \neq 0$) and inertia effects (*fictitious graviton*). This refers to the particle of interest itself, without relation to other matter fields, so that this can be globally removed by appropriate coordinate transformations. A next member of the basic multiplet of local $\widetilde{MS}_p$-SUSY -*fictitious gravitino*, will be arisen under infinitesimal transformations of local supersymmetry, because the $\widetilde{MS}_p$-SUSY is so constructed as to make these two particles just as being the two bosonic and fermionic states in the curved background spaces $\widetilde{M}_4$ and $\underline{\widetilde{M}}_2$, respectively, or vice versa. Whereas, in order to become on the same footing with the distorted space $\underline{\widetilde{M}}_2$, the space $\widetilde{M}_4$ refers only to the accelerated proper reference frame of a particle. With these physical requirements, a standard Lagrangian consisted of the gravitation field Lagrangian plus a part which contains the Rarita-Schwinger field, and coupling of supergravity with matter superfields evidently no longer holds. Instead we are now looking for an alternative way of implications of local $\widetilde{MS}_p$-SUSY for the model of accelerated motion and inertial effects. For example, we may with equal justice start from the reverse, which as we mentioned before is also expected. If one starts with a constant parameter ϵ (20) and performs a local Lorentz transformation, which can only be implemented if MS and space-time are curved (deformed/distorted) ($\widetilde{M}_2$, $\widetilde{M}_4$), then this parameter will in general become space-time dependent as a result of this Lorentz transformation, which readily implies *local* $\widetilde{MS}_p$-SUSY. In going into practical details of the realistic local $\widetilde{MS}_p$-SUSY model, it remains to derive the explicit form of the

doi: https://doi.org/10.52526/25792776-23.70.2-212

vierbien $e_{\hat{m}}^{\,\hat{a}}(\varrho) \equiv (e_m^{\;a}(\varrho),\, e_{\underline{m}}^{\;\underline{a}}(\varrho))$, which describes *fictitious graviton* as a function of *local rate* $\varrho(\eta, m, f)$ of instantaneously change of the velocity $v^{(\pm)}$ of massive (m) test particle under the unbalanced net force (f). We, of course, does not profess to give any clear-cut answers to all the problems addressed, the complete picture of which, of course, is largely beyond the scope of present paper. In particular, the latter does not allows us to offer here a straightforward recipe for deducing the vierbien $e_{\hat{m}}^{\,\hat{a}}(\varrho)$ in the framework of quantum field theory of $\widetilde{MS}_p$-supergravity, where the accelerated frame has to be described as the frame that has torsion. This will be a separate topic for research in a subsequent paper. Therefore, cutting short where our analysis is leading to, instead we now turn to recently derived by Ter-Kazarian (2012) the vierbien $e_{\hat{m}}^{\,\hat{a}}(\varrho)$ in the framework of classical physics. Together with other usual aspects of the theory, the latter illustrates a possible solution to the problems of inertia behind spacetime deformations. It was argued that a deformation/(distortion of local internal properties) of $\underline{M}_2$ is the origin of inertia effects that can be observed by us. Consequently, the next member of the basic multiplet of local $\widetilde{MS}_p$-SUSY -*fictitious gravitino*, $\psi_{\hat{m}}^{\;\alpha}(\varrho)$, will be arisen under infinitesimal transformations of local supersymmetry (64), provided by the local parameters $\zeta^M(a)$ (45).

For brevity reason, we shall forbear here review of certain essential theoretical aspects of a general *distortion of local internal properties of MS_p*, formulated in the framework of classical physics by Ter-Kazarian (2012). But for the self-contained arguments, the interested reader is invited to consult the original paper.

6. Model building in the 4D background Minkowski space-time

In this section, we briefly discuss the RTI in particular case when the relativistic test particle accelerated in the background flat M_4 space under an unbalanced net force other than gravitational, but we refer to the original paper for more details. To make the remainder of our discussion a bit more concrete, it proves necessary to provide, further, a constitutive ansatz of simple, yet tentative, linear *distortion transformations* of the basis $\underline{e}_{\underline{m}}$ at the point of interest in flat space $\underline{M}_2$, which can be written in terms of *local rate* $\varrho(\eta, m, f)$ of instantaneously change of the measure $\underline{v}^{\underline{m}}$ of massive test particle under the unbalanced net force (f) (Ter-Kazarian, 2012):

$$
\begin{array}{l}
e_{(\tilde{+})}(\varrho) = D^{\;\underline{m}}_{(\tilde{+})}(\varrho)\, \underline{e}_{\underline{m}} = \underline{e}_{(+)} - \varrho(\eta, m, f)\, \underline{v}^{(-)}\, \underline{e}_{(-)}, \\
e_{(\tilde{-})}(\varrho) = D^{\;\underline{m}}_{(\tilde{-})}(\varrho)\, \underline{e}_{\underline{m}} = \underline{e}_{(-)} + \varrho(\eta, m, f)\, \underline{v}^{(+)}\, \underline{e}_{(+)},
\end{array} \tag{72}
$$

Clearly, these transformations imply a violation of relation $e^2_{\underline{\mu}}(\varrho) \neq 0$ for the null vectors $\underline{e}_{\underline{m}}$. The norm in the resulting general deformed/distorted space $\widetilde{\mathcal{M}}_2$ now can be rewritten in terms of space-time variables as

$$
id = e\,\vartheta \equiv d\widetilde{\hat{q}} = \widetilde{e}_0 \otimes d\widetilde{t} + \widetilde{e}_q \otimes d\widetilde{q}, \tag{73}
$$

where $\widetilde{e}_0$ and $\widetilde{e}_q$ are, respectively, the temporal and spatial basis vectors in $\widetilde{\mathcal{M}}_2$:

$$
\widetilde{e}_0(\varrho) = \tfrac{1}{\sqrt{2}} \left[e_{(\tilde{+})}(\varrho) + e_{(\tilde{-})}(\varrho) \right], \quad \widetilde{e}_q(\varrho) = \tfrac{1}{\sqrt{2}} \left[e_{(\tilde{+})}(\varrho) - e_{(\tilde{-})}(\varrho) \right]. \tag{74}
$$

Hence, in the framework of the space-time deformation/distortion theory (Ter-Kazarian, 2012), we can compute the general metric $\widetilde{g}$ as

$$
\widetilde{g} = g_{\tilde{r}\tilde{s}}\, d\widetilde{q}^{\tilde{r}} \otimes d\widetilde{q}^{\tilde{s}}, \tag{75}
$$

provided

$$
g_{\tilde{0}\tilde{0}} = (1 + \tfrac{\varrho v_q}{\sqrt{2}})^2 - \tfrac{\varrho^2}{2}, \quad g_{\tilde{1}\tilde{1}} = -(1 - \tfrac{\varrho v_q}{\sqrt{2}})^2 + \tfrac{\varrho^2}{2}, \quad g_{\tilde{1}\tilde{0}} = g_{\tilde{0}\tilde{1}} = -\sqrt{2}\varrho. \tag{76}
$$

We suppose that a second observer, who makes measurements using a frame of reference $\widetilde{S}_{(2)}$ which is held stationary in curved (deformed/distorted) master space $\widetilde{\mathcal{M}}_2$, uses for the test particle the corresponding space-time coordinates $\widetilde{q}^{\tilde{r}} \left((\widetilde{q}^{\tilde{0}}, \widetilde{q}^{\tilde{1}}) \equiv (\widetilde{t}, \widetilde{q}) \right)$. The very concept of the local *absolute acceleration* (in Newton's terminology) is introduced by Ter-Kazarian (2012), brought about via the Fermi-Walker transported frames as

$$
\vec{a}_{abs} \equiv \vec{e}_q \frac{d(\varrho)}{\sqrt{2} ds_q} = \vec{e}_q \left| \frac{de_{\hat{0}}}{ds} \right| = \vec{e}_q\, |\mathbf{a}|. \tag{77}
$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

Here we choose the system $S_{(2)}$ in such a way as the axis $\vec{e}_q$ lies along the net 3-acceleration $(\vec{e}_q\,||\,\vec{e}_a)$, $(\vec{e}_a = \vec{a}_{net}/|\vec{a}_{net}|)$, $\vec{a}_{net}$ is the local net 3-acceleration of an arbitrary observer with proper linear 3-acceleration, $\vec{a}$, and proper 3-angular velocity, $\vec{\omega}$, measured in the rest frame: $\vec{a}_{net} = \frac{d\vec{u}}{ds} = \vec{a} \wedge \vec{u} + \vec{\omega} \times \vec{u}$, where $\mathbf{u}$ is the 4-velocity. A magnitude of $\vec{a}_{net}$ can be computed as the simple invariant of the absolute value $|\frac{d\mathbf{u}}{ds}|$ as measured in rest frame:

$$|\mathbf{a}| = |\tfrac{d\mathbf{u}}{ds}| = \left(\tfrac{du^l}{ds},\, \tfrac{du_l}{ds}\right)^{1/2}. \tag{78}$$

Following Misner et al. (1973), Synge (1960), we also define the orthonormal frame, $e_{\underline{a}}$, carried by an accelerated observer, who moves with proper linear 3-acceleration, $\vec{a}(s)$, and proper 3-rotation, $\vec{\omega}(s)$. Particular frame components are $e_{\underline{a}}$, where $\underline{a} = \hat{0}, \hat{1}$, etc. Let the zeroth leg of the frame $e_{\hat{0}}$ be 4-velocity $\mathbf{u}$ of the observer that is tangent to the worldline at a given event $x^l(s)$ and we parameterize the remaining spatial triad frame vectors $e_{\hat{i}}$, orthogonal to $e_{\hat{0}}$, also by (s). The spatial triad $e_{\hat{i}}$ rotates with proper 3-rotation $\vec{\omega}(s)$. The 4-velocity vector naturally undergoes Fermi-Walker transport along the curve C, which guarantees that $e_{\hat{0}}(s)$ will always be tangent to C determined by $x^l = x^l(s)$:

$$\frac{de_{\underline{a}}}{ds} = -\Phi\, e_{\underline{a}} \tag{79}$$

where the antisymmetric rotation tensor Φ splits into a Fermi-Walker transport part Φ_{FW} and a spatial rotation part Φ_{SR}:

$$\Phi_{FW}^{lk} = a^l u^k - a^k u^l, \quad \Phi_{SR}^{lk} = u_m \omega_n \varepsilon^{mnlk}. \tag{80}$$

The 4-vector of rotation ω^l is orthogonal to 4-velocity u^l, therefore, in the rest frame it becomes $\omega^l(0, \vec{\omega})$, and ε^{mnlk} is the Levi-Civita tensor with $\varepsilon^{0123} = -1$. So, the resulting metric (75) is reduced to

$$d\widetilde{s}_q^2 = \Omega^2(\overline{\varrho})\, ds_q^2, \quad \Omega(\overline{\varrho}) = 1 + \overline{\varrho}^2,\ \overline{\varrho}^2 = v^2\varrho^2, \quad \underline{v}^2 = \underline{v}^{(+)}\underline{v}^{(-)}, \quad \varrho = \sqrt{2}\int_0^{s_q} |\mathbf{a}| ds'_q. \tag{81}$$

Combining (37) and (77), we obtain

$$\varrho = \frac{i}{\gamma_q^2}\left|\left(\underline{\theta}\,\sigma^3 \frac{d\bar{\xi}}{ds_q} - \frac{d\xi}{ds_q}\sigma^3\bar{\underline{\theta}}\right)\right|, \tag{82}$$

where $\gamma_q = (1 - v_q^2)^{-1/2}$. The resulting *inertial force* $\vec{f}_{(in)}$ has computed by Ter-Kazarian (2012) as

$$\vec{f}_{(in)} = -m\,\Gamma^1_{\tilde{r}\tilde{s}}(\varrho)\frac{d\tilde{q}^{\tilde{r}}}{d\tilde{s}_q}\frac{d\tilde{q}^{\tilde{s}}}{d\tilde{s}_q} = -\frac{m\vec{a}_{abs}}{\Omega^2(\overline{\varrho})\,\gamma_q}, \tag{83}$$

Whereupon, in case of absence of rotation, the relativistic inertial force reads

$$\vec{f}_{(in)} = -\frac{1}{\Omega^2(\overline{\varrho})\,\gamma_q\gamma}[\vec{F} + (\gamma - 1)\frac{\vec{v}(\vec{v}\cdot\vec{F})}{|\vec{v}|^2}]. \tag{84}$$

Note that the inertial force arises due to *nonlinear* process of deformation of MS_p, resulting after all to *linear* relation (84). So, this also ultimately requires that MS_p should be two dimensional, because in this case we may reconcile the alluded *nonlinear* and *linear* processes by choosing the system $S_{(2)}$ in only allowed way mentioned above. At low velocities $v_q \simeq |\vec{v}| \simeq 0$ and tiny accelerations we usually experience, one has $\Omega(\overline{\varrho}) \simeq 1$, therefore the (84) reduces to the conventional non-relativistic law of inertia

$$\vec{f}_{(in)} = -m\vec{a}_{abs} = -\vec{F}. \tag{85}$$

At high velocities $v_q \simeq |\vec{v}| \simeq 1$ $(\Omega(\overline{\varrho}) \simeq 1)$, if $(\vec{v}\cdot\vec{F}) \neq 0$, the inertial force (84) becomes

$$\vec{f}_{(in)} \simeq -\frac{1}{\gamma}\vec{e}_v(\vec{e}_v\cdot\vec{F}), \tag{86}$$

and it vanishes in the limit of the photon $(|\vec{v}| = 1, m = 0)$. Thus, it takes force to disturb an inertia state, i.e. to make the *absolute acceleration* $(\vec{a}_{abs} \neq 0)$. The *absolute acceleration* is due to the real deformation/distortion of the space $\underline{M}_2$. The *relative* $(d(\tau_2\varrho)/ds_q = 0)$ acceleration (in Newton's terminology) (both magnitude and direction), to the contrary, has nothing to do with the deformation/distortion of the space $\underline{M}_2$ and, thus, it cannot produce an inertia effects.

doi: https://doi.org/10.52526/25792776-23.70.2-212

7. Beyond the hypothesis of locality

In SR an assumption is required to relate the ideal inertial observers to actual observers that are all noninertial, i.e., accelerated. Therefore, it is a long-established practice in physics to use the hypothesis of locality (Hehl & Ni, 1990, Hehl et al., 1991, Li & Ni, 1979, Mashhoon, 2002, 2011, Misner et al., 1973, Ni, 1977, Ni & Zimmermann, 1978), for extension of the Lorentz invariance to accelerated observers in Minkowski space-time. The standard geometrical structures, referred to a noninertial coordinate frame of accelerating and rotating observer in Minkowski space-time, were computed on the base of the assumption that an accelerated observer is pointwise inertial, which in effect replaces an accelerated observer at each instant with a momentarily comoving inertial observer along its wordline. This assumption is known to be an approximation limited to motions with sufficiently low accelerations, which works out because all relevant length scales in feasible experiments are very small in relation to the huge acceleration lengths of the tiny accelerations we usually experience, therefore, the curvature of the wordline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small. However, it seems quite clear that such an approach is a work in progress, which reminds us of a puzzling underlying reality of inertia, and that it will have to be extended to describe physics for arbitrary accelerated observers. Ever since this question has become a major preoccupation of physicists, see e.g. (Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Marzlin, 1996, Mashhoon, 95, 1988, 1990a,b, 2002, 2011) and references therein. The hypothesis of locality represents strict restrictions, because in other words, it approximately replaces a noninertial frame of reference $\widetilde{S}_{(2)}$, which is held stationary in the deformed/distorted space $\widetilde{\mathcal{M}}_2 \equiv \underline{V}_2^{(\varrho)}\,(\varrho \neq 0)$, where $\underline{V}_2$ is the 2D semi-Riemann space, with a continuous infinity set of the inertial frames $\{S_{(2)}, S'_{(2)}, S''_{(2)}, ...\}$ given in the flat $\underline{M}_2\,(\varrho = 0)$. In this situation the use of the hypothesis of locality is physically unjustifiable. Therefore, it is worthwhile to go beyond the hypothesis of locality with special emphasis on distortion of $\underline{M}_2$ ($\underline{M}_2 \longrightarrow \underline{V}_2^{(\varrho)}$), which we might expect will essentially improve the standard results. Here, following (Mashhoon, 2002, Misner et al., 1973), we introduced a geodesic coordinate system - the coordinates relative to the accelerated observer (the laboratory coordinates), in the neighborhood of the accelerated path. We choose the zeroth leg of the frame, $\breve{e}_{\hat{0}}$, as before, to be the unit vector $\mathbf{u}$ that is tangent to the worldline at a given event $x^\mu(s)$, where (s) is a proper time measured along the accelerated path by the standard (static inertial) observers in the underlying global inertial frame. Following (Maluf & Faria, 2008, Maluf et al., 2007, Marzlin, 1996, Mashhoon, 95, 1988, 1990a,b, 2002, 2011), in analogy with the Faraday tensor, one can identify the antisymmetric acceleration tensor $\Phi_{ab} \longrightarrow (-\mathbf{a}, \omega)$, with $\mathbf{a}(s)$ as the translational acceleration $\Phi_{0i} = -a_i$, and $\omega(s)$ as the frequency of rotation of the local spatial frame with respect to a nonrotating (Fermi- Walker transported) frame $\Phi_{ij} = -\varepsilon_{ijk}\,\omega^k$. The invariants constructed out of Φ_{ab} establish the acceleration scales and lengths. The hypothesis of locality holds for huge proper acceleration lengths $|I|^{-1/2} \gg 1$ and $|I^*|^{-1/2} \gg 1$, where the scalar invariants are given by $I = (1/2)\,\Phi_{ab}\,\Phi^{ab} = -\vec{a}^2 + \vec{\omega}^2$ and $I^* = (1/4)\,\Phi^*_{ab}\,\Phi^{ab} = -\vec{a}\cdot\vec{\omega}$ ($\Phi^*_{ab} = \varepsilon_{abcd}\,\Phi^{cd}$) (Mashhoon, 95, 1988, 1990a,b, 2002, 2011). Suppose the displacement vector $z^\mu(s)$ represents the position of the accelerated observer. According to the hypothesis of locality, at any time (s) along the accelerated worldline the hypersurface orthogonal to the worldline is Euclidean space and we usually describe some event on this hypersurface ("local coordinate system") at x^μ to be at $\widetilde{x}^\mu$, where x^μ and $\widetilde{x}^\mu$ are connected via $\widetilde{x}^{\,0} = s$ and

$$x^\mu = z^\mu(s) + \widetilde{x}^{\,i}\,\bar{e}^{\,\mu}_{\,\hat{i}}(s). \tag{87}$$

Consequently, the standard metric of semi-Riemannian 4D background space $V_4^{(0)}$ in noninertial system of the accelerating and rotating observer, computed on the base of hypothesis of locality (see also Mashhoon (95, 1988, 1990a,b, 2002, 2011)), is:

$$\begin{aligned}\widetilde{g} = \eta_{\mu\nu}\,dx^\mu \otimes dx^\nu = \Big[(1 + \vec{a}\cdot\vec{\widetilde{x}})^2 + (\vec{\omega}\cdot\vec{\widetilde{x}})^2 - (\vec{\omega}\cdot\vec{\omega})(\vec{\widetilde{x}}\cdot\vec{\widetilde{x}})\Big]\,d\widetilde{x}^0 \otimes d\widetilde{x}^0 - \\ 2\,(\vec{\omega}\wedge\vec{\widetilde{x}})\cdot d\vec{\widetilde{x}} \otimes d\widetilde{x}^0 - d\vec{\widetilde{x}} \otimes d\vec{\widetilde{x}},\end{aligned} \tag{88}$$

We see that the hypothesis of locality leads to the 2D semi-Riemannian space, $V_{\underline{2}}^{(0)}$, with the incomplete metric $\widetilde{g}\quad(\varrho = 0)$:

$$\widetilde{g} = \left[(1 + \widetilde{q}^{\,1}\widetilde{\varphi}_0)^2 - (\widetilde{q}^{\,1}\widetilde{\varphi}_1)^2\right]\,d\widetilde{q}^{\,0} \otimes d\widetilde{q}^{\,0} - 2\,(\widetilde{q}^{\,1}\widetilde{\varphi}_1)\,d\widetilde{q}^{\,1} \otimes d\widetilde{q}^{\,0} - d\widetilde{q}^{\,1} \otimes d\widetilde{q}^{\,1}, \tag{89}$$

provided,

$$\widetilde{q}^{\,1}\widetilde{\varphi}_0 = \widetilde{x}^{\,i}\,\Phi^0_i, \quad \widetilde{q}^{\,1}\widetilde{\varphi}_1 = \widetilde{x}^{\,i}\,\Phi^j_i\,\widetilde{e}^{\,-1}_j. \tag{90}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

Therefore, our strategy now is to deform the metric (89) by carrying out an additional deformation of semi-Riemannian 4D background space $V_4^{(0)} \longrightarrow \widetilde{M}_4 \equiv V_4^{(\varrho)}$, in order it becomes on the same footing with the complete metric $\widetilde{g} \quad (\varrho \neq 0)$ (75) of the distorted space $\widetilde{\underline{\mathcal{M}}}_2 \equiv \underline{V}_2^{(\varrho)}$. Let the Latin letters $\hat{r}, \hat{s}, ... = 0, 1$ be the anholonomic indices referred to the anholonomic frame $e_{\hat{r}} = e^{\tilde{s}}{}_{\hat{r}}\, \partial_{\tilde{s}}$, defined on the $\underline{V}_2^{(\varrho)}$, with $\partial_{\tilde{s}} = \partial/\partial\, \widetilde{q}^{\tilde{s}}$ as the vectors tangent to the coordinate lines. So, a smooth differential 2D-manifold $\underline{V}_2^{(\varrho)}$ has at each point $\widetilde{q}^s$ a tangent space $\widetilde{T}_{\tilde{q}}\underline{V}_2^{(\varrho)}$, spanned by the frame, $\{e_{\hat{r}}\}$, and the coframe members $\vartheta^{\hat{r}} = e_s{}^{\hat{r}}\, d\widetilde{q}^{\tilde{s}}$, which constitute a basis of the covector space $\widetilde{T}^{\star}_{\tilde{q}}\underline{V}_2^{(\varrho)}$. All this nomenclature can be given for $\underline{V}_2^{(0)}$ too. Then, we may compute corresponding vierbein fields $\widetilde{e}_r{}^{\hat{s}}$ and $e_r{}^{\hat{s}}$ from the equations

$$g_{\tilde{r}\tilde{s}} = \widetilde{e}_{\tilde{r}}{}^{\hat{r}'}\, \widetilde{e}_{\tilde{s}}{}^{\hat{s}'}\, o_{\hat{r}'\hat{s}'}, \quad g_{\tilde{r}\tilde{s}}(\varrho) = e_{\tilde{r}}{}^{\hat{r}'}(\varrho)\, e_{\tilde{s}}{}^{\hat{s}'}(\varrho)\, o_{\hat{r}'\hat{s}'}, \tag{91}$$

with $\widetilde{g}_{rs}$ (89) and $g_{\tilde{r}\tilde{s}}(\varrho)$ (76). Hence

$$\begin{array}{l} \widetilde{e}_{\tilde{0}}{}^{\hat{0}} = 1 + \vec{a}\cdot\vec{\widetilde{x}}, \quad \widetilde{e}_{\tilde{0}}{}^{\hat{1}} = \vec{\omega}\wedge\vec{\widetilde{x}}, \quad \widetilde{e}_{\tilde{1}}{}^{\hat{0}} = 0, \quad \widetilde{e}_{\tilde{1}}{}^{\hat{1}} = 1, \\ e_{\tilde{0}}{}^{\hat{0}}(\varrho) = 1 + \frac{\varrho v_q}{\sqrt{2}}, \quad e_{\tilde{0}}{}^{\hat{1}}(\varrho) = \frac{\varrho}{\sqrt{2}}, \quad e_{\tilde{1}}{}^{\hat{0}}(\varrho) = -\frac{\varrho}{\sqrt{2}}, \quad e_{\tilde{1}}{}^{\hat{1}}(\varrho) = 1 - \frac{\varrho v_q}{\sqrt{2}}. \end{array} \tag{92}$$

A deformation $V_4^{(0)} \longrightarrow V_4^{(\varrho)}$ is equivalent to a straightforward generalization of (87) as

$$x^\mu \longrightarrow x^\mu_{(\varrho)} = z^\mu_{(\varrho)}(s) + \widetilde{x}^i\, e^\mu{}_{\hat{i}}(s), \tag{93}$$

provided, as before, $\widetilde{x}^\mu$ denotes the coordinates relative to the accelerated observer in 4D background space $V_4^{(\varrho)}$. A displacement vector from the origin is then $dz^\mu_\varrho(s) = e^\mu{}_{\hat{0}}(\varrho)\, d\widetilde{x}^0$, Inverting $e_r{}^{\hat{s}}(\varrho)$ (92), we obtain $e^\mu{}_{\hat{a}}(\varrho) = \pi_{\hat{a}}{}^{\hat{b}}(\varrho)\, \overline{e}^\mu{}_{\hat{b}}$, where

$$\begin{array}{l} \pi^{\hat{0}}_{\hat{0}}(\varrho) \equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1}(1 - \frac{\varrho v_q}{\sqrt{2}})\,(1 + \vec{a}\cdot\vec{\widetilde{x}}), \quad \pi^{\hat{i}}_{\hat{0}}(\varrho) \equiv -(1 + \frac{\varrho^2}{2\gamma_q^2})^{-1}\frac{\varrho}{\sqrt{2}}\,\widetilde{e}^i\,(1 + \vec{a}\cdot\vec{\widetilde{x}}), \\ \pi^{\hat{0}}_{\hat{i}}(\varrho) \equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1}\left[(\vec{\omega}\wedge\vec{\widetilde{x}})(1 - \frac{\varrho v_q}{\sqrt{2}}) - \frac{\varrho}{\sqrt{2}}\right]\widetilde{e}_i^{-1}, \quad \pi^{\hat{j}}_{\hat{i}}(\varrho) = \delta^j_i\,\pi(\varrho), \\ \pi(\varrho) \equiv (1 + \frac{\varrho^2}{2\gamma_q^2})^{-1}\left[(\vec{\omega}\wedge\vec{\widetilde{x}})\,\frac{\varrho}{\sqrt{2}} + 1 + \frac{\varrho v_q}{\sqrt{2}}\right]. \end{array} \tag{94}$$

Thus,

$$dx^\mu_\varrho = dz^\mu_\varrho(s) + d\widetilde{x}^i\, e^\mu{}_{\hat{i}} + \widetilde{x}^i\, de^\mu{}_{\hat{i}}(s) = (\tau^{\hat{b}}\, d\widetilde{x}^0 + \pi^{\hat{b}}_{\hat{i}}\, d\widetilde{x}^i)\,\overline{e}^\mu{}_{\hat{b}}, \tag{95}$$

where

$$\tau^{\hat{b}} \equiv \pi^{\hat{b}}_{\hat{0}} + \widetilde{x}^i\left(\pi^{\hat{a}}_{\hat{i}}\Phi^b_a + \frac{d\pi^{\hat{b}}_{\hat{i}}}{ds}\right). \tag{96}$$

Hence, in general, the metric in noninertial frame of arbitrary accelerating and rotating observer in Minkowski space-time is

$$\widetilde{g}(\varrho) = \eta_{\mu\nu}\, dx^\mu_\varrho \otimes dx^\nu_\varrho = W_{\mu\nu}(\varrho)\, d\widetilde{x}^\mu \otimes d\widetilde{x}^\nu, \tag{97}$$

which can be conveniently decomposed according to

$$\begin{array}{l} W_{00}(\varrho) = \pi^2\left[(1 + \vec{a}\cdot\vec{\widetilde{x}})^2 + (\vec{\omega}\cdot\vec{\widetilde{x}})^2 - (\vec{\omega}\cdot\vec{\omega})(\vec{\widetilde{x}}\cdot\vec{\widetilde{x}})\right] + \gamma_{00}(\varrho), \\ W_{0i}(\varrho) = -\pi^2\,(\vec{\omega}\wedge\vec{\widetilde{x}})^i + \gamma_{0i}(\varrho), \quad W_{ij}(\varrho) = -\pi^2\,\delta_{ij} + \gamma_{ij}(\varrho), \end{array} \tag{98}$$

and that

$$\begin{array}{l} \gamma_{00}(\varrho) = \pi\,\left[(1 + \vec{a}\cdot\vec{\widetilde{x}})\zeta^0 - (\vec{\omega}\wedge\vec{\widetilde{x}})\cdot\vec{\zeta}\right] + (\zeta^0)^2 - (\vec{\zeta})^2, \quad \gamma_{0i}(\varrho) = -\pi\,\zeta^i + \tau^{\hat{0}}\,\pi^{\hat{0}}_{\hat{i}}, \\ \gamma_{ij}(\varrho) = \pi^{\hat{0}}_{\hat{i}}\,\pi^{\hat{0}}_{\hat{j}}, \quad \zeta^0 = \pi\,\left(\tau^{\hat{0}} - 1 - \vec{a}\cdot\vec{\widetilde{x}}\right), \quad \vec{\zeta} = \pi\,\left(\vec{\tau} - \vec{\omega}\wedge\vec{\widetilde{x}}\right). \end{array} \tag{99}$$

As we expected, according to (97)- (99), the matric $\widetilde{g}(\varrho)$ is decomposed in the following form:

$$g(\varrho) = \pi^2(\varrho)\,\widetilde{g} + \gamma(\varrho), \tag{100}$$

where $\gamma(\varrho) = \gamma_{\mu\nu}(\varrho)\, d\widetilde{x}^\mu \otimes d\widetilde{x}^\nu$ and $\Upsilon(\varrho) = \pi^{\hat{a}}_{\hat{a}}(\varrho) = \pi(\varrho)$. In general, the geodesic coordinates are admissible as long as

$$\left(1 + \vec{a}\cdot\vec{\widetilde{x}} + \frac{\zeta^0}{\pi}\right)^2 > \left(\vec{\omega}\wedge\vec{\widetilde{x}} + \frac{\vec{\zeta}}{\pi}\right)^2. \tag{101}$$

doi: https://doi.org/10.52526/25792776-23.70.2-212

The equations (88) and (97) say that the vierbein fields, with entries $\eta_{\mu\nu}\,\overline{e}^{\,\mu}_{\ \hat{a}}\,\overline{e}^{\,\nu}_{\ \hat{b}} = o_{\hat{a}\hat{b}}$ and $\eta_{\mu\nu}\,e^{\,\mu}_{\ \hat{a}}\,e^{\,\nu}_{\ \hat{b}} = \gamma_{\hat{a}\hat{b}}$ lead to the relations

$$\widetilde{g} = o_{\hat{a}\hat{b}}\,\widetilde{\vartheta}^{\hat{a}} \otimes \widetilde{\vartheta}^{\hat{b}}, \quad g = o_{\hat{a}\hat{b}}\,\vartheta^{\hat{a}} \otimes \vartheta^{\hat{b}} = \gamma_{\hat{a}\hat{b}}\,\widetilde{\vartheta}^{\hat{a}} \otimes \widetilde{\vartheta}^{\hat{b}}, \tag{102}$$

which readily leads to the coframe fields:

$$\begin{array}{l} \widetilde{\vartheta}^{\hat{b}} = \overline{e}_{\ \mu}^{\ \hat{b}}\,dx^{\mu} = \widetilde{e}^{\hat{b}}_{\ \mu}\,d\widetilde{x}^{\,\mu}, \quad \widetilde{e}^{\hat{b}}_{\ 0} = N^{b}_{0}, \quad \widetilde{e}^{\hat{b}}_{\ i} = N^{b}_{i}, \\ \vartheta^{\hat{b}} = \overline{e}_{\ \mu}^{\ \hat{b}}\,dx^{\mu}_{\varrho} = e^{\hat{b}}_{\ \mu}\,d\widetilde{x}^{\,\mu} = \pi^{\hat{b}}_{\ \hat{a}}\,\widetilde{\vartheta}^{\hat{a}}, \quad e^{\hat{b}}_{\ 0} = \tau^{\hat{b}}, \quad e^{\hat{b}}_{\ i} = \pi^{\hat{b}}_{\ \hat{i}}. \end{array} \tag{103}$$

Here $N^0_0 = N \equiv \left(1 + \vec{a}\cdot\vec{\widetilde{x}}\right)$, $N^0_i = 0$, $N^i_0 = N^i \equiv \left(\vec{\omega}\cdot\vec{\widetilde{x}}\right)^i$, $N^j_i = \delta^j_i$. In the standard (3 + 1)-decomposition of space-time, N and N^i are known as *lapse function* and *shift vector*, respectively (Gronwald & Hehl, 1996). Hence, we may easily recover the frame field $e_{\hat{a}} = e^{\mu}_{\ \hat{a}}\,\widetilde{e}_{\mu} = \pi_{\hat{a}}^{\ \hat{b}}\,\widetilde{e}_{\hat{b}}$ by inverting (103):

$$\begin{array}{l} e_{\hat{0}}(\varrho) = \frac{\pi(\varrho)}{\pi(\varrho)\,\tau^{\hat{0}}(\varrho) - \pi^{\hat{0}}_{\hat{k}}(\varrho)\,\tau^{\hat{k}}(\varrho)}\,\widetilde{e}_0 - \frac{\tau^{\hat{i}}(\varrho)}{\pi(\varrho)\,\tau^{\hat{0}}(\varrho) - \pi^{\hat{0}}_{\hat{k}}(\varrho)\,\tau^{\hat{k}}(\varrho)}\,\widetilde{e}_i, \\ e_{\hat{i}}(\varrho) = -\frac{\pi^{\hat{0}}_{\hat{i}}(\varrho)}{\pi(\varrho)\,\tau^{\hat{0}}(\varrho) - \pi^{\hat{0}}_{\hat{k}}(\varrho)\,\tau^{\hat{k}}(\varrho)}\,\widetilde{e}_0 + \pi^{-1}(\varrho)\left[\delta^j_i + \frac{\tau^{j}(\varrho)\,\pi^{\hat{0}}_{\hat{i}}(\varrho)}{\pi(\varrho)\,\tau^{\hat{0}}(\varrho) - \pi^{\hat{0}}_{\hat{k}}(\varrho)\,\tau^{\hat{k}}(\varrho)}\right]\widetilde{e}_j. \end{array} \tag{104}$$

A *generalized transport* for deformed frame $e_{\hat{a}}$, which includes both the Fermi-Walker transport and distortion of $\underline{M}_2$, can be written in the form

$$\frac{de^{\,\mu}_{\ \hat{a}}}{ds} = \widetilde{\Phi}_a^{\ b}\,e^{\mu}_{\ \hat{b}}, \tag{105}$$

where a *deformed acceleration tensor* $\widetilde{\Phi}_a^{\ b}$ concisely is given by

$$\widetilde{\Phi} = \frac{d\,\ln\pi}{ds} + \pi\,\Phi\,\pi^{-1}. \tag{106}$$

Thus, we derive the tetrad fields $e_r^{\ \hat{s}}(\varrho)$ (92) and $e^{\mu}_{\ \hat{a}}(\varrho)$ (104) as a function of *local rate* ϱ of instantaneously change of a constant velocity (both magnitude and direction) of a massive particle in M_4 under the unbalanced net force, describing corresponding *fictitious graviton*. Then, the *fictitious gravitino*, $\psi_{\hat{m}}^{\ \alpha}(\varrho)$, will be arisen under infinitesimal transformations of local supersymmetry (64), provided by the local parameters $\zeta^M(a)$ (45).

8. Involving the background semi-Riemann space V_4; Justification for the introduction of the WPE

We can always choose *natural coordinates* $X^{\alpha}(T, X, Y, Z) = (T,\,\vec{X})$ with respect to the axes of the local free-fall coordinate frame $S^{(l)}_4$ in an immediate neighbourhood of any space-time point $(\widetilde{x}_p) \in V_4$ in question of the background semi- Riemann space, V_4, over a differential region taken small enough so that we can neglect the spatial and temporal variations of gravity for the range involved. The values of the metric tensor $\widetilde{g}_{\mu\nu}$ and the affine connection $\widetilde{\Gamma}^{\lambda}_{\mu\nu}$ at the point $(\widetilde{x}_p)$ are necessarily sufficient information for determination of the natural coordinates $X^{\alpha}(\widetilde{x}^{\mu})$ in the small region of the neighbourhood of the selected point (Weinberg, 1972). Then the whole scheme outlined in the previous subsections (a) and (b) will be held in the frame $S^{(l)}_4$. The general *inertial force* computed by Ter-Kazarian (2012) reads

$$\widetilde{\vec{f}}_{(in)} = -\frac{m\vec{a}_{abs}}{\Omega^2(\overline{\varrho})\,\gamma_q} = -\frac{\vec{e}_f}{\Omega^2(\overline{\varrho})\,\gamma_q}\left|f^{\alpha}_{(l)} - m\frac{\partial X^{\alpha}}{\partial\widetilde{x}^{\sigma}}\Gamma^{\sigma}_{\mu\nu}\frac{d\widetilde{x}^{\mu}}{dS}\frac{d\widetilde{x}^{\nu}}{dS}\right|. \tag{107}$$

Whereas, as before, the two systems S_2 and $S^{(l)}_4$ can be chosen in such a way as the axis $\vec{e}_q$ of $S_{(2)}$ lies $(\vec{e}_q = \vec{e}_f)$ along the acting net force $\vec{f} = \vec{f}_{(l)} + \vec{f}_{g(l)}$, while the time coordinates in the two systems are taken the same, $q^0 = t = X^0 = T$. Here $\vec{f}_{(l)}$ is the SR value of the unbalanced relativistic force other than gravitational and $\vec{f}_{g(l)}$ is the gravitational force given in the frame $S^{(l)}_4$. Despite of totally different and independent sources of gravitation and inertia, at $f^{\alpha}_{(l)} = 0$, the (107) establishes the independence of free-fall $(v_q = 0)$ trajectories of the mass, internal composition and structure of bodies. This furnishes a justification for the introduction of the WPE. A remarkable feature is that, although the inertial force has a nature different than the gravitational force, nevertheless both are due to a distortion of the local inertial properties of, respectively, 2D $\underline{M}_2$ and 4D-background space.

doi: https://doi.org/10.52526/25792776-23.70.2-212

9. The inertial effects in the background post Riemannian geometry

If the nonmetricity tensor $N_{\lambda\mu\nu} = -\mathcal{D}_\lambda\, g_{\mu\nu} \equiv -g_{\mu\nu\,;\lambda}$ does not vanish, the general formula for the affine connection written in the space-time components is (Poplawski, 2009)

$$\Gamma^\rho{}_{\mu\,\nu} = \overset{\circ}{\Gamma}{}^\rho{}_{\mu\,\nu} + K^\rho{}_{\mu\nu} - N^\rho{}_{\mu\nu} + \tfrac{1}{2} N_{(\mu}{}^\rho{}_{\nu)}, \tag{108}$$

where the metric alone determines the torsion-free Levi-Civita connection $\overset{\circ}{\Gamma}{}^\rho{}_{\mu\nu}$, $K^\rho{}_{\mu\nu}:\;\; = 2Q_{(\mu\nu)}{}^\rho + Q^\rho{}_{\mu\nu}$ is the non-Riemann part - the affine *contortion tensor*. The torsion, $Q^\rho{}_{\mu\nu} = \frac{1}{2} T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{[\mu\,\nu]}$ given with respect to a holonomic frame, $d\,\vartheta^\rho = 0$, is a third-rank tensor, antisymmetric in the first two indices, with 24 independent components. We now compute the relativistic inertial force for the motion of the matter, which is distributed over a small region in the U_4 space and consists of points with the coordinates x^μ, forming an extended body whose motion in the space, U_4, is represented by a world tube in space-time. Suppose the motion of the body as a whole is represented by an arbitrary timelike world line γ inside the world tube, which consists of points with the coordinates $\tilde{X}^\mu(\tau)$, where τ is the proper time on γ. Define

$$\delta x^\mu = x^\mu - \tilde{X}^\mu, \;\; \delta x^0 = 0, \;\; u^\mu = \tfrac{d\,\tilde{X}^\mu}{d\,s}. \tag{109}$$

The *Papapetrou equation of motion for the modified momentum* (Bergmann & Thompson, 1953, Møller, 1958, Papapetrou, 1974, Poplawski, 2009) is

$$\tfrac{\overset{\circ}{\mathcal{D}}\,\Theta^\nu}{\mathcal{D}\,s} = -\tfrac{1}{2}\; \overset{\circ}{R}{}^\nu{}_{\mu\sigma\rho}\, u^\mu\, J^{\sigma\rho} - \tfrac{1}{2}\, N_{\mu\rho\lambda}\, K^{\mu\rho\lambda:\,\nu}, \tag{110}$$

where $K^\mu_{\nu\lambda}$ is the contortion tensor,

$$\Theta^\nu = P^\nu + \tfrac{1}{u^0}\; \overset{\circ}{\Gamma}{}^{\;\nu}_{\mu\,\rho}\,(u^\mu\, J^{\rho 0} + N^{0\mu\rho}) - \tfrac{1}{2u^0}\, K_{\mu\rho}{}^\nu\, N^{\mu\rho 0} \tag{111}$$

is referred to as the *modified 4-momentum*, $P^\lambda = \int \tau^{\lambda 0}\, d\,\Omega$ is the ordinary 4-momentum, $d\,\Omega := d\,x^4$, and the following integrals are defined:

$$\begin{array}{l} M^{\mu\rho} = u^0 \int \tau^{\mu\rho}\, d\,\Omega, \quad M^{\mu\nu\rho} = -u^0 \int \delta x^\mu\, \tau^{\nu\rho}\, d\,\Omega, \quad N^{\mu\nu\rho} = u^0 \int s^{\mu\nu\rho}\, d\,\Omega, \\ J^{\mu\rho} = \int (\delta x^\mu\, \tau^{\rho 0} - \delta x^\rho\, \tau^{\mu 0} + s^{\mu\rho 0})\, d\,\Omega = \frac{1}{u^0}(-M^{\mu\rho 0} + M^{\rho\mu 0} + N^{\mu\rho 0}), \end{array} \tag{112}$$

where $\tau^{\mu\rho}$ is the energy-momentum tensor for particles, $s^{\mu\nu\rho}$ is the spin density. The quantity $J^{\mu\rho}$ is equal to $\int (\delta x^\mu\, \tau^{kl} - \delta x^\rho\, \tau^{\mu\lambda} + s^{\mu\rho\lambda})\, d\,S_\lambda$ taken for the volume hypersurface, so it is a tensor, which is called the *total spin tensor*. The quantity $N^{\mu\nu\rho}$ is also a tensor. The relation $\delta x^0 = 0$ gives $M^{0\nu\rho} = 0$. It was assumed that the dimensions of the body are small, so integrals with two or more factors δx^μ multiplying $\tau^{\nu\rho}$ and integrals with one or more factors δx^μ multiplying $s^{\nu\rho\lambda}$ can be neglected. The *Papapetrou equations of motion for the spin* (Bergmann & Thompson, 1953, Møller, 1958, Papapetrou, 1974, Poplawski, 2009) is

$$\tfrac{\overset{\circ}{\mathcal{D}}}{\mathcal{D}s}\, J^{\lambda\nu} = u^\nu\, \Theta^\lambda - u^\lambda\, \Theta^\nu + K^\lambda{}_{\mu\rho}\, N^{\nu\mu\rho} + \tfrac{1}{2}\, K_{\mu\rho}{}^\lambda\, N^{\mu\nu\rho} - K^\nu{}_{\mu\rho}\, N^{\lambda\mu\rho} - \tfrac{1}{2}\, K_{\mu\rho}{}^\nu\, N^{\mu\rho\lambda}. \tag{113}$$

Computing from (110), in general, the relativistic inertial force, exerted on the extended spinning body moving in the RC space U_4, can be found to be

$$\begin{array}{l} \vec{f}_{(in)}(x) = -\frac{m\vec{a}_{abs}(x)}{\Omega^2(\bar{\varrho})\,\gamma_q} = -m\, \frac{\vec{e}_f}{\Omega^2(\bar{\varrho})\,\gamma_q} \Big| \frac{1}{m}\, f^\alpha_{(l)} - \frac{\partial X^\alpha}{\partial\, x^\mu} \Big[\overset{\circ}{\Gamma}{}^{\;\mu}_{\nu\lambda}\, u^\nu\, u^\lambda + \\ \frac{1}{u^0}\; \overset{\circ}{\Gamma}{}^{\;\mu}_{\nu\rho}\,(u^\nu\, J^{\rho 0} + N^{0\nu\rho}) - \frac{1}{2u^0}\, K_{\nu\rho}{}^\mu\, N^{\nu\rho 0} + \frac{1}{2}\; \overset{\circ}{R}{}^\mu{}_{\nu\sigma\rho}\, u^\nu\, J^{\sigma\rho} + \frac{1}{2}\, N_{\nu\rho\lambda}\, K^{\nu\rho\lambda:\,\mu} \Big] \Big|. \end{array} \tag{114}$$

10. Concluding remarks

In the framework of local $\widetilde{MS}_p$-SUSY, we address the inertial effects. The local $\widetilde{MS}_p$-SUSY can only be implemented if $\underline{\widetilde{M}}_2$ and $\widetilde{M}_4$ are curved (deformed). Whereas the space $\widetilde{M}_4$, in order to become on the same footing with the distorted space $\underline{\widetilde{M}}_2$, refers to the accelerated reference frame of a particle, without relation to other matter fields. Thus, unlike gravitation, a curvature of space-time arises entirely due to the inertial properties of the Lorentz-rotated frame of interest, i.e. a *fictitious gravitation* which can be globally removed by appropriate coordinate transformations. The superspace $(z^M,\, \Theta,\, \bar{\Theta})$ is a direct sum extension of

doi: https://doi.org/10.52526/25792776-23.70.2-212

background double spaces $\widetilde{M}_4 \oplus \widetilde{\underline{M}}_2$, with an inclusion of additional fermionic coordinates $(\Theta(\underline{\theta}, \theta)$, $\bar{\Theta}(\bar{\underline{\theta}}, \bar{\theta}))$ induced by the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, which refer to $\widetilde{\underline{M}}_2$. Thereby thanks to the embedding $\widetilde{\underline{M}}_2 \hookrightarrow \widetilde{M}_4$, the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, in turn, induce the spinors $\theta(\underline{\theta}, \bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta}, \bar{\underline{\theta}})$, as to $\widetilde{M}_4$. The $\widetilde{MS}_p$-SUSY was conceived as a quantum field theory whose action included the Einstein-Hilbert term, where the graviton coexists with a fermionic field called gravitino, described by the Rarita-Scwinger kinetic term. The two fields differ in their spin: 2 for the graviton, 3/2 for the gravitino. The spin 3/2 contact term in total Lagrangian arises from equations of motion for the torsion tensor, and that the original Lagrangian itself takes the simpler interpretation of a minimally coupled spin (2, 3/2) theory. The only source of graviton and gravitino is the acceleration of a particle, because the $\widetilde{MS}_p$-SUSY is so constructed as to make these two particles just as being the two bosonic and fermionic states of a particle of interest in the background spaces M_4 and $\underline{M}_2$, respectively, or vice versa. The different 4D $N = 1$ supergravity multiplets all contain the graviton and the gravitino, but differ by their systems of auxiliary fields. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The dynamical variables of superspace formulation are the frame field $E^A(z)$ and spin-connection Ω. The superspace has at each point a tangent superspace spanned by the frame field $E^A(z) = dz^M E_M^A(z)$, defined as a 1-form over superspace, with coefficient superfields, generalizing the usual frame, namely supervierbien $E_M^A(z)$. Covariant derivatives with respect to local Lorentz transformations are constructed by means of the spin connection, which is a 1-form in superspace as well. There is no general prescription for deducing necessary covariant constraints which if imposed upon the superfields of super-vielbein and spin-connection will eliminate the component fields. However, some usual constraints are found using tangent space and supergeneral coordinate transformations of the torsion and curvature covariant tensors, given in appropriate super-gauge. A coupling of supergravity with matter superfields evidently no longer holds in resulting theory. Instead, we argue that a deformation/(distortion of local internal properties) of MS_p is the origin of inertia effects that can be observed by us. In going into practical details of the realistic local $\widetilde{MS}_p$-SUSY model, we briefly review of certain aspects of a general distortion of local internal properties of MS_p and derive the explicit form of the vierbien $e_{\hat{m}}^{\ a}(\varrho) \equiv (e_m^{\ a}(\varrho), e_{\underline{m}}^{\ a}(\varrho))$, which describes *fictitious graviton* as a function of *local rate* $\varrho(\eta, m, f)$ of instantaneously change of the velocity $v^{(\pm)}$ of massive (m) test particle under the unbalanced net force (f). We discuss the inertia effects by going beyond the hypothesis of locality. This allows to improve essentially the relevant geometrical structures referred to the noninertial frame in Minkowski space-time for an arbitrary velocities and characteristic acceleration lengths. Despite the totally different and independent physical sources of gravitation and inertia, this approach furnishes justification for the introduction of the WPE. Consequently, we relate the inertia effects to the more general post-Riemannian geometry. We derive a general expression of the relativistic inertial force exerted on the extended spinning body moving in the Rieman-Cartan space.

References

Atwood D., et al. 1984, Phys. Rev. Lett., 52, 1673

Bakke K., Furtado C., 2010, Phys. Rev. D, 82, 084025

Beck A., Havas P., 1989, The Collected Papers of Albert Einstein. Princeton University Press, Princeton, NJ, vol. 2; pp. 252–311. The quote is on p. 302.

Bergmann P., Thompson R., 1953, Phys. Rev., 89, 400

Binetruy P., Girardi G., Rrimm R., 2001, Physics Reports, 343, 255

Bini D., Cherubini C., Mashhoon B., 2004, Class. and Quant. Gr., 21, 3893

Bonse U., Wroblewski T., 1983, Phys. Rev. Lett., 51, 1401

Brans C., 1977, Phys. Rev. Lett., 39, 856

Capozziello S., Lambiase G., 2000, Eur. Phys. J C, 16, 155

Cardall C., Fuller G., 1997, Phys. Rev. D, 55, 7960

Cocconi G., Salpeter E., 1960, Phys. Rev. Lett., 4, 176

Colella R., Overhauser A., Werner S., 1975, Phys. Rev. Lett., 34, 1472

Dicke R., 1961, Phys. Rev. Lett., 7, 359

Drake S., 1978, Galileo at work. Chicago, University of Chicago Press

doi: https://doi.org/10.52526/25792776-23.70.2-212

Drever R., 1961, Phil. Mag., 6, 683

Everitt C. W. F., et al. 2009, Space Sci. Rev., 148, 53

Everitt C. W. F., et al. 2011, Phys. Rev. Lett., 106, 221101

Faller J., et al. 1990, Phys. Rev. Lett., 64, 825

Fayet P., Ferrara S., 1977, Physics Reports, 32, 249

Gabriel M., Haugan M., 1990, Phys. Rev. D, 41, 2943

Gasperini M., 1988, Phys. Rev. D, 38, 2635

Gillies G., 1997, Rep. Prog. Phys., 60, 151

Gronwald F., Hehl F., 1996, On the Gauge Aspects of Gravity. Proc. of the 14th Course of the School of Cosmology and Gravitation on Quantum Gravity, held at Erice, Italy. Eds. Bergmann P G, de Sabbata V, and Treder H.-J., World Scientific, Singapore

Halprin A., Leung C., 1991, Phys. Rev. Lett., 67, 1833

Haugan M., Kauffmann T., 1995, Phys. Rev. D, 52, 3168

Haugan M., Lämmerzahl C., 2001, in Gyrod, Clocks, Interferometers...: Testing Relativistic Gravity in Space. C. Lämmerzahl, C.W.F. Everitt, F.W. Hehl (eds.): Springer–Velag, p.195

Hayasaka H., Takeuchi S., 1989, Phys. Rev. Lett., 63, 2701

Hehl F., Ni W.-T., 1990, Phys. Rev. D, 42, 2045

Hehl F., Lemke J., Mielke E., 1991, Two lectures on fermions and gravity. In: Geometry and Theoretical Physics, Debrus J and Hirshfeld A C (eds.) (Springer, Berlin) p.56

Heifetz M., et al. 2009, Space Sci. Rev., 148, 411

Hughes V. W., Robinson H. G., Beltran-Lopez V., 1960, Phys. Rev. Lett., 4, 342

Imanishi A., et al. 1991, J. Phys. Soc. Jpn., 60, 1150

Infeld L., 1950, Albert Einstein. Charles Scribner's Sons , New York, p. 47

Jacob M., 1987, Supersymmetry and Supergravity. North Holland, Amsterdam

Keiser G., et al. 2009, Space Sci. Rev., 148, 383

Lämmerzahl C., Bordé C., 2001, in Gyrod, Clocks, Interferometers...: Testing Relativistic Gravity in Space. C. Lämmerzahl, C.W.F. Everitt, F.W. Hehl (eds.): Springer–Velag, p.466

Li W.-Q., Ni W.-T., 1979, Chin. J. Phys, 16, 214

Luo J., et al. 2002, Phys. Rev. D, 65, 042005

Maluf J., Faria F., 2008, Ann. der Physik, 520, 326

Maluf J., Faria F., Ulhoa S., 2007, Class. Quantum Grav., 24, 2743

Marzlin K.-P., 1996, Phys. Lett. A, 215, 1

Mashhoon B., 95, Phys. Rev. Lett., 198, 9

Mashhoon B., 1988, Phys. Rev. Lett., 61, 2639

Mashhoon B., 1990a, Phys. Lett. A, 143, 176

Mashhoon B., 1990b, Phys. Lett. A, 145, 147

Mashhoon B., 2002, Ann. der Physik, 514, 532

Mashhoon B., 2011, Ann. der Physik, 523, 226

Mashhoon B., Wesson P., 2012, Annalen Phys., 524, 63

Misner C., Thorne K., Wheeler J., 1973, Gravitation. Freeman, San Francisco

Møller C., 1958, Ann. Phys. (NY), 4, 347

Muhlfelder B., et al. 2009, Space Sci. Rev., 148, 429

Newton I., 1687, Philosophiae Naturalis Principia Mathematica. http://plato.stanford.edu/entries/newton-principia

Ni W.-T., 1977, Chinese J. Phys., 15, 51

Ni W.-T., 1983, Precision Measurement and Gravitational Experiments. (National Tsing Hua University, Taiwan, China

doi: https://doi.org/10.52526/25792776-23.70.2-212

Ni W.-T., 2005a, Int. J. Mod. Phys. D, 14, 901

Ni W.-T., 2005b, Chin. Phys. Lett., 22, 33

Ni W.-T., 2005c, Chin. Phys. Lett., 22, 33

Ni W.-T., 2008, Prog. Theor. Phys. Suppl., 172, 49

Ni W.-T., 2011, Phys. Rev. Lett., 107, 051103

Ni W.-T., Zimmermann M., 1978, Phys. Rev. D, 17, 1473

Nilles H., 1984, Physics Reports, 110, 1

Nitschke J., Wilmarth P., 1990, Phys. Rev. Lett., 64, 2115

Pan Y., Ren J., 2011, Int. J Theor. Phys., 50, 724

Pantaleone J., Halprin A., Leung C., 1993, Phys. Rev. D, 47, R4199(R)

Papapetrou A., 1974, Lectures on general relativity. (Reidel D)(ISBN 9027705402)

Piniz D., Roy M., Wudka J., 1997, Phys. Rev. D, 56, 2403

Poplawski N., 2009, Spacetime and fields. arXiv[gr-qc/0911.0334]

Prestage J., et al. 1985, Phys. Rev. Lett., 54, 2387

Quinn T., Picard A., 1990, Nature, 343, 732

Schucking E., 2009, arXiv:0903.3768[physics.hist-ph]

Schucking E., Surowitz E., 2012, arXiv:gr-qc/0703149

Silbergleit A., et al. 2009, Space Sci. Rev., 148, 397

Silenko A., Teryaev O., 2007, Phys. Rev. D, 76, 061101(R)

Synge J., 1960, Relativity: The General Theory. North-Holland, Amsterdam

Ter-Kazarian G., 2012, Advances in Mathematical Physics, 2012, Article ID 692030, 41 pages,doi:10.1155/2012/692030, Hindawi Publ. Corporation

Ter-Kazarian G., 2023, Communications of BAO, 70, ??

Ter-Kazarian G., 2024, To appear in Grav. Cosmol., 30, No 1

Turyshev S., 2008, Annu. Rev. Nucl. Part. Sci., 58, 207

Weinberg S., 1972, Gravitation and Cosmology. J. W. and Sons, New York

Wess J., Bagger J., 1983, Supersymmetry and Supergravity. Princeton University Press, Princeton, New Jersey

West P., 1987, Introduction to Supersymmetry and Supergravity. World Scientific, Singapure

Will C., 2006, Living Rev. Relativity, 9, 3

Zhou Z., et al. 2002, Phys. Rev. D, 66, 022002

de Sabbata V., Gasperini M., 1981, Nuovo Cimento A, 66, 479

van Nieuwenhuizen P., 1981, Physics Reports, 68, 189

van Nieuwenhuizen P., Freedman D., Ferrara S., 1976, Phys. Rev. D, 13, 3214

doi: https://doi.org/10.52526/25792776-23.70.2-212