Three-Body Problem and an Unrecognized Oscillatory Motion of the Earth

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Abstract

Building on the classical three-body problem, this paper introduces the concept of a "center of acceleration" in the Sun–Earth–Moon system. This center is shown to move with the frequency of the Moon's orbit and induces an additional radial oscillation in the Earth's trajectory, independent of barycentric motion. Using Dopplergram data from the Solar Dynamics Observatory (SDO), this oscillation was measured to have an amplitude exceeding 25,000 kilometers.

Keywords: *Celestial mechanics, Three-body problem, Acceleration center, Earth-Moon system, Radial velocities, Dopplergrams, Keplerian deviation*

1. Introduction

The three-body problem is one of the most celebrated and complex topics in celestial mechanics, dating back to Newton Newton (1687) and extensively explored by later scholars Barrow-Green (1997a), Musielak & Quarles (2014), Szebehely (1967). The classic formulation describes three point masses—m_1, m_2, m_3—interacting gravitationally. While analytical solutions exist only for special cases, this study revisits the three-body dynamics with a focus on the force vectors' intersection, proposing a novel point, the "center of acceleration"—as a key dynamic variable.

In contrast to the center of mass, the center of acceleration is defined as the instantaneous intersection point of all net gravitational force vectors. This point typically differs from the barycenter and moves dynamically in response to the Moon's position. Equation (1) determines its Cartesian coordinates:

$$c_l = \frac{m_1 r_{23} + m_2 r_{13} + m_3 r_{12}}{m_1 + m_2 + m_3} \qquad (1)$$

The acceleration center is particularly close to Earth in the Sun–Earth–Moon system. While Lagrange Barrow-Green (1997b) equated this point to the center of mass under idealized conditions, here its position is shown to vary and correlate with the small, previously unrecognized oscillatory drift of Earth.

2. Theoretical Considerations

Figure 1 illustrates the motion of the center of acceleration across the lunar cycle. As the Moon moves from the new to the full phase, the acceleration center shifts behind Earth's velocity vector, tilting Earth's motion slightly toward the Sun. In the waning lunar phase, the reverse occurs: Earth's velocity vector tilts outward, inducing a minor drift away from the Sun.

This radial drift is not related to barycentric motion, but arises directly from the repositioning of the acceleration center. The Earth-Moon barycenter follows this modulation. Although subtle, this oscillation is detectable via high-resolution solar Doppler measurements and could have implications for long-term ephemerides and precision astrometry.

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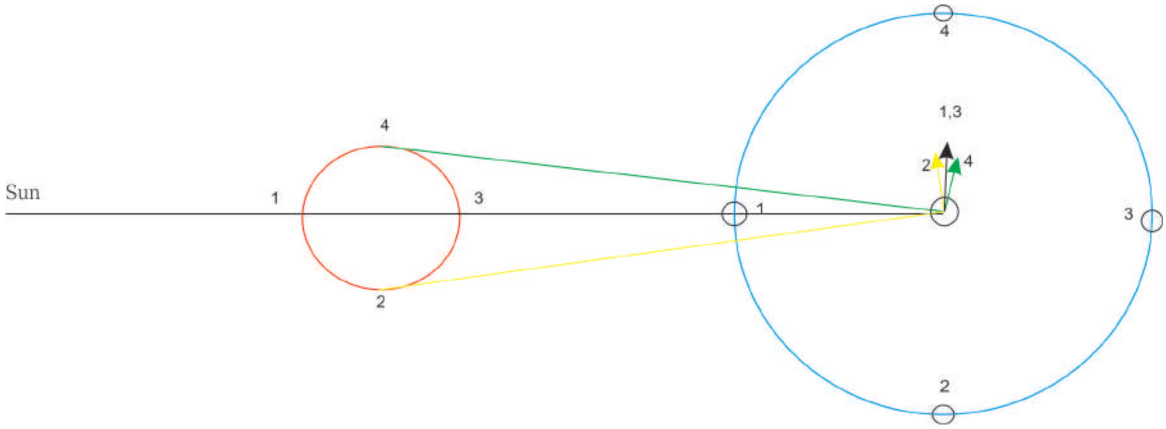

Figure 1. Schematic view of the Sun–Earth–Moon interaction and the center of acceleration.

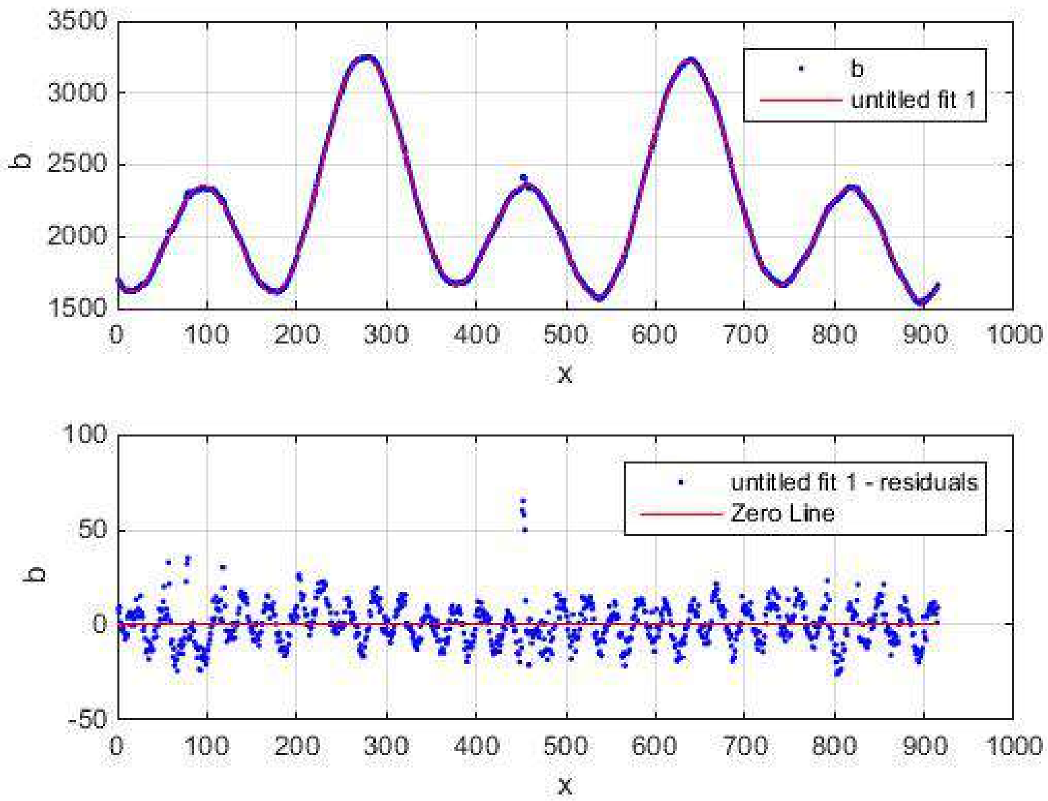

Figure 2. Average radial velocities from SDO Dopplergrams (2011–2013). Bottom: Residuals after multi-term sinusoidal fit, highlighting 29-day periodic drift.

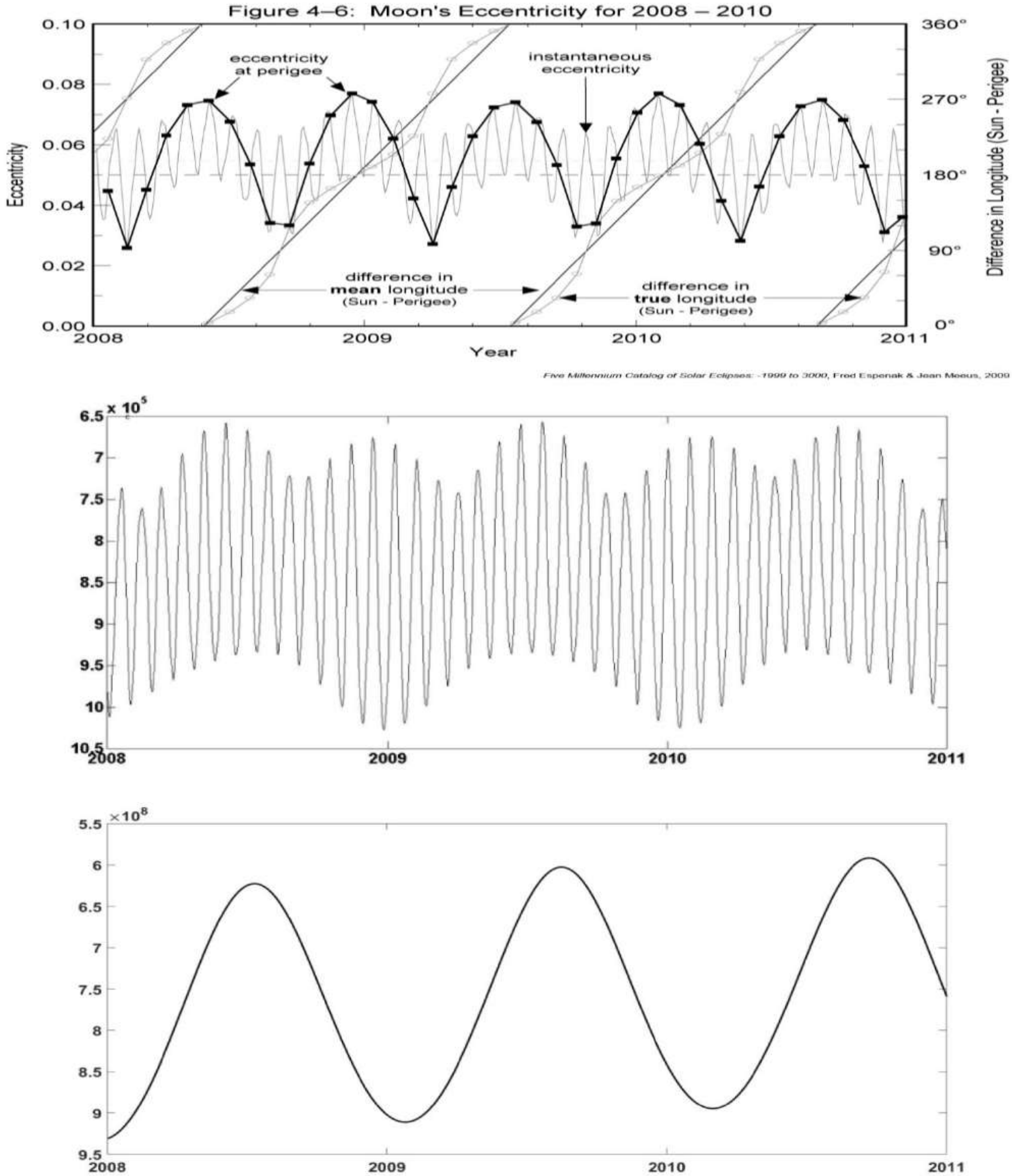

Figure 3. Eccentricity variation in the Moon's orbit (2008–2010, after Espenak (2012)). Middle: Earth–acceleration center distance. Bottom: Earth–Jupiter distance. All three curves demonstrate phase alignment.

3. Data Analysis

SDO Dopplergrams from July 2011 to December 2013 were analyzed at 23:00 UT daily. Using a 121×121 pixel region at the solar disc center, radial velocity data were averaged and fitted with a seven-term sine function:

$$f(x) = \sum_{i=1}^{7} A_i \sin(k_i x + \phi_i) \tag{2}$$

The residuals reveal a dominant frequency of $\sim$ 29.26 days, consistent with the lunar synodic period. The peak-to-peak variation in Earth's radial velocity is about 40 m/s, indicating a drift amplitude of approximately 25,500 km:

$$D = 20\frac{m}{s} \times 14.5 \text{ days} \times 86400s \approx 25,000 \text{ km} \tag{3}$$

This observed oscillation strongly deviates from a pure Keplerian orbit of the Earth-Moon barycenter, supporting the theoretical model.

4. Correlation with Lunar Eccentricity and Jupiter's Influence

Although this study does not directly model the Moon's motion, it identifies a striking correlation: the eccentricity of the Moon's orbit appears to be synchronized with the distance of Earth to the center of acceleration and Jupiter. As shown in Figure 3, all three parameters oscillate in-phase.

Historical work by Gutzwiller **?** and Espenak **?** observed periodic variations in lunar eccentricity. Here, these variations align with the minima in both the Earth–acceleration center distance and the Earth–Jupiter distance, suggesting an extended dynamical coupling.

This leads to a broader hypothesis. Drawing from the Milankovitch cycle—where Earth's orbital eccentricity varies on ∼100,000-year timescales—one could propose the existence of an undiscovered massive object at $R \approx 2100$ AU, deduced from Kepler's Third Law:

$$R^3 = P^2 \quad \Rightarrow \quad P = 100{,}000 \text{ years} \Rightarrow R \approx 2100 \text{ AU} \tag{4}$$

Further research is planned to explore this idea and use the precise positions of Mars and Venus to detect additional evidence of Earth's radial oscillations.

5. Conclusion

This study presents a novel theoretical and observational framework for detecting a radial oscillation in Earth's motion caused by the moving center of acceleration in the Sun–Earth–Moon system. Based on Doppler data from the SDO satellite, the amplitude of this motion is estimated to exceed 25,000 km.

These findings suggest that long-term ephemerides and precision measurements should consider this subtle but measurable drift when modeling Earth's orbit.

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