Communications of BAO, Vol. 72, Issue 2, 2025, pp. 206-225

Quantum interference of a de Broglie wave of a Dirac particle beyond the `hypothesis of locality´. Part I: Dirac equation

G.Ter-Kazarian*

Byurakan Astrophysical Observatory, Byurakan, Aragatsotn Province, Armenia

Abstract

This is the first of three articles that explore the possibility of quantum mechanical inertial properties of the Dirac particle beyond the `hypothesis of locality´. This is done within the framework of the *Master Space*-Teleparallel Supergravity ($\widetilde{MS}_p$-TSG) (Ter-Kazarian, 2025) theory, which we recently proposed to account for inertial effects (Ter-Kazarian, 2026). The `hypothesis of locality´ used for extension of the Lorentz invariance to accelerated observers within the Special Relativity. This hypothesis in effect bypasses acceleration and replaces the accelerated observer by a continuous infinity of hypothetical momentarily comoving inertial observers along its wordline. Despite the successes for the tiny accelerations we usually experience, when the curvature of the wordline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small, however, the basic conceptual framework of this assumption has been considered by many scientists to be unsatisfactory. In general case, this is actually untenable and represents strict restrictions, and that the hypothesis of locality will have to be extended to describe physics for arbitrarily accelerated observers. This immediately leads to the disturbing fact within the $\widetilde{MS}_p$-TSG theory that the metric of a two-dimensional semi-Riemannian space, calculated in the non-inertial frame of reference of an accelerating and rotating observer, becomes incomplete. To recover the complete metric, therefore, our strategy in the latest paper (Ter-Kazarian, 2026) was to go beyond this hypothesis by invoking a general deformation of the flat master space, $\mathrm{MS}_p \to \widetilde{MS}_p$. Continuing along this line, in present article, we compute the object of anholonomicity and connection defined with respect to the anholonomic frame. Based on these premises, we derive the explicit form of the Dirac equation for an observer in a reference frame that is accelerated and rotating.

Keywords: *Teleparallel Supergravity–Spacetime Deformation–Inertia Effects–Quantum interference*

1. Introduction

The theoretical studies of the relativistic quantum theory in a curved spacetime have predicted a number of interesting manifestations of the spin-gravity coupling for a Dirac particle, see e.g. (Audretsch & Schafer, 1978, Cai & Papini, 1991, 1992, Fischbach et al., 1981, Hehl & Ni, 1990, Obukhov, 2001, 2002, Ryder, 1998, Singh & Papini, 2000, Varjú & Ryder, 1998, 2000, de Oliveira & Tiomno, 1962). In most cases, the various approximate schemes were used for the case of the weak gravitational field. The exact results for an arbitrary static spacetime geometry are reported by (Obukhov, 2002). Inertial effects are expected to have a significant influence on a determinations of spatial distances and temporal durations that are associated with the effective establishment of a sufficiently local frame of reference. Such a local coordinate system is what one actually uses in laboratory. For a performing the laboratory measurements, it is necessary to give a theoretical description of the measurements of accelerated observers. This is done via the hypothesis of locality. It is a long-established practice to use in the framework of Special Relativity (SR) the `hypothesis of locality´ for extension of the Lorentz invariance to accelerated observers, see e.g. (Hehl & Ni, 1990, Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Marzlin, 1996, Mashhoon, 2002, 2011, Misner et al., 1973, Synge, 1960) and references therein. This in effect bypasses acceleration and replaces the accelerated observer by a continuous infinity of hypothetical momentarily comoving inertial observers along its wordline.

*gago_50@yahoo.com

doi:https://doi.org/10.52526/25792776-25.72.2-206

In other words, the influence of inertial effects can be neglected on the length and time scales characteristic of elementary local observations. An accelerated observer measures the same physical results as a standard inertial observer that has the same position and velocity at the time of measurement. The curved path of the observer is substituted by the straight line tangential to the curve at the time of measurement (Mashhoon, 2002). The accelerated observer carries an orthonormal frame along its trajectory such that at each instant of its proper time τ, the local tetrad frame is common to both the accelerated observer and the momentarily comoving inertial observer (up to a spatial rotation). These observers refer their respective measurements to the local frame. The results of measurements performed by accelerated observers can then be related to those of inertial observers via the hypothesis of locality. At a certain distance from the accelerated worldline, successive spacelike hyperplanes orthogonal to the worldline, which are Euclidean spaces, instead of advancing with increasing time τ, will be retrogressing (Misner et al., 1973). If we go beyond the time τ_1, for example, coordinate assignments would start to overlap for the time τ_2. Therewith the principle construction (78), which introduces problems with the coordinate extension of the lab frame, is only possible in a Minkowski spacetime. The acceleration can indeed be accurately described within Minkowski spacetime using Fermi-Walker transport. The problem is different - in constructing laboratory coordinates. That is, by the framed construction (78), one expressed the Cartesian coordinates of an event in terms of the lab coordinates. This is done for two different times at the worldline $\mathcal{C}$. In a Riemannian spacetime one has to take the spacelike geodesics emanating perpendicularly from $\tilde{\mathcal{P}}$ for the construction of the lab coordinates thereby finding an expression which, for $\mathcal{P}$ sufficiently near to $\tilde{\mathcal{P}}$, contains additionally higher order deviations caused by the curvature of spacetime. At this distance, and at greater distances, the concept of "coordinates relative to the accelerated observer" becomes ambiguous and has to be abandoned. Since this cannot be accepted, the "local coordinates" approximate a Lorentz coordinate system in the immediate neighborhood of the observer. Therefore, in general, the Fermi-Walker transport cannot be extended to the whole spacetime manifold due to limitations imposed by spacetime curvature. The hypothesis of locality requires that the intrinsic length and time scales of the phenomena under observation be negligibly small relative to the corresponding acceleration scales associated with the observer. The charts for introduced coordinates cannot be global for accelerated observers. In fact, such geodesic coordinates are admissible as long as the condition (81) is valid. However, a classical measuring device must have spatial and temporal extension far exceeding the intrinsic length and time scales, respectively. The above considerations imply an upper limit on the magnitude of merely translational acceleration of a standard classical device. A similar limitation applies to the rotational frequency as well. This problem is inherent to any accelerated observer, and it can only be remedied by making the laboratory sufficiently small. The lab coordinate system is only useful in the immediate vicinity of the laboratory observer (Hehl et al., 1991). The hypothesis of locality requires that the proper length and time scales of the observed phenomena be negligible compared to the corresponding acceleration scales associated with the observer. The basic distinction between an accelerated observer in Minkowski spacetime and a momentarily comoving inertial observer is the existence of acceleration scales associated with the noninertial observer. These scales are intrinsic measures of the rate of variation of the local reference frame of the observer along the accelerated path.

That is, the hypothesis of locality, as well as its restricted version, so-called, clock hypothesis, which is a hypothesis of locality only concerned about the measurement of time, are permissible if condition $\lambda \ll L$ is valid, where λ is the intrinsic length scale of the phenomenon under observation, L is the acceleration length. Here λ could be the wavelength of electromagnetic radiation or the Compton wavelength of a particle, and L is the natural length that can be formed from the acceleration and the speed of light in vacuum; thus, $L = c^2/g$ for translational acceleration g, while $L = c/\omega$ for rotation of frequency ω. As long as $\lambda \ll L$ for the tiny accelerations we usually experience, the curvature of the wordline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small. Thus, a consistency can be achieved only in a rather limited neighborhood around the observer with linear dimensions that are negligibly small compared to the characteristic acceleration length of the observer. In this case, noninertial reference frames in Minkowski spacetime that undergo Fermi-Walker transport are useful in the analysis, for example, of inertia-induced quantum interference of a de Broglie wave of a Dirac particle (Hehl & Ni, 1990). The authors have put the special-relativistic Dirac equation into a noninertial reference frame by standard methods, confining ourselves strictly to flat Minkowski spacetime. Despite these successes, however, the basic conceptual framework of the `hypothesis of locality´ for extension of the Lorentz invariance to accelerated observers within the framework of SR, has been considered by many scientists to be unsatisfactory, see e.g. (Hehl et al., 1991, Mashhoon, 2002). In general case, this is actually untenable and represents strict restrictions, and that the hypothesis of locality will have to be extended

doi:https://doi.org/10.52526/25792776-25.72.2-206

to describe physics for arbitrarily accelerated observers. This can only be done with the help of a deep knowledge of the most striking phenomenon of inertia.

In doing so, we proposed the theory of global *master space* (MS_p) induced supersymmetry (MS_p-SUSY) (Ter-Kazarian, 2024a), which reveals the physical processes underlying the standard Lorenz code of motion and its deformation tested in experiments for ultra-high energy cosmic ray and TeV-γ photons observed. This theory, among other things, explores the first part of the phenomenon of inertia. This calls for a complete reconsideration of our standard ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property. The nature of the origin of *physical space-time* of SR is revealed, which turns out to be *a direct consequence of motion*. Namely, we have derived the relative *temporal* and *spatial* coordinates of *physical space-time* of SR as a function of parameters of underlying physical reality. This is a valuable hint for proposing the theory of $\widetilde{MS}_p$-TSG (Ter-Kazarian, 2025) (see also (Ter-Kazarian, 2024b,c,d)), as a local extension of the global theory of MS_p-SUSY. The $\widetilde{MS}_p$-TSG reviews the *acceleration and inertial effects*. The present article splits naturally into three parts. In the first, a local MS_p-SUSY is conceived as a theory of $\widetilde{MS}_p$-SG. The action of simple $\widetilde{MS}_p$-SG includes the Hilbert term for a *fictitious* graviton coexisting with a *fictitious* fermionic field of gravitino described by the Rarita-Scwinger kinetic term. Whereas a coupling of supergravity with matter superfields no longer holds. In the second, using Palatini's formalism generalized for the $\widetilde{MS}_p$-SG, we reinterpret a flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as a theory of $\widetilde{MS}_p$-TSG having the gauge *translation* group in tangent bundle. Its spin connection is related only to the inertial properties of the frame, not to gravitation. Whereas the Hilbert action vanishes and the gravitino action loses its spin connections, so we find that the accelerated reference frame has Weitzenböck torsion induced by gravitinos. In the third, our idea is that the *universality* of gravitation and inertia attribute to the single mechanism of origin from geometry but having a different nature. We have ascribed, therefore, the inertia effects to the geometry itself but as having a nature other than 4D Riemannian space. We briefly discuss a general deformation of the flat master space ($MS_p \to \widetilde{MS}_p$), in order to show that the source of graviton and gravitino is certainly this. We supplement the $\widetilde{MS}_p$-TSG theory by considering the consequences for the Newtonian limit, the uniform acceleration field and the relativistic inertial force in Minkowski and semi-Riemannian spaces. The relativistic Weak Principle of Equivalence is a consequence of the theory, at which inertial effects gradually decrease at large Lorentz factors and vanished in the photon limit. Thus, the MS_p-SUSY and $\widetilde{MS}_p$-TSG provide valuable theoretical clue for a complete revision of our ideas about the Lorentz code of motion, as well as the acceleration and inertia effects, to be now referred to as the *intrinsic* property of a particle of interest devoid of any matter influence. This is a result of the first importance for a really comprehensive entire theory of inertia. In the subsequent paper (Ter-Kazarian, 2026) our main interest still is to complement the theory of $\widetilde{MS}_p$-TSG with two more consequences. First, within the framework of $\widetilde{MS}_p$-TSG, the locality hypothesis introduces strict restrictions, replacing the curved $\widetilde{MS}_p$ with the flat MS_p. Our strategy, therefore, go beyond the hypothesis of locality to recover $\widetilde{MS}_p$ by invoking a general deformation $MS_p \to \widetilde{MS}_p$. This significantly improves the standard metric and other relevant geometrical structures referred to a noninertial frame in Minkowski spacetime for relativistic velocities and an arbitrary characteristic acceleration lengths. Second, we derive the relativistic inertial force in semi-Riemannian space, and the inertial force acting on an extended rotating body moving in Riemann-Cartan space.

This article is the first of three papers that explore the quantum mechanical inertial properties of the Dirac particle beyond the `hypothesis of locality´. This is done within the framework of the *Master Space*-Teleparallel Supergravity ($\widetilde{MS}_p$-TSG) (Ter-Kazarian, 2025) theory, which we recently proposed to account for inertial effects (Ter-Kazarian, 2026). Here we compute the object of anholonomicity and the connection defined with respect to the anholonomic frame, and based on this, we derive the Dirac equation in an accelerated and rotating frame of reference beyond the `hypothesis of locality´.

We proceed according to the following structure. To start with, in section 2 we briefly revisit the main points of going beyond the `hypothesis of locality´. In section 3 we compute the object of anholonomicity (subsect. 3.2) and the connection (subsect. 3.2) defined with respect to the anholonomic frame. On these premises, in section 4 we derive the Dirac equation in an accelerated and rotating frame of reference beyond the `hypothesis of locality´. As the concluding remarks, in section 5 , we review the key points of this report. It is worthwhile to clarify some technical details collected in Appendix. Unless indicated otherwise, the natural units, $h = c = 1$ are used throughout.

doi:https://doi.org/10.52526/25792776-25.72.2-206

2. Beyond the hypothesis of locality

We consider only mass points, then the non-inertial frame of reference in the Minkowski space of SR is represented by a curvilinear coordinate system, since it is conventionally accepted to use the names `curvilinear coordinate system´ and `non-inertial system´ interchangeably. To make this article understandable, the interested reader is referred to the original papers (Ter-Kazarian, 2024a, 2025, 2026) (see also (Ter-Kazarian, 2024b,c,d)). In addition, in this section we will briefly review the main points of the procedures of going beyond the hypothesis of locality. A notable conceptual element is the concept of 2D master space, $\mathrm{MS}_p(\equiv \underline{M}_2)$, which is a 2D Minkowski space endowed with a physical structure with its own internal geometric properties (see App.A/(1)). The MS_p embedded in background 4D Minkowski space, M_4, is the unmanifested, irreplaceable, individual companion of the particle of interest. All quantities related to the master space (flat or curved) will be underlined.

In the framework of $\widetilde{MS}_p$-TSG theory (Ter-Kazarian, 2025), the master space $\widetilde{MS}_p \equiv \underline{V}_2^{(\varrho)}$ is the 2D semi - Riemannian space, arisen from an instantaneous change in the velocity ($\underline{v}^{(\pm)}$) of a massive test particle (at *local rate* $\varrho(\underline{x}) \neq 0$) under the unbalanced external net force in non-inertial frame of reference in Minkowski space.

Consider the accelerated motion of a relativistic test particle in Minkowski 4D background flat space, M_4, under the unbalanced net force other than gravitational. As mentioned above, the hypothesis of locality assumes the equivalence of an accelerated observer and an instantaneously moving inertial observer, i.e. it links the measurements of the accelerated observer with the measurements of the inertial observer. This immediately leads to a disturbing fact within the $\widetilde{MS}_p$-TSG theory that the non-inertial reference frame $S^{(\varrho)}_{(2)}$, which is held stationary in the deformed master space $\underline{V}_2^{(\varrho)}$ ($\varrho \neq 0$), is replaced with a continuous infinity set of the inertial frames $\{S^{(0)}_{(2)}, S'^{(0)}_{(2)}, S''^{(0)}_{(2)}, ...\}$ given in $\underline{V}_2^{(\varrho)}$ ($\varrho = 0$). In other words, the hypothesis of locality leads to the 2D semi-Riemannian space, $V_2^{(0)}(\varrho = 0)$, with the incomplete metric of $\widetilde{g}$ (see (92)). Namely, it replaces the space $\widetilde{MS}_p \equiv \underline{V}_2^{(\varrho)}$ with the $\underline{V}_2^{(0)}$. Therefore, our further strategy is to consider the two-steps deformation with $\underline{V}_2 \equiv \underline{V}_2^{(0)}$ and $\underline{\mathcal{M}}_2 \equiv \underline{V}_2^{(\varrho)}$,

$$\Omega(\varrho) : \underline{M}_2 \to \underline{V}_2^{(\varrho)}, \tag{1}$$

which is composed of the two deformations as follows:

$$\begin{aligned} &\Omega : \underline{M}_2 \to \underline{V}_2^{(0)}, \\ &\widetilde{\Omega}(\varrho) : \underline{V}_2^{(0)} \to \underline{V}_2^{(\varrho)}, \end{aligned} \tag{2}$$

where the *world-deformation* tensors $\Omega(\varrho)$ and $\widetilde{\Omega}(\varrho)$ are the functions of *local rate*, $\varrho(\underline{x})$ of instantaneously change of a constant velocity (both magnitude and direction) of a massive particle in 4D Minkowski space under the unbalanced net force. Keeping in mind above said, it is worth going beyond the hypothesis of locality with special emphasis on the specific deformations (1), (2), which we might expect will essentially improve the standard results. Constructing Cartesian coordinates based on accelerated and rotating laboratory, let $\mathcal{S}(\mathcal{P})$ be the spacelike hyperplane associated to each event (point) $\mathcal{P}$ on the timelike world line at x^μ of the accelerated observer, orthogonal to it. The accelerated observer carries the orthonormal frame $e_{\hat{a}}$. Defining $\overline{x}^0 = c\overline{t} = s$ and $\overline{x}^1, \overline{x}^2, \overline{x}^3$ as Cartesian coordinates using the triad $e_{\hat{i}}(s)$ with the observer at the origin: $\overline{x}^\mu = (\overline{x}^0, \overline{x}^1, \overline{x}^2, \overline{x}^3)$ are the *local coordinates* relative to the accelerated observer. The tetrad $e_{\hat{\mu}}(s)$ can be parallel transported from $\mathcal{P}$ to all neighboring points on $\mathcal{S}(\mathcal{P})$, which defines the orthonormal tetrad field $\overline{e}_{\hat{\mu}}(\overline{x}^\nu)$. This local coordinate system is used in the laboratory, while the world line is the line of the reference clock. The tetrad field $\overline{e}_{\hat{\mu}}(\overline{x}^\nu)$ is anholonomic. Define the coordinate tetrad $\overline{e}_\mu = \overline{\partial}_\mu = \partial/\partial\overline{x}^\mu$. The orthonormal frame $e_{\hat{a}}$, carried by an accelerated observer, now can be written with respect to curvilinear or Cartesian coordinates, respectively:

$$\begin{aligned} e_{\hat{a}} &= \lambda_{(a)}{}^{\mu}\, e_\mu = \overline{\lambda}_{(a)}{}^{\mu}\, \overline{e}_\mu, \\ \vartheta^{\hat{b}} &= \lambda^{(b)}{}_{\nu}\, \vartheta^\nu = \overline{\lambda}^{(b)}{}_{\nu}\, \overline{\vartheta}^\nu, \end{aligned} \tag{3}$$

with $\vartheta^\mu = dx^\mu$, $\overline{\vartheta}^\mu = d\overline{x}^\mu$. The coframe members $\{\vartheta^{\hat{b}}\}$ are the objects of dual counterpart: $e_{\hat{a}} \rfloor \vartheta^{\hat{b}} = \delta^{\hat{b}}_{\hat{a}}$. Following (Hehl et al., 1991, Mashhoon, 2002, Misner et al., 1973), let us introduce a *geodesic* coordinate system $X^\mu(s)$, which is in general valid in a sufficiently narrow worldtube along the timelike world line of the observer. Suppose the displacement vector $\overline{x}^\mu(s)$ represents the position of the accelerated observer.

doi:https://doi.org/10.52526/25792776-25.72.2-206

According to the hypothesis of locality, at any time (s) along the accelerated world line the spacelike $\mathcal{S}(\mathcal{P})$ hyperplane orthogonal to the world line is Euclidean space and we usually describe some event on this hyperplane at x^μ to be at X^μ, where x^μ and X^μ are connected via $X^0 = s$ and

$$x^\mu = \overline{x}^\mu(s) + X^k\,\lambda_{(k)}{}^\mu(s). \tag{4}$$

This gives

$$dx^\mu = d\overline{x}^\mu(s) + dX^i\,\lambda_{(i)}{}^\mu(s) + X^i\,d\lambda_{(i)}{}^\mu(s), \tag{5}$$

where the displacement vector from the origin reads $d\overline{x}^\mu = \lambda_{(0)}{}^\mu(s)\,dX^0$. Consequently, (79) yields the standard metric of semi-Riemannian 4D background space $V_4^{(0)}$, in noninertial system of the accelerating and rotating observer, computed on the basis of hypothesis of locality (Hehl & Ni, 1990, Hehl et al., 1991) (see also (Mashhoon, 2002, 2011)):

$$\begin{aligned} ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (dX^0)^2\Big[(1+\vec{a}\cdot\vec{X})^2 \\ -(\vec{\omega}\times\vec{X})^2\Big] - 2dX^0\,d\vec{X}\cdot(\vec{\omega}\times\vec{X}) - d\vec{X}\cdot d\vec{X}. \end{aligned} \tag{6}$$

From (80) it is seen that such geodesic coordinates are admissible as long as

$$(1+\vec{a}\cdot\vec{X})^2 > (\bar{\omega}\times\vec{X})^2. \tag{7}$$

Thus in the discussion of the admissibility of the geodesic coordinates, two independent acceleration lengths must be considered: the translational acceleration length c^2/a and the rotational acceleration length c/ω that appear in equation (81). While the components of the orthonormal frame field read

$$\begin{aligned} &\lambda_{(0)}{}^0 = \tfrac{1}{1+\vec{a}\cdot\vec{X}}, \quad \lambda_{(0)}{}^k = -\tfrac{[\vec{\omega}\times\vec{X}]^k}{1+\vec{a}\cdot\vec{X}}, \\ &\lambda_{(i)}{}^j = \delta_i^j, \quad \lambda_{(i)}{}^0 = 0, \end{aligned} \tag{8}$$

and the components of the dual coframe field are

$$\begin{aligned} &\lambda^{(0)}{}_0 = (1+\vec{a}\cdot\vec{X}), \quad \lambda^{(0)}{}_i = 0, \\ &\lambda^{(i)}{}_0 = [\vec{\omega}\times\vec{X}]^i, \quad \lambda^{(i)}{}_j = \delta_j^i. \end{aligned} \tag{9}$$

All the nomenclature given above and in the previous section can be extended in a plausible way to the flat MS_p. Define the orthonormal frame, $\underline{e}_{\underline{\hat{a}}}$ $(\underline{\hat{a}} = \underline{\hat{0}}, \underline{\hat{1}})$, carried by an accelerated observer, who moves in a non-inertial frame of reference. Arbitrary curvilinear coordinates of a non-inertial frame of reference in the flat MS_p will be denoted by $\underline{x}^{\underline{\mu}}(\underline{s})$, $\underline{s} = s$ being the proper time. The components of the orthonormal frame field are $\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}} := \underline{e}_{\underline{\hat{a}}}{}^{\underline{\mu}}$, where $\underline{e}_{\underline{\hat{a}}} = \underline{e}_{\underline{\hat{a}}}{}^{\underline{\mu}}\,\underline{e}_{\underline{\mu}}$ $(\underline{e}_{\underline{\mu}} = \underline{\partial}_{\underline{\mu}} = \partial/\partial\underline{x}^{\underline{\mu}})$. The spacetime indices $\underline{\mu}, \underline{\nu}$... and $SO(1,1)$ indices $\underline{a}, \underline{b}, ...$ run from $\underline{0}$ to $\underline{1}$. The time axis must be the time axis of a comoving inertial frame in which the observer is momentarily at rest, i.e. the zeroth leg of the frame $\underline{e}_{\underline{\hat{0}}}$ be 2-velocity $\underline{u}^{\underline{\mu}}$ of the observer that is tangent to the world line at a given point $\underline{\mathcal{P}}$. The spatial frame vector $\underline{e}_{\underline{\hat{1}}}$, orthogonal to $\underline{e}_{\underline{\hat{0}}}$, is also parameterized by $(\underline{s})$. Constructing Cartesian coordinates based on laboratory, let $\underline{\mathcal{S}}(\underline{\mathcal{P}})$ be the spacelike hyperplane associated to each event (point) $\underline{\mathcal{P}}$ on the timelike world line at $\underline{x}^{\underline{\mu}}$ of the accelerated observer, orthogonal to it. Defining $\overline{\underline{x}}^0 = c\overline{\underline{t}} = \underline{s}$ and $\overline{\underline{x}}^{\underline{1}}$ as Cartesian coordinates using the $\underline{e}_{\underline{\hat{1}}}(\underline{s})$ with the observer at the origin: $\overline{\underline{x}}^{\underline{\mu}} = (\overline{\underline{x}}^0, \overline{\underline{x}}^1)$ are the *local coordinates* relative to the accelerated observer. The tetrad $\underline{e}_{\underline{\hat{\mu}}}(\underline{s})$ can be parallel transported from $\underline{\mathcal{P}}$ to all neighboring points on $\underline{\mathcal{S}}(\underline{\mathcal{P}})$, which defines the orthonormal tetrad field $\overline{\underline{e}}_{\underline{\hat{\mu}}}(\overline{\underline{x}}^{\underline{\nu}})$. The tetrad field $\overline{\underline{e}}_{\underline{\hat{\mu}}}(\overline{\underline{x}}^{\underline{\nu}})$ is anholonomic. Define the coordinate tetrad $\overline{\underline{e}}_{\underline{\mu}} = \overline{\underline{\partial}}_{\underline{\mu}} = \partial/\partial\overline{\underline{x}}^{\underline{\mu}}$. The orthonormal frame, $\underline{e}_{\underline{\hat{a}}}$, can be written with respect to curvilinear or Cartesian coordinates, respectively:

$$\begin{aligned} &\underline{e}_{\underline{\hat{a}}} = \underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}\,\underline{e}_{\underline{\mu}} = \overline{\underline{\lambda}}_{(\underline{a})}{}^{\underline{\mu}}\,\overline{\underline{e}}_{\underline{\mu}}, \\ &\underline{\vartheta}^{\underline{\hat{b}}} = \underline{\lambda}^{(\underline{b})}{}_{\underline{\nu}}\,\underline{\vartheta}^{\underline{\nu}} = \overline{\underline{\lambda}}^{(\underline{b})}{}_{\underline{\nu}}\,\overline{\underline{\vartheta}}^{\underline{\nu}}, \end{aligned} \tag{10}$$

with $\underline{\vartheta}^{\underline{\mu}} = d\underline{x}^{\underline{\mu}}$, $\overline{\underline{\vartheta}}^{\underline{\mu}} = d\overline{\underline{x}}^{\underline{\mu}}$. The coframe members $\{\underline{\vartheta}^{\underline{\hat{b}}}\}$ are the objects of dual counterpart: $\underline{e}_{\underline{\hat{a}}} \rfloor \underline{\vartheta}^{\underline{\hat{b}}} = \delta_a^b$.

Let $(\underline{X}^{\underline{\mu}}(\underline{X}^{\underline{0}}, \underline{X}^{\underline{1}})$ be *geodesic local coordinates* relative to the accelerated observer in the neighborhood of the accelerated path in MS_p, with spacetime components satisfying the embedding map

$$\begin{aligned} &d\underline{X}^{\underline{0}} = dX^0, \quad d\underline{X}^{\underline{1}} = |d\vec{X}|, \\ &\vec{n} = \tfrac{d\vec{X}}{d\underline{X}^{\underline{1}}} = \tfrac{d\vec{X}}{|d\vec{X}|}, \quad \vec{n}\cdot\vec{n} = 1. \end{aligned} \tag{11}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

Then, in view of (82) and (83), the components of the orthonormal frame field, $\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}$, read

$$\begin{aligned} &\underline{\lambda}_{(\underline{0})}{}^{\underline{0}} = \tfrac{1}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}, \quad \underline{\lambda}_{(\underline{0})}{}^{\underline{1}} = -\tfrac{[\vec{\omega}\times\vec{X}]^{\underline{1}}}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}, \\ &\underline{\lambda}_{(\underline{1})}{}^{\underline{1}} = 1, \quad \underline{\lambda}_{(\underline{1})}{}^{\underline{0}} = 0. \end{aligned} \tag{12}$$

while the components of the dual coframe field, $\underline{\lambda}^{(\underline{a})}{}_{\underline{\mu}}$, become

$$\begin{aligned} &\underline{\lambda}^{(0)}{}_{\underline{0}} = (1 + (\vec{a}\cdot\vec{X})^{\underline{1}}), \quad \underline{\lambda}^{(0)}{}_{\underline{1}} = 0, \\ &\underline{\lambda}^{(1)}{}_{\underline{0}} = [\vec{\omega}\times\vec{X}]^{\underline{1}}, \quad \underline{\lambda}^{(1)}{}_{1} = 1. \end{aligned} \tag{13}$$

The acceleration of the observer along the accelerated path, who carries an orthonormal tetrad frame $\underline{e}_{\,\hat{\underline{a}}} = (\underline{e}_{\,\hat{\underline{0}}}, \underline{e}_{\,\hat{\underline{1}}})$, therefore, can be expressed in the frame basis:

$$\frac{d\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}(\underline{s})}{d\underline{s}} = \underline{\Phi}_{(\underline{a})}{}^{(\underline{b})}(\underline{s})\,\underline{\lambda}_{(\underline{b})}{}^{\underline{\mu}}(\underline{s}), \tag{14}$$

where the inertial accelerations are represented by a second rank antisymmetric tensor $\underline{\Phi}_{(\underline{a})}{}^{(\underline{b})}(\underline{s})$ under global $SO(1,1)$ transformations. The $\underline{\Phi}_{(\underline{a})(\underline{b})}$ can be interpreted as the inertial accelerations of the frame along the timelike curve $\underline{\mathcal{C}}$ (the translational acceleration and the frequency of rotation of the frame):

$$\begin{aligned} &\underline{\Phi}_{(\underline{1})}{}^{(0)}\underline{X}^{\underline{1}} = (\vec{a}\cdot\vec{X})^{\underline{1}} = |\vec{a}\cdot\vec{X}|, \\ &\underline{\Phi}_{(\underline{1})}{}^{(\underline{1})}\underline{X}^{\underline{1}} = [\vec{\omega}\times\vec{X}]^{\underline{1}} = |\vec{\omega}\times\vec{X}|. \end{aligned} \tag{15}$$

According to the hypothesis of locality, at any time ($\underline{s}$) along the accelerated world line the spacelike $\underline{\mathcal{S}}(\underline{\mathcal{P}})$ hyperplane orthogonal to the world line is Euclidean space and we usually describe some event on this hyperplane at $\underline{x}^{\underline{\mu}}$ to be at $\underline{X}^{\underline{\mu}}$, where $\underline{x}^{\underline{\mu}}$ and $\underline{X}^{\underline{\mu}}$ are connected via $\underline{X}^{\underline{0}} = \underline{s}$ and

$$\underline{x}^{\underline{\mu}} = \underline{\overline{x}}^{\mu}(s) + \underline{X}^{\,1}\,\underline{\lambda}_{(\underline{1})}{}^{\mu}(s). \tag{16}$$

This gives

$$d\underline{x}^{\underline{\mu}} = d\underline{\overline{x}}^{\underline{\mu}}(\underline{s}) + d\underline{X}^{\,1}\,\underline{\lambda}_{(\underline{1})}{}^{\underline{\mu}}(\underline{s}) + \underline{X}^{\,1}\,d\underline{\lambda}_{(\underline{1})}{}^{\underline{\mu}}(\underline{s}), \tag{17}$$

where the displacement vector from the origin reads $d\underline{\overline{x}}^{\mu}(s) = \underline{\lambda}_{(\underline{0})}{}^{\mu}(s)\,d\underline{X}^{\,0}$. The (91) yields the metric

$$d\underline{s}^2 = g_{\underline{\mu}\underline{\nu}}\,d\underline{x}^{\underline{\mu}}d\underline{x}^{\underline{\nu}} = \underline{\vartheta}^{\underline{0}}\otimes\underline{\vartheta}^{\underline{0}} - \underline{\vartheta}^{\underline{1}}\otimes\underline{\vartheta}^{\underline{1}}. \tag{18}$$

In doing so, we calculated the orthonormal frame, $\underline{e}_{\,\hat{\underline{a}}}$, and corresponding coframe, $\underline{\vartheta}^{\hat{\underline{b}}}$ members, carried by an accelerated observer, which by virtue of (86) and (87) are equal to

$$\begin{aligned} &\underline{e}_{\hat{\underline{0}}} = \tfrac{1}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}\,\{\underline{e}_{\,\underline{0}} - [\vec{\omega}\times\vec{X}]^{\underline{1}}\,\underline{e}_{\,\underline{1}}\}, \\ &\underline{e}_{\hat{\underline{1}}} = \underline{e}_{\,\underline{1}}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} &\underline{\vartheta}^{\hat{\underline{0}}} = (1 + (\vec{a}\cdot\vec{X})^{\underline{1}})\,d\underline{X}^{\underline{0}}, \\ &\underline{\vartheta}^{\hat{\underline{1}}} = d\underline{X}^{\underline{1}} + [\vec{\omega}\times\vec{X}]^{\underline{1}}\,d\underline{X}^{\underline{0}}, \end{aligned} \tag{20}$$

respectively. Hence the metric (92) of 2D semi-Riemannian space,$V_2^{(0)}$, in noninertial system of the accelerating and rotating observer, computed on the basis of hypothesis of locality reads

$$\begin{aligned} &d\underline{s}^2 = (d\underline{X}^{\underline{0}})^2[(1 + (\vec{a}\cdot\vec{X})^{\underline{1}})^2 + (\vec{\omega}\times\vec{X})^{\underline{1}}(1- \\ &(\vec{\omega}\times\vec{X})^{\underline{1}})] - (d\underline{X}^{\underline{1}})^2 - 2d\underline{X}^{\underline{0}}d\underline{X}^{\underline{1}}[(\vec{\omega}\times\vec{X})^{\underline{1}}(1- \\ &(\vec{\omega}\times\vec{X})^{\underline{1}})]^{1/2}. \end{aligned} \tag{21}$$

Thus we see that the hypothesis of locality leads to the 2D semi-Riemannian space,$V_2^{(0)}$ with the incomplete metric (92). To recover the complete metric of $\underline{V}_2^{(\varrho)}$, therefore, our further strategy is to consider the deformation (2). The deformation tensor $\widetilde{\Omega}_{\underline{\mu}}{}^{\nu}(\varrho) = \underline{\pi}_{\underline{\mu}}{}^{\lambda}(\varrho)\,\underline{\pi}_{\underline{\lambda}}{}^{\nu}(\varrho)$, yields the deformations of linear holonomic

doi:https://doi.org/10.52526/25792776-25.72.2-206

basis. Accordingly, we must find the first deformation matrices, $\underline{\pi}(\varrho) := (\underline{\pi}_{(\hat{a})}{}^{\hat{b}})(\varrho)$, which yield the local tetrad deformations

$$\begin{array}{l} \underline{e}_{(\hat{c})} = \underline{\pi}_{(\hat{c})}{}^{\hat{a}}\, \underline{e}_{\hat{a}}, \quad \underline{\vartheta}^{(\hat{c})} = \underline{\pi}^{(\hat{c})}{}_{\hat{b}}\, \underline{\vartheta}^{\hat{b}}, \\ \underline{e}\, \underline{\vartheta} = \underline{e}_{(\hat{a})} \otimes \underline{\vartheta}^{(\hat{a})} = \Omega^{\hat{a}}{}_{\hat{b}}\, \underline{e}_{\hat{a}} \otimes \underline{\vartheta}^{\hat{b}}, \end{array} \tag{22}$$

where $\Omega^{\hat{a}}{}_{\hat{b}}(\varrho) = \underline{\pi}_{(\hat{c})}{}^{\hat{a}}(\varrho)\, \underline{\pi}^{(\hat{c})}{}_{\hat{b}}(\varrho)$ is referred to as the anholonomic *deformation tensor*. The resulting deformed metric of the space $\underline{V}_2^{(\varrho)}$ can be split as

$$g_{\tilde{\mu}\tilde{\nu}}(\varrho) = \Upsilon^2(\varrho)\, g_{\underline{\mu\nu}} + \underline{\gamma}_{\tilde{\mu}\tilde{\nu}}(\varrho), \tag{23}$$

provided

$$\begin{array}{l} \underline{\gamma}_{\tilde{\mu}\tilde{\nu}} = [\underline{\gamma}_{(\hat{a})(\hat{b})} - \Upsilon^2(\varrho)\, o_{ab}]\, \underline{e}^{(\hat{a})}{}_{\tilde{\mu}}\, \underline{e}^{(\hat{b})}{}_{\tilde{\nu}}, \\ \underline{\gamma}_{(\hat{c})(\hat{d})} = o_{ab}\, \underline{\pi}_{(\hat{c})}{}^{\hat{a}}\, \underline{\pi}_{(\hat{d})}{}^{\hat{b}}, \end{array} \tag{24}$$

where $\Upsilon(\varrho) = \underline{\pi}_{(\hat{a})}{}^{\hat{a}}(\varrho)$ and $\underline{\gamma}_{(\hat{a})(\hat{b})}(X)$ are the second deformation matrices. The complete metric in $\underline{V}_2^{(\varrho)}$, reads

$$\begin{array}{l} d\underline{\tilde{s}}^2 = g_{\tilde{\mu}\tilde{\nu}} d\underline{X}^{\tilde{\mu}} d\underline{X}^{\tilde{\nu}} = \underline{\vartheta}^{(\hat{0})}(\varrho) \otimes \underline{\vartheta}^{(\hat{0})}(\varrho) \\ -\underline{\vartheta}^{(\hat{1})}(\varrho) \otimes \underline{\vartheta}^{(\hat{1})}(\varrho), \end{array} \tag{25}$$

with the components of metric tensor $g_{\tilde{\mu}\tilde{\nu}}(\varrho)$. This equation gives the coframe members

$$\begin{array}{l} \underline{\vartheta}^{(\hat{0})}(\varrho) = \frac{b_0(\varrho)}{(1+\vec{a}\cdot\vec{X})}\, \underline{\vartheta}^{\hat{0}}, \\ \underline{\vartheta}^{(\hat{1})}(\varrho) = \frac{1}{(1+\vec{a}\cdot\vec{X})}\, [b_1(\varrho)\, \underline{\vartheta}^{\hat{1}} \\ -\left(b_2(\varrho) + b_1(\varrho)\, (\vec{\omega}\times\vec{X})^1\right)\, \underline{\vartheta}^{\hat{0}}], \end{array} \tag{26}$$

with the following notation used:

$$\begin{array}{l} b_1(\varrho) \equiv (-g_{\tilde{1}\tilde{1}})^{1/2}, \quad b_2(\varrho) = \frac{g_{\tilde{1}\tilde{0}}+g_{\tilde{0}\tilde{1}}}{2(-g_{\tilde{1}\tilde{1}})^{1/2}(\varrho)}, \\ b_0(\varrho) = (g_{\tilde{0}\tilde{0}} + b_2(\varrho)^2)^{1/2}, \\ \varrho(\tilde{s}) = \sqrt{2}\int_0^{\tilde{s}} |\vec{a}\wedge\vec{u} + \vec{\omega}\times\vec{u}|d\tilde{s}'. \end{array} \tag{27}$$

The relations $\underline{e}_{(\hat{a})} \rfloor \underline{\vartheta}^{(\hat{b})} = \delta^b_a$, give the frame members

$$\begin{array}{l} \underline{e}_{(\hat{0})}(\varrho) = b_0^{-1}(\varrho)\, \Big[(1+\vec{a}\cdot\vec{X})\, \underline{e}_{\hat{0}} \\ + \left(\frac{b_2(\varrho)}{b_1(\varrho)} + (\vec{\omega}\times\vec{X})^1\right)\, \underline{e}_{\hat{1}}\Big], \\ \underline{e}_{(\hat{1})}(\varrho) = \frac{1}{b_1(\varrho)}\, \underline{e}_{\hat{1}}. \end{array} \tag{28}$$

The elements of first deformation matrices

$$\underline{\pi}_{(\hat{a})}{}^{\hat{c}} = \underline{e}_{\hat{a}} \rfloor \underline{\vartheta}^{\hat{c}}, \quad \underline{\pi}^{(\hat{b})}{}_{\hat{c}} = \underline{\vartheta}^{(\hat{b})} \rfloor \underline{e}^{\hat{c}} \tag{29}$$

are then written

$$\begin{array}{l} \underline{\pi}_{(\hat{0})}{}^{\hat{0}}(\varrho) = \frac{1+\vec{a}\cdot\vec{X}}{b_0}, \quad \underline{\pi}_{(\hat{1})}{}^{\hat{0}}(\varrho) = 0, \\ \underline{\pi}_{(\hat{1})}{}^{\hat{1}}(\varrho) = \frac{1}{b_1}, \quad \underline{\pi}_{(\hat{0})}{}^{\hat{1}}(\varrho) = \frac{b_2+b_1(\vec{\omega}\times\vec{X})^1}{b_0 b_1}, \end{array} \tag{30}$$

and

$$\begin{array}{l} \underline{\pi}^{(\hat{0})}{}_{\hat{0}}(\varrho) = \frac{b_0}{1+\vec{a}\cdot\vec{X}}, \quad \underline{\pi}^{(\hat{0})}{}_{\hat{1}}(\varrho) = 0, \\ \underline{\pi}^{(\hat{1})}{}_{\hat{0}}(\varrho) = -\frac{b_2+b_1(\vec{\omega}\times\vec{X})^1}{1+\vec{a}\cdot\vec{X}}, \quad \underline{\pi}^{(\hat{1})}{}_{\hat{1}}(\varrho) = b_1. \end{array} \tag{31}$$

respectively. Using the following embedding relations as a converting guide:

$$\begin{array}{l} g_{\tilde{0}\tilde{0}}(d\underline{X}^{\underline{0}})^2 = g_{\tilde{0}\tilde{0}}(dX^0)^2, \quad g_{\tilde{1}\tilde{1}}(d\underline{X}^{\underline{1}})^2 \\ = g_{\tilde{1}\tilde{1}}(d\vec{X}\cdot d\vec{X}), \quad g_{\tilde{1}\tilde{0}} d\underline{X}^{\underline{1}} = g_{\tilde{i}\tilde{0}} d\underline{X}^{\underline{i}}, \\ g_{\tilde{i}\tilde{0}} = g_{\tilde{0}\tilde{i}} = n_i g_{\tilde{0}\tilde{1}} = n_i g_{\tilde{1}\tilde{0}}, \quad b_{2i} = n_i b_2, \end{array} \tag{32}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

from (30) and (31), we obtain

$$\begin{array}{ll}\pi_{(\hat{0})}{}^{\hat{0}}(\varrho)=\frac{1+\vec{a}\cdot\vec{X}}{b_0}, & \pi_{(\hat{i})}{}^{\hat{0}}(\varrho)=0,\\ \pi_{(\hat{i})}{}^{\hat{j}}(\varrho)=\frac{1}{b_1}\,\delta^j_i, & \pi_{(\hat{0})}{}^{\hat{i}}(\varrho)=\frac{b_{2i}+b_1(\vec{\omega}\times\vec{X})^i}{b_0b_1},\end{array} \tag{33}$$

and

$$\begin{array}{ll}\pi^{(\hat{0})}{}_{\hat{0}}(\varrho)=\frac{b_0}{1+\vec{a}\cdot\vec{X}}, & \pi^{(\hat{0})}{}_{\hat{i}}(\varrho)=0,\\ \pi^{(\hat{i})}{}_{\hat{0}}(\varrho)=-\frac{b_{2i}+b_1(\vec{\omega}\times\vec{X})^i}{1+\vec{a}\cdot\vec{X}}, & \pi^{(\hat{i})}{}_{\hat{j}}(\varrho)=b_1\,\delta^i_j.\end{array} \tag{34}$$

Then, by means of (26), (28), (93), (94), we obtain the generalized frame and coframe members referred to the 4D background space as follows:

$$\begin{array}{l}e_{(\hat{0})}=b_0^{-1}\left\{(1+\vec{a}\cdot\vec{X})\,e_{\hat{0}}+\left[\frac{b_{2i}}{b_1}+(\vec{\omega}\times\vec{X})^i\right]\,e_{\hat{i}}\right\},\\ e_{(\hat{i})}=b_1^{-1}\,e_{\hat{i}},\end{array} \tag{35}$$

and

$$\begin{array}{l}\vartheta^{(\hat{0})}=\frac{b_0}{1+\vec{a}\cdot\vec{X}}\,\vartheta^{\hat{0}},\\ \vartheta^{(\hat{i})}=b_1\,\vartheta^{\hat{i}}-\frac{1}{1+\vec{a}\cdot\vec{X}}\,[b_{2i}+b_1(\vec{\omega}\times\vec{X})^i]\,\vartheta^{\hat{0}},\end{array} \tag{36}$$

respectively. The orthonormal frame $e_{(\hat{a})}(\varrho)$ and coframe $\vartheta^{(\hat{b})}(\varrho)$, carried by an accelerated observer, can as well be written with respect to curvilinear coordinates:

$$e_{(\hat{a})}(\varrho)=e_{(a)}{}^{\mu}(\varrho)\,e_{\mu},\quad \vartheta^{(\hat{b})}(\varrho)=e^{(b)}{}_{\nu}(\varrho)\,\vartheta^{\nu}, \tag{37}$$

whereas

$$e_{(a)}{}^{\mu}(\varrho)=\pi_{(\hat{a})}{}^{\hat{c}}(\varrho)\,\lambda_{(c)}{}^{\mu},\quad e^{(b)}{}_{\nu}(\varrho)=\pi^{(\hat{b})}{}_{\hat{c}}(\varrho)\,\lambda^{(c)}{}_{\nu}. \tag{38}$$

The (37) and (38) give

$$e_{(\hat{0})}=b_0^{-1}(e_0+\tfrac{b_2^i}{b_1}e_i),\quad e_{(\hat{i})}=b_0^{-1}e_i, \tag{39}$$

and

$$\vartheta^{(\hat{0})}=b_0\vartheta^0,\quad \vartheta^{(\hat{i})}=b_1\left(\vartheta^i-\tfrac{b_2^i}{b_1}\vartheta^0\right)), \tag{40}$$

respectively, with the components of the orthonormal frame field and their reciprocals

$$\begin{array}{ll}e_{(\hat{0})}{}^{0}(\varrho)=b_0^{-1}, & e_{(\hat{0})}{}^{i}(\varrho)=(b_0b_1)^{-1}b_2^i,\\ e_{(\hat{i})}{}^{0}(\varrho)=0, & e_{(\hat{i})}{}^{j}(\varrho)=b_1^{-1}(\varrho)\,\delta^j_i,\end{array} \tag{41}$$

and

$$\begin{array}{ll}e^{(\hat{0})}{}_{0}(\varrho)=b_0, & e^{(\hat{0})}{}_{i}(\varrho)=0,\\ e^{(\hat{i})}{}_{0}(\varrho)=-b_{2i}, & e^{(\hat{i})}{}_{j}(\varrho)=b_1\,\delta^i_j.\end{array} \tag{42}$$

The equations (39) and (40) may be written in a more convenient way if we introduce new deformation coefficients b_3 and b_4 as follows:

$$b_3\equiv\frac{b_0}{(1+\vec{a}\cdot\vec{X})},\quad \frac{b_2^j}{b_1}\equiv-b_4v^j, \tag{43}$$

provided, $v^j=(\vec{\omega}\times\vec{X})^j$. The (39) and (40) then become

$$e_{(\hat{0})}=\frac{1}{b_3(1+\vec{a}\cdot\vec{X})}\left(e_0-b_4v^ie_i\right),\quad e_{(\hat{i})}=b_0^{-1}e_i, \tag{44}$$

and

$$\begin{array}{l}\vartheta^{(\hat{0})}=b_3(1+\vec{a}\cdot\vec{X})\vartheta^0,\\ \vartheta^{(\hat{i})}=b_1\left(\vartheta^i+b_4v^i\vartheta^0\right)).\end{array} \tag{45}$$

In the limit, $(\pi)\to 1$, of the hypothesis of locality ((82), (83)), the deformation coefficients b_1, b_3, b_4 tend to 1, and hence the (44) and (45) restore the established essential contributions by (Hehl & Ni, 1990, Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Mashhoon, 2002, 2011). Thus, we derived the

doi:https://doi.org/10.52526/25792776-25.72.2-206

tetrad fields $e_{(\hat{a})}^{\ \mu}(\varrho)$ (41) and $e^{(\hat{b})}_{\ \nu}(\varrho)$ (42) as a function of *local rate* (ϱ) of instantaneously change of a constant velocity (both magnitude and direction) of a massive particle in M_4 under the unbalanced net force, describing corresponding *fictitious graviton* in the $\widetilde{MS}_p$-TSG theory. Whereas the *fictitious gravitino* described by the Rarita-Scwinger kinetic term, $\psi_\mu^{\ \alpha}(X)$, will be arisen under infinitesimal transformations of local supersymmetry (Ter-Kazarian, 2025).

Accordingly, the complete metric in noninertial frame of arbitrary accelerating and rotating observer in Minkowski spacetime reads

$$\begin{array}{l} d\tilde{s}^2(\varrho) = g_{\mu\nu}(\varrho)dX^\mu dX^\nu = \vartheta^{(\hat{0})}\otimes\vartheta^{(\hat{0})} - \\ \vartheta^{(\hat{i})}\otimes\vartheta^{(\hat{i})} = [(\pi^{(\hat{0})}_{\ \hat{0}})^2 - (\pi^{(\hat{i})}_{\ \hat{0}})^2]\,\vartheta^{\hat{0}}\otimes\vartheta^{\hat{0}} \\ -\pi^{(\hat{i})}_{\ \hat{j}}\,\pi^{(\hat{i})}_{\ \hat{k}}\,\vartheta^{\hat{j}}\otimes\vartheta^{\hat{k}} - 2\pi^{(\hat{i})}_{\ \hat{0}}\,\pi^{(\hat{i})}_{\ \hat{j}}\,\vartheta^{\hat{0}}\otimes\vartheta^{\hat{j}} \\ = (b_0^2 - b_2^2)\vartheta^0\otimes\vartheta^0 + 2b_1 b_2^i\,\vartheta^0\otimes\vartheta^i - b_1^2\,\vartheta^i\otimes\vartheta^i \\ = [b_3^2(1+\vec{a}\cdot\vec{X})^2 - (b_1 b_4)^2(\vec{\omega}\times\vec{X})^2]\,(dX^0)^2 \\ -2b_1 b_4\,(\vec{\omega}\times\vec{X})^i\,dX^0 dX^i - b_1^2\,dX^i dX^i. \end{array} \tag{46}$$

This gives a generalization of the condition (81) of admissibility of geodesic coordinates:

$$b_3^2(1+\vec{a}\cdot\vec{X})^2 > (b_1 b_4)^2(\vec{\omega}\times\vec{X})^2. \tag{47}$$

In the limit of the hypothesis of locality, the (101) and (47) are reduced to the corresponding results of (Hehl & Ni, 1990, Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Mashhoon, 2002, 2011).

The metric (101) can be conveniently decomposed

$$g_{\mu\nu}(\varrho) = \Upsilon^2(\varrho)\,g_{\mu\nu} + \gamma_{\mu\nu}(\varrho), \tag{48}$$

provided,

$$\begin{array}{l} \gamma_{\mu\nu}(\varrho) = [\gamma_{(\hat{a})(\hat{b})} - \Upsilon^2(\varrho)\,o_{((a)(b))}]\,e^{(\hat{a})}_{\ \mu}\,e^{(\hat{b})}_{\ \nu}, \\ \gamma_{(\hat{c})(\hat{d})} = o_{(a)(b)}\,\pi_{(\hat{c})}^{\ \ \hat{a}}\,\pi_{(\hat{d})}^{\ \ \hat{b}}, \end{array} \tag{49}$$

where $\Upsilon(\varrho) = \pi_{(\hat{a})}^{\ \ \hat{a}}(\varrho)$, and $\gamma_{(\hat{c})(\hat{d})}$ are the second deformation matrices, with the elements

$$\begin{array}{l} \Upsilon(\varrho) = b_3^{-1} + b_1^{-1}, \\ \gamma_{(\hat{0})(\hat{0})} = b_3^{-2}\left[1 - \frac{(1+b_1 b_4)^2}{(1+\vec{a}\cdot\vec{X})^2}\,\vec{v}^2\right], \\ \gamma_{(\hat{0})(\hat{i})} = \gamma_{(\hat{i})(\hat{0})} = \frac{b_4}{b_3(1+\vec{a}\cdot\vec{X})}\,v^i, \\ \gamma_{(\hat{i})(\hat{j})} = -b_1^{-2}\,\delta_{ij}. \end{array} \tag{50}$$

3. The object of anholonomicity and the connection

3.1. The computation of all commutators in standard case

In standard case, we are given the anholonomic frame $e_{(\hat{\mu})}$, $(\hat{\mu} = \hat{0}, \hat{1}, \hat{2}, \hat{3})$

$$e_{(\hat{0})} = \tfrac{1}{1+\vec{a}\cdot\vec{X}}(e_0 - [\vec{\omega}\times\vec{X}]^k)e_i), \quad e_{(\hat{i})} = e_i,$$

where $e_0 = \partial/\partial X^0$ and $e_i = \partial/\partial X^i\,(i = 1,2,3)$, and commutator

$$[e_{(\hat{0})}, e_{(\hat{i})}] = C_{(\hat{0})(\hat{i})}^{\ \ \ \ \ (\hat{\lambda})}e_{(\hat{\lambda})},$$

with the object of anholonomicity, $C_{(\hat{\mu})(\hat{\nu})}^{\ \ \ \ \ (\hat{\lambda})}$, of the commutation table. Denote $\gamma \equiv \frac{1}{1+\vec{a}\cdot\vec{X}}$. Write explicitly

$$e_{(\hat{0})} = \gamma\left(\partial_0 - (\vec{\omega}\times\vec{X})^k\partial_k\right) = \gamma\partial_0 - \gamma(\vec{\omega}\times\vec{X})^k\partial_k.$$

Then

$$[e_{(\hat{0})}, e_{(\hat{i})}] = [\gamma\partial_0 - \gamma(\vec{\omega}\times\vec{X})^k\partial_k, \partial_i].$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

We use the identity

$$[\partial_\mu, \partial_\nu] = -(\partial_\nu f(x))\partial_\mu.$$

The proof is as follows: denote $V = f\partial_\mu, \quad W = \partial_\nu$, and consider their commutator acting on a scalar function g

$$[V, W](g) = V(W(g)) - W(V(g)).$$

Compute

$$V(W(g)) = (f\partial_\mu)(\partial_\nu g) = f\partial_\mu\partial_\nu g,$$
$$W(V(g)) = \partial_\nu(f\partial_\mu g) = (\partial_\nu f)\partial_\mu g + f\partial_\nu\partial_\mu g,$$

and subtract to find the commutator

$$[V, W](g) = f\partial_\mu\partial_\nu g - ((\partial_\nu f)\partial_\mu g + f\partial_\nu\partial_\mu g)\,[V, W].$$

Note that the second-order derivatives cancel

$$[V, W](g) = -(\partial_\nu f)\partial_\mu g,$$

and the commutator as a differential operator is

$$[V, W] = -(\partial_\nu f)\partial_\mu.$$

So we obtain

$$[\gamma\partial_0, \partial_i] = -(-\gamma^2 a_i)\partial_0 = \gamma^2 a_i \partial_0.$$

Denote

$$f^k := \gamma(\vec{\omega} \times \vec{X})^k = \gamma\epsilon^k{}_{mn}\omega^m X^n.$$

Then

$$[\gamma(\vec{\omega} \times \vec{X})^k\partial_k, \partial_i] = [f^k\partial_k, \partial_i] = -(\partial_i f^k)\partial_k = (\partial_i\gamma)(\vec{\omega} \times \vec{X})^k + \gamma\partial_i(\vec{\omega} \times \vec{X})^k.$$

We already have $\partial_i\gamma = -\gamma^2 a_i$, and $(\vec{\omega} \times \vec{X})^k = \epsilon^k{}_{mn}\omega^m X^n \Rightarrow \partial_i(\vec{\omega} \times \vec{X})^k = \epsilon^k{}_{mi}\omega^m$. So

$$\partial_i f^k = -\gamma^2 a_i \epsilon^k{}_{mn}\omega^m X^n + \gamma\epsilon^k{}_{mi}\omega^m.$$

Then the commutator is

$$-[\gamma(\vec{\omega} \times \vec{X})^k\partial_k, \partial_i] = (\partial_i f^k)\partial_k = \left(-\gamma^2 a_i \epsilon^k{}_{mn}\omega^m X^n + \gamma\epsilon^k{}_{mi}\omega^m\right)\partial_k.$$

Combining both terms, we obtain

$$[e_{(\hat{0})}, e_{(\hat{i})}] = \gamma^2 a_i\partial_0 - \gamma^2 a_i(\vec{\omega} \times \vec{X})^k\partial_k + \gamma\epsilon^k{}_{mi}\omega^m\partial_k.$$

Rewrite the commutator in the $e_{(\hat{\mu})}$ basis. Now substitute

$$\partial_0 = \gamma^{-1}e_{(\hat{0})} + (\vec{\omega} \times \vec{X})^j e_{(\hat{j})}, \quad \partial_k = e_{(\hat{k})}.$$

Compute each term

Term 1:

$$\gamma^2 a_i\partial_0 = \gamma^2 a_i\left(\gamma^{-1}e_{(\hat{0})} + (\vec{\omega} \times \vec{X})^j e_{(\hat{j})}\right)$$
$$= \gamma a_i e_{(\hat{0})} + \gamma^2 a_i(\vec{\omega} \times \vec{X})^j e_{(\hat{j})}.$$

Term 2:

$$-\gamma^2 a_i(\vec{\omega} \times \vec{X})^k\partial_k = -\gamma^2 a_i(\vec{\omega} \times \vec{X})^k e_{(\hat{k})}.$$

Term 3:

$$\gamma\epsilon^k{}_{mi}\omega^m\partial_k = \gamma\epsilon^k{}_{mi}\omega^m e_{(\hat{k})}.$$

Now combine all

$$[e_{(\hat{0})}, e_{(\hat{i})}] = \gamma a_i e_{(\hat{0})} + \gamma^2 a_i(\vec{\omega} \times \vec{X})^j e_{(\hat{j})}$$
$$-\gamma^2 a_i(\vec{\omega} \times \vec{X})^k e_{(\hat{k})} + \gamma\epsilon^k{}_{mi}\omega^m e_{(\hat{k})}.$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

Note that the second and third terms cancel each other, as they are equal and opposite. So we're left with

$$e_{(\hat{0})}, e_{(\hat{i})}] = \gamma a_i e_{(\hat{0})} + \gamma \epsilon^k{}_{mi}\omega^m e_{(\hat{k})}.$$

Reading off the structure coefficients, compare

$$[e_{(\hat{0})}, e_{(\hat{i})}] = C_{(\hat{0})(\hat{i})}^{(\hat{\lambda})} e_{(\hat{\lambda})},$$

which gives the non-zero components of the object of anholonomicity

$$C_{(\hat{0})(\hat{i})}^{(\hat{0})} = \gamma a_i = \frac{a_i}{1+\vec{a}\cdot\vec{X}},$$
$$C_{(\hat{0})(\hat{i})}^{(\hat{k})} = \gamma \epsilon^k{}_{mi}\omega^m = \frac{\epsilon^k{}_{mi}\omega^m}{1+\vec{a}\cdot\vec{X}}.$$

By antisymmetry of the Lie bracket, we also have $C_{(\hat{i})(\hat{0})}^{(\hat{\lambda})} = -C_{(\hat{0})(\hat{i})}^{(\hat{\lambda})}$. All other components vanish. Since

$$a_i = g_{ij}a^j = -\delta_{ij}a^j = -a^i,$$

then

$$C_{(\hat{0})(\hat{i})}^{(\hat{0})} = -\frac{a^i}{1+\vec{a}\cdot\vec{X}} = -C_{(\hat{i})(\hat{0})}^{(\hat{0})},$$

and

$$C_{(\hat{0})(\hat{i})}^{(\hat{k})} = g_{kl}\frac{\epsilon^l{}_{mi}\omega^m}{1+\vec{a}\cdot\vec{X}} = -\frac{\epsilon_{kmi}\omega^m}{1+\vec{a}\cdot\vec{X}} = -\frac{\epsilon_{ikm}\omega^m}{1+\vec{a}\cdot\vec{X}} = \frac{\epsilon_{kim}\omega^m}{1+\vec{a}\cdot\vec{X}}.$$

3.2. The general derivation of the coefficients $C_{(\hat{\mu})(\hat{\nu})}^{\lambda}$

Rewrite frame vectors in terms of coordinate basis

$$e_{(\hat{0})} = b_0^{-1}\left(e_0 - \bar{v}^k e_k\right) = b_0^{-1}\left(\partial_0 - \bar{v}^k\partial_k\right),$$
$$e_{(\hat{i})} = b_1^{-1}e_i = b_1^{-1}\partial_i.$$

Compute commutators $[e_{(\hat{\mu})}, e_{(\hat{\nu})}]$.

Since the coordinate basis vectors ∂_μ commute $[\partial_\mu, \partial_\nu] = 0$. We will use the identity

$$[f(x)A, B] = f[A,B] + (Af)B - (Bf)A.$$

We compute

$$[e_{(\hat{0})}, e_{(\hat{i})}] = \left[\frac{1}{b_0}(\partial_0 - \bar{v}^k\partial_k), \frac{1}{b_1}\partial_i\right]$$
$$= -\frac{1}{b_0b_1}(\partial_i\bar{v}^k)\partial_k - \frac{1}{b_0}\left(\bar{v}^k\partial_k\left(\frac{1}{b_1}\right)\right)\partial_i - \left(\partial_i\left(\frac{1}{b_0}\right)\right)\frac{1}{b_1}(\partial_0 - \bar{v}^k\partial_k).$$

Now write this in terms of frame vectors $e_{(\hat{\mu})}$. Term-by-term conversion is

First term-

$$-\frac{1}{b_0b_1}(\partial_i\bar{v}^k)\partial_k = -\frac{1}{b_0b_1}(\partial_i\bar{v}^k)b_1e_{(\hat{k})} = -\frac{1}{b_0}(\partial_i\bar{v}^k)e_{(\hat{k})}.$$

Second term-

$$-\frac{1}{b_0}\left(\bar{v}^k\partial_k\left(\frac{1}{b_1}\right)\right)\partial_i = -\frac{1}{b_0}\left(\bar{v}^k\partial_k\left(\frac{1}{b_1}\right)\right)b_1e_{(\hat{i})}.$$

Note

$$\partial_k\left(\frac{1}{b_1}\right) = \bar{v}^k\left(-\frac{1}{b_1^2}\partial_k b_1\right) = -\frac{1}{b_1^2}\bar{v}^k\partial_k.$$

So the coefficient becomes

$$\left(-\frac{1}{b_1^2}\bar{v}^k\partial_k b_1\right)b_1 = \frac{1}{b_0b_1}\bar{v}^k\partial_k b_1,$$

and the second term is written

$$\bar{v}^k\partial_k b_1 \cdot e_{(\hat{i})}.$$

Third term-

$$-\left(\partial_i\left(\frac{1}{b_0}\right)\right)\frac{1}{b_1}(\partial_0 - \bar{v}^k\partial_k) = -\left(\partial_i\left(\frac{1}{b_0}\right)\right)\frac{1}{b_1}b_0e_{(\hat{0})}.$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

Now

$$\left(\frac{1}{b_0}\right) = -\frac{1}{b_0^2}\partial_i b_0.$$

So

$$-\left(-\frac{1}{b_0^2}\partial_i b_0\right)\frac{1}{b_1}b_0 = \frac{1}{b_0 b_1}\partial_i b_0,$$

and third term is

$$\frac{1}{b_0 b_1}\partial_i b_0 \cdot e_{(\hat{0})}.$$

Hence

$$[e_{(\hat{0})}, e_{(\hat{i})}] = \frac{1}{b_0 b_1}\partial_i b_0 \cdot e_{(\hat{0})} + \frac{1}{b_0 b_1}\bar{v}^k \partial_k b_1 \cdot e_{(\hat{i})} - \frac{1}{b_0}(\partial_i \bar{v}^k) e_{(\hat{k})}.$$

Summary of non-zero anholonomicity coefficients is

$$\begin{array}{ll} C_{(\hat{0})(\hat{i})}^{\quad(\hat{0})} = \frac{1}{b_0 b_1}\partial_i b_0 & C_{(\hat{0})(\hat{i})}^{\quad(\hat{i})} = \frac{1}{b_0 b_1}\bar{v}^k \partial_k b_1 \\ C_{(\hat{0})(\hat{i})}^{\quad(\hat{k})} = -\frac{1}{b_0}\partial_i \bar{v}^k, & \end{array}$$

with antisymmetry: $C_{(\hat{i})(\hat{0})}^{\quad(\hat{\lambda})} = -C_{(\hat{0})(\hat{i})}^{\quad(\hat{\lambda})}$.

Now compute

$$[e_{(\hat{i})}, e_{(\hat{j})}] = \left[\frac{1}{b_1}\partial_i, \frac{1}{b_1}\partial_j\right].$$

Let $f = \frac{1}{b_1}$, so

$$e_{(\hat{j})}] = [f\partial_i, f\partial_j].$$

Use the identity

$$[fA, fB] = f^2[A, B] + f(Af)B - f(Bf)A.$$

Apply it

$$[f\partial_i, f\partial_j] = f^2[\partial_i, \partial_j] + f(\partial_i f)\partial_j - f(\partial_j f)\partial_i.$$

Since $[\partial_i, \partial_j] = 0$, we get

$$[e_{(\hat{i})}, e_{(\hat{j})}] = f(\partial_i f)\partial_j - f(\partial_j f)\partial_i.$$

We now summarize all non-zero components $C_{(\hat{\mu})(\hat{\nu})}^{\quad(\hat{\lambda})}$ of the anholonomicity object.

From the commutators

$$[e_{(\hat{0})}, e_{(\hat{i})}] = +\frac{1}{b_1}\partial_i \ln b_0 \cdot e_{(\hat{0})} - \left(\bar{v}^k \partial_k b_1^{-1}\right) e_{(\hat{i})} + \frac{1}{b_0}\partial_i \bar{v}^k \cdot e_{(\hat{k})}.$$

and

$$[e_{(\hat{i})}, e_{(\hat{j})}] = (\partial_j \ln b_1)\, e_{(\hat{i})} - (\partial_i \ln b_1)\, e_{(\hat{j})},$$

we obtain

$$\begin{array}{l} C_{(\hat{0})(\hat{i})}^{\quad(\hat{0})} = +\frac{1}{b_1}\partial_i \ln b_0 \\ C_{(\hat{0})(\hat{i})}^{\quad(\hat{i})} = -\bar{v}^k \partial_k b_1^{-1} \\ C_{(\hat{0})(\hat{i})}^{\quad(\hat{k})} = +\frac{1}{b_0}\partial_i \bar{v}^k \\ C_{(\hat{i})(\hat{j})}^{\quad(\hat{i})} = \partial_j \ln b_1, \quad C_{(\hat{i})(\hat{j})}^{\quad(\hat{j})} = -\partial_i \ln b_1. \end{array}$$

Lowering the upper index by a metric $o_{(\hat{\rho})(\hat{\lambda})}$, by using an orthogonal basis $o = (diag{+1}, -1, -1, -1)$, we summarize all non vanishing components $C_{(\hat{\mu})(\hat{\nu})(\hat{\lambda})}$ of the anholonomicity as follows:

$$\begin{array}{l} C_{(\hat{0})(\hat{i})(\hat{0})} = -C_{(\hat{i})(\hat{0})(\hat{0})} = \frac{1}{b_1^{-1}}\partial_i \ln b_0, \\ C_{(\hat{0})(\hat{i})(\hat{j})} = -C_{(\hat{i})(\hat{0})(\hat{j})} = -\frac{\partial_i v_j}{b_0} + \vec{v}\cdot(\nabla b_1^{-1})\delta_{ij}, \\ C_{(\hat{i})(\hat{j})(\hat{k})} = -C_{(\hat{j})(\hat{i})(\hat{k})} = -(\partial_i b_1^{-1})\delta_{jk} \\ +(\partial_j b_1^{-1})\delta_{ik}, \quad C_{(\hat{i})(\hat{j})(\hat{0})} = C_{(\hat{\mu})(\hat{\nu})(\hat{\lambda})} = 0. \end{array} \tag{51}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

3.3. The connection components $\Gamma_{(\hat\lambda)(\hat\nu)(\hat\mu)}$

We compute the connection components, $\Gamma_{(\hat\lambda)(\hat\nu)(\hat\mu)}$, defined with respect to the anholonomic frame (**??**), from the given structure coefficients (51) and the formula

$$\Gamma_{(\hat\lambda)(\hat\nu)(\hat\mu)} = \frac{1}{2}\left(C_{(\hat\lambda)(\hat\nu)(\hat\mu)} + C_{((\hat\lambda)(\hat\mu))(\hat\nu)} - C_{(\hat\nu)(\hat\mu)(\hat\lambda)}\right).$$

It is straightforward to calculate the connection coefficients

$$\begin{array}{l}
\Gamma_{(\hat 0)(\hat i)(\hat 0)} = -\Gamma_{(\hat i)(\hat 0)(\hat 0)} = \frac{1}{b_1^{-1}}\partial_i \ln b_0, \\
\Gamma_{(\hat i)(\hat j)(\hat 0)} = \frac{1}{2b_0}(\partial_i \bar v_j - \partial_j \bar v_i) \\
= \frac{b_4}{b_0}\partial_i v_j + \frac{1}{2b_0}\left[(\partial_i b_4)v_j - (\partial_j b_4)v_i\right], \\
\Gamma_{(\hat i)(\hat j)(\hat k)} = -(\partial_i b_1^{-1})\delta_{jk} + (\partial_j b_1^{-1})\delta_{ik}, \\
\Gamma_{(\hat 0)(\hat i)(\hat j)} = -\Gamma_{(\hat i)(\hat 0)(\hat j)} = -\frac{1}{2b_0}(\partial_i \bar v_j + \partial_j \bar v_i) \\
+\vec{\bar v}\cdot(\nabla b_1^{-1})\delta_{ij} = -\frac{1}{2b_0}\left[(\partial_i b_4)v_j + (\partial_j b_4)v_i\right] \\
+\vec{\bar v}\cdot(\nabla b_1^{-1})\delta_{ij}, \quad \Gamma_{(\hat 0)(\hat 0)(\hat 0)} = \Gamma_{(\hat\mu)(\hat\nu)(\hat i)} = 0.
\end{array} \tag{52}$$

In the limit of the hypothesis of locality: $(\pi) \to 1$, the deformation coefficients $b_1, b_3 \equiv b_0/(1+\vec a\cdot\vec X), b_4$ tend to 1, and hence the (51) and (52) restore the standard contributions by Hehl et al. (1991). Actually,

$$\begin{array}{l}
\Gamma_{(\hat 0)(\hat i)(\hat 0)} = C_{(\hat 0)(\hat i)(\hat 0)} \\
\to\ \Gamma_{\hat 0\,\hat i\,\hat 0} = C_{\hat 0\,\hat i\,\hat 0} = \frac{a_i}{(1+\vec a\cdot\vec X)}.
\end{array} \tag{53}$$

Since $[a_i = g_{ij}a^j = -\delta_{ij}a^j = -a^i$, then

$$\Gamma_{\hat 0\,\hat i\,\hat 0} = -\Gamma_{\hat i\,\hat 0\,\hat 0} = -\frac{a^i}{(1+\vec a\cdot\vec X)}. \tag{54}$$

Next,

$$\begin{array}{l}
\Gamma_{(\hat 0)(\hat i)(\hat j)} \to \Gamma_{\hat 0\,\hat i\,\hat j} = \frac{\partial_i v_j}{(1+\vec a\cdot\vec X)} = o_{jk}\frac{\partial_i v^k}{(1+\vec a\cdot\vec X)} \\
= o_{jk}\frac{\varepsilon^k{}_{mi}\omega^m}{(1+\vec a\cdot\vec X)} = -\frac{\varepsilon_{mij}\omega^m}{(1+\vec a\cdot\vec X)} = -\frac{\varepsilon_{ijm}\omega^m}{(1+\vec a\cdot\vec X)}.
\end{array} \tag{55}$$

All other components vanish

$$\Gamma_{\hat\mu\,\hat\nu\,\hat i} = \Gamma_{\hat 0\,\hat 0\,\hat 0} = 0.$$

4. Dirac equation in an accelerated and rotating frame beyond the hypothesis of locality

In the Minkowski spacetime of SR in Cartesian coordinates $\overline{x}^{\mu'} = (\overline{x}^0, \overline{x}^1, \overline{x}^2, \overline{x}^3)$, the Dirac equation for a massive fermion reads

$$i\gamma^{\mu'}\overline{\partial}_{\mu'}\Psi' = m\Psi', \tag{56}$$

with the Dirac matrices $\gamma^{\mu'}$. From now on we will essentially follow Bjorken-Drell (Bjorken & Drell, 1964), in particular we use its conventions for the Dirac matrices: $\gamma^0 = \beta$ and $\gamma^i = \beta\alpha^i$. In its most fundamental form, the Dirac equation in a locally accelerated and rotating frame of reference of the observer, obtained from first principles, is a generalization of the equation (56):

$$i\gamma^{\hat\mu}D_{(\hat\mu)}\Psi = m\Psi, \tag{57}$$

where the anholonomic Dirac matrices are defined by $\gamma^{\hat\mu} := e^{\hat\mu}{}_\nu\gamma^\nu$, and $\gamma^{\hat\mu}\gamma^{\hat\nu} + \gamma^{\hat\nu}\gamma^{\hat\mu} = 2o^{\hat\mu\hat\nu}$. The partial derivative in the Dirac equation is simply replaced by the covariant derivative

$$D_{(\hat\mu)} := \partial_{(\hat\mu)} + \Gamma_{(\hat\mu)}, \tag{58}$$

where the quantities $\Gamma_{(\hat\mu)}$ are related to the connection coefficients

$$\Gamma_{(\hat\mu)} := -\frac{i}{4}\sigma^{\hat\lambda\hat\nu}\Gamma_{(\hat\lambda)(\hat\nu)(\hat\mu)}, \tag{59}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

with the six matrices $\sigma^{\hat\lambda\hat\nu}$ of the infinitesimal generators of the Lorentz group

$$\sigma^{\hat\lambda\hat\nu} := \tfrac{i}{2}[\gamma^{\hat\lambda}, \gamma^{\hat\nu}]. \tag{60}$$

After calculation, by virtue of (52), and simplification of (59), we find

$$\begin{array}{l}\Gamma_{(\hat 0)} = -\frac{1}{2}\vec a_1 \cdot \alpha - \frac{i}{2}\vec\omega_1 \cdot \vec\sigma, \\ \Gamma_{(\hat k)} = \vec a_k \cdot \vec\alpha + i\,\vec\omega_k \cdot \vec\sigma,\end{array} \tag{61}$$

provided,

$$\begin{array}{l}\vec a_1 \equiv \frac{1}{b_1}(\nabla \ln b_0), \\ \vec\omega_1 \equiv \frac{1}{b_0}\left[b_4\vec\omega - \frac{1}{2}(\nabla \ln b_4)\times\vec v\right], \\ a_{ik} = a_{ki} \equiv \frac{1}{4b_0}\left[(\partial_i b_4)v_k + (\partial_k b_4)v_i\right] \\ -\frac{1}{2}(\vec v\cdot\nabla b_1^{-1})\delta_{ik}, \\ \omega_{ik} = -\omega_{ki} \equiv \frac{1}{2}\varepsilon_{kli}(\partial_i b_1^{-1})),\end{array} \tag{62}$$

where ε_{ijk} is the three-dimensiona1 Levi-Civita symbol with $\varepsilon_{123} = 1$. Collecting (58) and (61) together, we find for the deformed spinor covariant derivatives

$$\begin{array}{l}D_{(\hat 0)} = \frac{1}{b_0}\left\{\frac{\partial}{\partial X^0} - \frac{1}{2b_1}\nabla b_0\cdot\vec\alpha - i\,\vec\omega\cdot\vec L\right. \\ \left.-i\,\vec\omega_S\cdot\vec S\right\}, \quad \omega_S \equiv b_4\vec\omega - \frac{1}{2}\nabla(b_4)\times\vec v, \\ D_{(\hat i)} = b_1^{-1}\frac{\partial}{\partial X^i} + \vec a_i\cdot\vec\alpha + i\,\vec\omega_i\cdot\vec\sigma,\end{array} \tag{63}$$

where the orbital ($\vec L$) and spin ($\vec S$) operators respectively have the form

$$\vec L \equiv (\vec X \times \frac{\partial}{i\partial\vec X}) = (\vec X\times\vec p), \quad \vec S \equiv \tfrac{1}{2}\vec\sigma. \tag{64}$$

In the standard limit, $(\pi)\to 1$, of the hypothesis of locality, the deformation coefficients b_1, b_3, b_4, tend to 1, so that the (63) and (64) restore the results of (Hehl & Ni, 1990):

$$\begin{array}{l}\vec J \to \vec J = \vec L + \vec S \equiv (\vec X\times\frac{\partial}{i\partial\vec X}) + \frac{1}{2}\vec\sigma, \\ D_{(\hat 0)} \to D_{\hat 0} = \frac{1}{(1+\vec a\cdot\vec X)}\left(\frac{\partial}{\partial X^0} - \vec v\cdot\frac{\partial}{\partial\vec X} + \frac{1}{2}\vec a\cdot\vec\alpha\right. \\ \left.-i\,\vec\omega\cdot\vec J\right), \quad D_{(\hat i)} \to D_{\hat i} = \frac{\partial}{\partial X^i}.\end{array} \tag{65}$$

Substituting (63) into (57) and multiplying it by $\gamma^0\beta b_0$, we obtain the explicit form of the Dirac equation beyond the hypothesis of locality for an observer in a reference frame that is accelerated with a proper linear 3-acceleration $\vec a$ and rotating with proper 3-angular velocity $\vec\omega$:

$$\begin{array}{l}\left\{i\partial_0 - i\frac{1}{2b_1}(\nabla b_0\cdot\vec\alpha) + \vec\omega\cdot\vec L + \vec\omega_S\cdot\vec S\right. \\ -b_0 b_1^{-1}(\vec\alpha\cdot\vec p) + \frac{i}{2}(\nabla b_4 - 3b_4 b_0\nabla b_1^{-1})\cdot\vec v \\ \left.-ib_0(\vec\alpha\cdot\nabla b_1^{-1})\right\}\Psi = b_0\beta m\Psi.\end{array} \tag{66}$$

Here we employed the following intermediate calculations. Using the Clifford algebra of Dirac matrices $\{\alpha_j, \alpha_k\} = 2\delta_{jk}I$, we obtain

$$\begin{array}{l}\alpha^i\vec a_i\cdot\vec\alpha = a_{ii} = (\frac{1}{2b_0}\nabla b_4 - \frac{3}{2}b_4\nabla b_1^{-1})\cdot\vec v, \\ \hat T \equiv i\,\vec\omega_k\cdot\vec\sigma = \frac{1}{2}\begin{pmatrix}0 & T\\ T & 0\end{pmatrix}, \\ T = \varepsilon^{jli}(\delta^{ij}I + i\varepsilon^{ijk}\sigma^k)\partial_l b_1^{-1} \\ = i\varepsilon^{jli}\varepsilon^{ijk}\sigma^k\partial_l b_1^{-1}.\end{array} \tag{67}$$

The identity

$$\varepsilon^{jli}\varepsilon^{ijk} = \delta^{li}\delta^{ik} - \delta^{lk}\delta^{ii}, \tag{68}$$

gives $\varepsilon^{jli}\varepsilon^{ijk} = \delta^{lk} - 3\delta^{lk} = -2\delta^{lk}$, such that

$$T = -2i\delta^{lk}\sigma^k\partial_l b_1^{-1} = -2i\sigma^l\partial_l b_1^{-1}, \tag{69}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

and

$$\hat{T} = -\vec{\alpha} \cdot \nabla b_1^{-1}. \tag{70}$$

Hence, the Dirac equation (66) can be recast into the form

$$i\partial_0 \Psi = H\Psi, \tag{71}$$

with the deformed Dirac Hamiltonian

$$\begin{array}{l} H = b_3 \beta m(1 + \vec{a} \cdot \vec{X}) + \frac{b_3}{b_1}\, \vec{\alpha} \cdot \vec{p} - \vec{\omega} \cdot \vec{L} \\ -\vec{\omega}_S \cdot \vec{S} + \frac{i}{2}(\vec{a}_2 \cdot \vec{\alpha} + a_3) + \frac{b_3}{2b_1} \Big[(\vec{a} \cdot \vec{X})(\vec{p} \cdot \vec{\alpha}) \\ +(\vec{p} \cdot \vec{\alpha})(\vec{a} \cdot \vec{X})\Big]. \end{array} \tag{72}$$

where $\vec{a}_2 \equiv (\frac{1}{b_1}\nabla b_0 + 2b_0 \nabla b_1^{-1})$, and $a_3 \equiv [3b_4 b_0 \nabla b_1^{-1} \ -\nabla b_4] \cdot \vec{v}$. The Dirac Hamiltonian (72) can be conveniently rewritten

$$\begin{array}{l} H = w_1 \beta m(1 + \vec{a} \cdot \vec{X}) + w_3\, \vec{\alpha} \cdot \vec{p} - \vec{\omega} \cdot \vec{L} \\ -w_2\, \vec{\omega} \cdot \vec{S} + \frac{i}{2}(w_4 \vec{a} \cdot \vec{\alpha} + w_5) + \frac{w_3}{2} \Big[(\vec{a} \cdot \vec{X})(\vec{p} \cdot \vec{\alpha}) \\ +(\vec{p} \cdot \vec{\alpha})(\vec{a} \cdot \vec{X})\Big], \end{array} \tag{73}$$

provided, $w_1 \equiv b_3, \quad w_2\, \vec{\omega} \equiv \vec{\omega}_S, \quad w_3 \equiv \frac{b_3}{b_1}, w_4 \vec{a} \equiv \vec{a}_2, w_5 \equiv a_3$. The coefficients $w_i(X)(i = 1, 2, 3, 4, 5)$ are time-independent scalar functions of X, $(w_i(X) \in \mathbb{R}^3)$.

Let's briefly interpret the physical meaning of each term before relating this to curved spacetime, see e.g. (Brill & Wheeler, 1957, Hehl & Ni, 1990, Hehl et al., 1991, Maluf & Faria, 2008, Maluf et al., 2007, Mashhoon, 2002, 2011, Parker & Toms, 2009). Mass term, $w_1 \beta m(1 + \vec{a} \cdot \vec{X})$, represents a gravitational redshift of mass energy in a weak field. This matches the Tolman redshift.

Orbital term, $\vec{\omega} \cdot (\vec{X} \times \vec{p})$, describes Coriolis force and orbital coupling in rotating frames. This is effective term of energy shift due to rotation. In general relativity, this is the Lense–Thirring effect — frame dragging by rotating mass.

Spin-Rotation Coupling: $w_2\, \vec{\omega} \cdot \vec{\sigma}$. This is the Mashhoon effect. This is spin experiences torque in rotating frames, which is analogous to magnetic dipole in a magnetic field. Can be derived from the spin connection ω_μ^{ab} in curved spacetime: spin term $\sim \frac{1}{4}\gamma^a \gamma^b \omega_{ab\mu}$.

The symmetrized inertial boost term,
$\frac{w_3}{2} \Big[(\vec{a} \cdot \vec{X})(\vec{p} \cdot \vec{\alpha}) + (\vec{p} \cdot \vec{\alpha})(\vec{a} \cdot \vec{X})\Big]$, arises from a non-inertial, accelerated, frame and reflects: boosts in the Dirac equation, and the non-trivial tetrad field structure. This term can be derived from the Fermi normal coordinates for a uniformly accelerated observer.

Residual imaginary terms are artifacts that due to coordinate transformations in the non-inertial frames. These are usually negligible under the assumption that b_1, b_3, b_4 vary slowly with $\vec{X}$, so their gradients are small, and these corrections are suppressed. Moreover, the standard method of similarity transformation of the Hamiltonian allows one to choose a physically more suitable reference frame (see next sect.). The expectation values of physical observables remain real. No imaginary contamination remains in physical quantities. Thus the energy, momentum, probability, etc. remain real and consistent.

5. Concluding remarks

In this section we briefly reflect upon the main points of this report. This is the first of three papers that explore the possibility of quantum mechanical inertial properties of the Dirac particle beyond the `hypothesis of locality´. This is done within the framework of the *Master Space*-Teleparallel Supergravity ($\widetilde{MS}_p$-TSG) (Ter-Kazarian, 2025) theory, which we recently proposed taking into account inertial effects (Ter-Kazarian, 2026). The `hypothesis of locality´ used for extension of the Lorentz invariance to accelerated observers within the Special Relativity, in effect bypasses acceleration and replaces the accelerated observer by a continuous infinity of hypothetical momentarily comoving inertial observers along its wordline. Despite the successes for the tiny accelerations we usually experience, when the curvature of the wordline could be ignored and that the differences between observations by accelerated and comoving inertial observers will also be very small, however, the basic conceptual framework of this assumption has been considered by many scientists to be unsatisfactory. In general case, this is actually untenable and

doi:https://doi.org/10.52526/25792776-25.72.2-206

represents strict restrictions, and that the hypothesis of locality will have to be extended to describe physics for arbitrarily accelerated observers. This immediately leads to the disturbing fact within the $\widetilde{MS}_p$-TSG theory that the metric of a two-dimensional semi-Riemannian space, calculated in the non-inertial frame of reference of an accelerating and rotating observer, becomes incomplete. To recover the complete metric of $\underline{V}_2^{(\varrho)}$, therefore, our further strategy is to consider a general deformation of the flat master space, $\mathrm{MS}_p \to \widetilde{MS}_p$ (2). The deformation tensor yields the deformations of linear holonomic basis. Accordingly, we must find the first deformation matrices (29), (30), which yield the local tetrad deformations (22). This significantly improves the standard metric and other relevant geometrical structures referred to a noninertial frame in Minkowski spacetime for relativistic velocities and an arbitrary characteristic acceleration lengths. We compute the object of anholonomicity (the structure-constants) (subsect. 3.2) and the connection (subsect. 3.3) defined with respect to the anholonomic frame. On these premises, we finally obtain the explicit form of the Dirac equation for an observer in a reference frame that is accelerated with a three-acceleration $\vec{a}$ and rotating with angular frequency $\vec{\omega}$ (66).

References

Audretsch J., Schafer G., 1978, Gen. Rel. Grav., 9, 243

Bjorken J., Drell S., 1964, Relativistic Quantum Mechanic. McGraw-Hill, New York

Brill D., Wheeler J., 1957, Rev. of Mod. Phys., 29, 465

Cai Y., Papini G., 1991, Phys. Rev. Lett., 66, 1259

Cai Y., Papini G., 1992, Phys. Rev. Lett., 68, 3811

Fischbach E., Freeman B., Cheng W.-K., 1981, Phys. Rev. D, 23, 2157

Hehl F., Ni W.-T., 1990, Phys. Rev. D, 42, 2045

Hehl F., Lemke J., Mielke E., 1991, Two lectures on fermions and gravity". In: Geometry and Theoretical Physics, p.56. J.Debrus and A.C. Hirshfeld (eds.) Springer, Berlin Heidelberg

Maluf J., Faria F., 2008, Ann. der Physik, 520, 326

Maluf J., Faria F., Ulhoa S., 2007, Class. Quantum Grav., 24, 2743

Marzlin K.-P., 1996, Phys. Lett. A, 215, 1

Mashhoon B., 2002, Ann. der Physik, 514, 532

Mashhoon B., 2011, Ann. der Physik, 523, 226

Misner C., Thorne K., Wheeler J., 1973, Gravitation. Freeman, San Francisco

Obukhov Y. N., 2001, Phys. Rev. Lett., 86, 192

Obukhov Y. N., 2002, Fortschr. Phys., 50, issue 5, 711

Parker L., Toms D., 2009, Quantum Field Theory in Curved Spacetime. Cambridge University Pres

Ryder L., 1998, J. Phys. A: Math. Gen. A, 31, 2465

Singh D., Papini G., 2000, Nuovo Cim. B, 115, 223

Synge J., 1960, Relativity: The General Theory. North-Holland, Amsterdam

Ter-Kazarian G., 2024a, Grav. Cosmol., 30, issue 1, 8

Ter-Kazarian G., 2024b, Communications of BAO, 71, Issue 2,

Ter-Kazarian G., 2024c, Communications of BAO, 71, Issue 2, 249

Ter-Kazarian G., 2024d, Communications of BAO, 71, Issue 2, 289

Ter-Kazarian G., 2025, Grav. Cosmol., 31, issue 2, 145

Ter-Kazarian G., 2026, Grav. Cosmol., 32, Issue 1

Varjú K., Ryder L., 1998, Phys. Lett. A, 250, 263

Varjú K., Ryder L., 2000, Phys. Rev. D, 62, 024016

de Oliveira C., Tiomno J., 1962, Nuovo Cim., 24, 672

doi:https://doi.org/10.52526/25792776-25.72.2-206

Appendices

Appendix A Preliminaries

(1) The embedding. A smooth embedding map, generalized for curved spaces, becomes $\widetilde{f}:\ \underline{V}_2 \longrightarrow V_4$ defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{\widetilde{e}}_{\underline{0}} = \widetilde{e}_0, \quad \underline{\widetilde{x}}^{\underline{0}} = \widetilde{x}^0, \quad \underline{\widetilde{e}}_{\underline{1}} = \vec{\widetilde{n}}, \quad \underline{\widetilde{x}}^{\underline{1}} = |\vec{\widetilde{x}}|, \tag{74}$$

where $\underline{\widetilde{e}}_{\underline{m}}$ ($\underline{m} = \underline{0}, \underline{1}$) is the basis at the point of interest in $\underline{V}_2$, $\vec{\widetilde{x}} = \widetilde{e}_i \widetilde{x}^i = \vec{\widetilde{n}}|\vec{\widetilde{x}}|$ $(i = 1, 2, 3)$ (the middle letters of the Latin alphabet $(i, j, ...)$ will be reserved for space indices in V_4). From embedding map (74), we obtain the components of velocity of a particle $\underline{\widetilde{v}}^{(\pm)} = \frac{d\underline{x}^{(\pm)}}{d\underline{\widetilde{x}}^{\underline{0}}} = \frac{1}{\sqrt{2}}(\underline{\widetilde{v}}^{\underline{0}} \pm \underline{\widetilde{v}}^{\underline{1}})$, $\underline{\widetilde{v}}^{\underline{1}} = \frac{d\underline{\widetilde{x}}^{\underline{1}}}{d\underline{\widetilde{x}}^{\underline{0}}} = |\vec{\widetilde{v}}| = |\frac{d\vec{\widetilde{x}}}{d\widetilde{x}^0}|$, so that $\underline{\widetilde{u}} = \underline{\widetilde{e}}_{\underline{m}} \underline{\widetilde{v}}^{\underline{m}} = (\vec{\underline{\widetilde{v}}}_{\underline{0}}, \vec{\underline{\widetilde{v}}}_{\underline{1}})$, $\vec{\underline{\widetilde{v}}}_{\underline{0}} = \underline{\widetilde{e}}_0 \underline{\widetilde{v}}^{\underline{0}}$, $\vec{\underline{\widetilde{v}}}_{\underline{1}} = \underline{\widetilde{e}}_{\underline{1}} \underline{\widetilde{v}}^{\underline{1}} = \vec{\widetilde{n}}|\vec{\widetilde{v}}| = \vec{\widetilde{v}}$, therefore, $\underline{\widetilde{u}} = (\vec{\underline{\widetilde{v}}}_{\underline{0}}, \vec{\underline{\widetilde{v}}}_{\underline{1}}) = \widetilde{u} = (\widetilde{e}_0, \vec{\widetilde{v}})$. Thence, the components of the acceleration vector satisfy the following embedding relations $\underline{a}^{\underline{0}} = a^0$, $\underline{a}^{\underline{1}} = |\vec{a}|$. A comprehensive principle which underlies the global MS_p-SUSY theory hinges on the following: *the particle perseveres in its permanent state of superoscillations between the spaces M_4 and $\underline{M}_2$, unless acted upon by some external force,* i.e. the particle undergoes the SUSY - transformations at successive transitions from M_4 to $\underline{M}_2$ and back ($M_4 \rightleftharpoons \underline{M}_2$).

On the premises of (Ter-Kazarian, 2024a), we review the accelerated motion of a particle in terms of local $\widetilde{MS}_p$-SUSY transformations. That is, a *creation* of a sparticle in $\underline{V}_2$ means the transition of a particle from initial state defined on V_4 into intermediate sparticle state defined on $\underline{V}_2$, while an *annihilation* of a sparticle in $\underline{V}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes of a particle in V_4. The net result of each atomic double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$ to $\underline{V}_2$ and back is as if we had operated with *a local space-time translation* with acceleration, $\vec{a}$, in the original space V_4. Accordingly, the acceleration, $\vec{\underline{a}}$, occurs in $\underline{V}_2$ for transition $\underline{V}_2 \rightleftharpoons V_4$. Thus, the accelerated motion of boson $A(\widetilde{x})$ in V_4 is a chain of its successive transformations to the Weyl fermion $\underline{\chi}(\underline{\widetilde{x}})$ defined on $\underline{V}_2$ (accompanied with the auxiliary fields $\underline{\widetilde{F}}$) and back,

$$\rightarrow A(\widetilde{x}) \rightarrow \underline{\chi}^{(\underline{F})}(\underline{\widetilde{x}}) \rightarrow A(\widetilde{x}) \rightarrow \underline{\chi}^{(\underline{F})}(\underline{\widetilde{x}}) \rightarrow, \tag{75}$$

and the same interpretation holds for fermion $\chi(\widetilde{x})$.

(2) The vielbein field in M_4. In the M_4, the vielbein field is orthonormal anywhere:

$$e_{\hat{a}} \cdot e_{\hat{b}} = g_{\mu\nu} \lambda_{(a)}{}^{\mu} \lambda_{(b)}{}^{\nu} = o_{ab} = diag(+ - - -). \tag{76}$$

Arbitrary curvilinear coordinates of a non-inertial frame of reference in a flat Minkowski spacetime M_4 will be denoted by $x^\mu(s)$, with proper linear 3-acceleration $\vec{a}(s)$ and proper 3-rotation $\vec{\omega}(s)$, s being the proper time. To describe the acceleration scales mathematically, the notion of a reference system has to be generalized from curvilinear coordinate frame $e_\mu = \partial_\mu = \partial/\partial x^\mu$ to orthonormal frame $e_{\hat{a}}$. This tetrad can be decomposed with respect to the tangent vectors e_μ along the curvilinear coordinates, the natural basis, according to $\lambda_{(a)}{}^{\mu} := e_{\hat{a}}{}^{\mu}$, where $e_{\hat{a}} = e_{\hat{a}}{}^{\mu} e_\mu$. The spacetime indices $\mu, \nu ...$ and $SO(3,1)$ indices $a, b, ...$ run from 0 to 3. The time axis must be the time axis of a comoving inertial frame in which the observer is momentarily at rest, i.e. the zeroth leg of the frame $e_{\hat{0}}$ be 4-velocity u^μ of the observer that is tangent to the world line at a given point $\mathcal{P}$. The remaining spatial triad frame vectors $e_{\hat{i}}$, orthogonal to $e_{\hat{0}}$, are also parameterized by (s). The spatial triad $e_{\hat{i}}$ rotates with proper 3-rotation $\vec{\omega}(s)$. The set of tetrad fields for which $\lambda^{\mu}{}_{(0)}$ describes a congruence of timelike curves $\mathcal{C}$ is adapted to a class of observers characterized by the velocity field $u^\mu = \lambda_{(0)}{}^{\mu}$ and by the acceleration $a^\mu = \frac{Du^\mu}{ds} = \frac{D\lambda_{(0)}{}^{\mu}}{ds} = u^a \nabla_a \lambda_{(0)}{}^{\mu}$, where the covariant derivative is constructed out of the Christoffel symbols.

Constructing Cartesian coordinates based on accelerated and rotating laboratory, let $\mathcal{S}(\mathcal{P})$ be the space-like hyperplane associated to each event (point) $\mathcal{P}$ on the timelike world line at x^μ of the accelerated observer, orthogonal to it. The accelerated observer carries the orthonormal frame $e_{\hat{a}}$. Defining $\overline{x}^0 = c\overline{t} = s$ and $\overline{x}^1, \overline{x}^2, \overline{x}^3$ as Cartesian coordinates using the triad $e_{\hat{i}}(s)$ with the observer at the origin: $\overline{x}^\mu = (\overline{x}^0, \overline{x}^1, \overline{x}^2, \overline{x}^3)$ are the *local coordinates* relative to the accelerated observer. The tetrad $e_{\hat{\mu}}(s)$ can be parallel transported from $\mathcal{P}$ to all neighboring points on $\mathcal{S}(\mathcal{P})$, which defines the orthonormal tetrad field $\overline{e}_{\hat{\mu}}(\overline{x}^\nu)$. This local coordinate system is used in the laboratory, while the world line is the line of the reference clock. The tetrad

doi:https://doi.org/10.52526/25792776-25.72.2-206

field $\bar{e}_{\hat{\mu}}(\bar{x}^{\nu})$ is anholonomic. Define the coordinate tetrad $\bar{e}_{\mu}=\bar{\partial}_{\mu}=\partial/\partial\bar{x}^{\mu}$. The orthonormal frame $e_{\hat{a}}$, carried by an accelerated observer, now can be written with respect to curvilinear or Cartesian coordinates, respectively:

$$\begin{array}{l} e_{\hat{a}}=\lambda_{(a)}{}^{\mu}\,e_{\mu}=\bar{\lambda}_{(a)}{}^{\mu}\,\bar{e}_{\mu},\\ \vartheta^{\hat{b}}=\lambda^{(b)}{}_{\nu}\,\vartheta^{\nu}=\bar{\lambda}^{(b)}{}_{\nu}\,\bar{\vartheta}^{\nu}, \end{array} \tag{77}$$

with $\vartheta^{\mu}=dx^{\mu}$, $\bar{\vartheta}^{\mu}=d\bar{x}^{\mu}$. The coframe members $\{\vartheta^{\hat{b}}\}$ are the objects of dual counterpart: $e_{\hat{a}}\rfloor\vartheta^{\hat{b}}=\delta^{\hat{b}}_{\hat{a}}$. Let us introduce a *geodesic* coordinate system $X^{\mu}(s)$, which is in general valid in a sufficiently narrow worldtube along the timelike world line of the observer. Suppose the displacement vector $\bar{x}^{\mu}(s)$ represents the position of the accelerated observer. According to the hypothesis of locality, at any time (s) along the accelerated world line the spacelike $\mathcal{S}(\mathcal{P})$ hyperplane orthogonal to the world line is Euclidean space and we usually describe some event on this hyperplane at x^{μ} to be at X^{μ}, where x^{μ} and X^{μ} are connected via $X^{0}=s$ and

$$x^{\mu}=\bar{x}^{\mu}(s)+X^{k}\,\lambda_{(k)}{}^{\mu}(s). \tag{78}$$

This gives

$$dx^{\mu}=d\bar{x}^{\mu}(s)+dX^{i}\,\lambda_{(i)}{}^{\mu}(s)+X^{i}\,d\lambda_{(i)}{}^{\mu}(s), \tag{79}$$

where the displacement vector from the origin reads $d\bar{x}^{\mu}=\lambda_{(0)}{}^{\mu}(s)\,dX^{0}$. Consequently, (79) yields the standard metric of semi-Riemannian 4D background space $V_4^{(0)}$, in noninertial system of the accelerating and rotating observer, computed on the basis of hypothesis of locality (Hehl & Ni, 1990, Hehl et al., 1991) (see also (Mashhoon, 2002, 2011)):

$$\begin{array}{l} ds^{2}=g_{\mu\nu}dx^{\mu}dx^{\nu}=(dX^{0})^{2}\Big[(1+\vec{a}\cdot\vec{X})^{2}\\ +(\vec{\omega}\cdot\vec{X})^{2}-(\vec{\omega}\cdot\vec{\omega})(\vec{X}\cdot\vec{X})\Big]\\ -2dX^{0}\,d\vec{X}\cdot(\vec{\omega}\times\vec{X})-d\vec{X}\cdot d\vec{X}. \end{array} \tag{80}$$

From (80) it is seen that such geodesic coordinates are admissible as long as

$$(1+\vec{a}\cdot\vec{X})^{2}>(\vec{\omega}\times\vec{X})^{2}. \tag{81}$$

Thus in the discussion of the admissibility of the geodesic coordinates, two independent acceleration lengths must be considered: the translational acceleration length c^{2}/a and the rotational acceleration length c/ω that appear in equation (81). While the components of the orthonormal frame field read

$$\begin{array}{l} \lambda_{(0)}{}^{0}=\frac{1}{1+\vec{a}\cdot\vec{X}},\quad \lambda_{(0)}{}^{k}=-\frac{[\vec{\omega}\times\vec{X}]^{k}}{1+\vec{a}\cdot\vec{X}},\\ \lambda_{(i)}{}^{j}=\delta^{j}_{i},\quad \lambda_{(i)}{}^{0}=0, \end{array} \tag{82}$$

and the components of the dual coframe field are

$$\begin{array}{l} \lambda^{(0)}{}_{0}=(1+\vec{a}\cdot\vec{X}),\quad \lambda^{(0)}{}_{i}=0,\\ \lambda^{(i)}{}_{0}=[\vec{\omega}\times\vec{X}]^{i},\quad \lambda^{(i)}{}_{j}=\delta^{i}_{j}. \end{array} \tag{83}$$

(3) The vielbein field in MS_p. The components of the orthonormal frame field are $\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}:=\underline{e}_{\underline{\hat{a}}}{}^{\underline{\mu}}$, where $\underline{e}_{\underline{\hat{a}}}=\underline{e}_{\underline{\hat{a}}}{}^{\underline{\mu}}\,\underline{e}_{\underline{\mu}}$ ($\underline{e}_{\underline{\mu}}=\underline{\partial}_{\underline{\mu}}=\partial/\partial\underline{x}^{\underline{\mu}}$). The time axis must be the time axis of a comoving inertial frame in which the observer is momentarily at rest, i.e. the zeroth leg of the frame $\underline{e}_{\underline{\hat{0}}}$ be 2-velocity $\underline{u}^{\underline{\mu}}$ of the observer that is tangent to the world line at a given point $\underline{\mathcal{P}}$. The spatial frame vector $\underline{e}_{\underline{\hat{1}}}$, orthogonal to $\underline{e}_{\underline{\hat{0}}}$, is also parameterized by $(\underline{s})$. Constructing Cartesian coordinates based on laboratory, let $\underline{\mathcal{S}}(\underline{\mathcal{P}})$ be the spacelike hyperplane associated to each event (point) $\underline{\mathcal{P}}$ on the timelike world line at $\underline{x}^{\underline{\mu}}$ of the accelerated observer, orthogonal to it. Defining $\bar{\underline{x}}^{0}=c\underline{\bar{t}}=\underline{s}$ and $\bar{\underline{x}}^{1}$ as Cartesian coordinates using the $\underline{e}_{\underline{\hat{1}}}(\underline{s})$ with the observer at the origin: $\bar{\underline{x}}^{\underline{\mu}}=(\bar{\underline{x}}^{0},\bar{\underline{x}}^{1})$ are the *local coordinates* relative to the accelerated observer. The tetrad $\underline{e}_{\underline{\hat{\mu}}}(\underline{s})$ can be parallel transported from $\underline{\mathcal{P}}$ to all neighboring points on $\underline{\mathcal{S}}(\underline{\mathcal{P}})$, which defines the orthonormal tetrad field $\bar{\underline{e}}_{\underline{\hat{\mu}}}(\bar{\underline{x}}^{\underline{\nu}})$. The tetrad field $\bar{\underline{e}}_{\underline{\hat{\mu}}}(\bar{\underline{x}}^{\underline{\nu}})$ is anholonomic. Define the coordinate tetrad $\bar{\underline{e}}_{\underline{\mu}}=\bar{\underline{\partial}}_{\underline{\mu}}=\partial/\partial\bar{\underline{x}}^{\underline{\mu}}$. The orthonormal frame, $\underline{e}_{\underline{\hat{a}}}$, can be written with respect to curvilinear or Cartesian coordinates, respectively:

$$\begin{array}{l} \underline{e}_{\underline{\hat{a}}}=\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}\,\underline{e}_{\underline{\mu}}=\bar{\underline{\lambda}}_{(\underline{a})}{}^{\underline{\mu}}\,\bar{\underline{e}}_{\underline{\mu}},\\ \underline{\vartheta}^{\underline{\hat{b}}}=\underline{\lambda}^{(\underline{b})}{}_{\underline{\nu}}\,\underline{\vartheta}^{\underline{\nu}}=\bar{\underline{\lambda}}^{(\underline{b})}{}_{\underline{\nu}}\,\bar{\underline{\vartheta}}^{\underline{\nu}}, \end{array} \tag{84}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

with $\underline{\vartheta}^{\underline{\mu}} = d\underline{x}^{\underline{\mu}}$, $\overline{\underline{\vartheta}}^{\underline{\mu}} = d\overline{\underline{x}}^{\underline{\mu}}$. The coframe members $\{\underline{\vartheta}^{\hat{\underline{b}}}\}$ are the objects of dual counterpart: $\underline{e}_{\hat{\underline{a}}} \rfloor \underline{\vartheta}^{\hat{\underline{b}}} = \delta^b_a$.

Let $(\underline{X}^{\underline{\mu}}(\underline{X}^{\underline{0}}, \underline{X}^{\underline{1}})$ be *geodesic local coordinates* relative to the accelerated observer in the neighborhood of the accelerated path in MS_p, with spacetime components satisfying the embedding map

$$\begin{array}{l} d\underline{X}^{\underline{0}} = dX^0, \quad d\underline{X}^{\underline{1}} = |d\vec{X}|, \\ \vec{n} = \frac{d\vec{X}}{d\underline{X}^{\underline{1}}} = \frac{d\vec{X}}{|d\vec{X}|}, \quad \vec{n}\cdot\vec{n} = 1. \end{array} \tag{85}$$

Then, in view of (82) and (83), the components of the orthonormal frame field, $\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}$, read

$$\begin{array}{l} \underline{\lambda}_{(\underline{0})}{}^{\underline{0}} = \frac{1}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}, \quad \underline{\lambda}_{(\underline{0})}{}^{\underline{1}} = -\frac{[\vec{\omega}\times\vec{X}]^{\underline{1}}}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}, \\ \underline{\lambda}_{(\underline{1})}{}^{\underline{1}} = 1, \quad \underline{\lambda}_{(\underline{1})}{}^{\underline{0}} = 0. \end{array} \tag{86}$$

while the components of the dual coframe field, $\underline{\lambda}^{(\underline{a})}{}_{\underline{\mu}}$, become

$$\begin{array}{l} \underline{\lambda}^{(0)}{}_{\underline{0}} = (1 + (\vec{a}\cdot\vec{X})^{\underline{1}}), \quad \underline{\lambda}^{(0)}{}_{\underline{1}} = 0, \\ \underline{\lambda}^{(1)}{}_{\underline{0}} = [\vec{\omega}\times\vec{X}]^{\underline{1}}, \quad \underline{\lambda}^{(1)}{}_{1} = 1. \end{array} \tag{87}$$

The acceleration of the observer along the accelerated path, who carries an orthonormal tetrad frame $\underline{e}_{\hat{\underline{a}}} = (\underline{e}_{\hat{\underline{0}}}, \underline{e}_{\hat{\underline{1}}})$, therefore, can be expressed in the frame basis:

$$\frac{d\underline{\lambda}_{(\underline{a})}{}^{\underline{\mu}}(\underline{s})}{d\underline{s}} = \underline{\Phi}_{(\underline{a})}{}^{(\underline{b})}(\underline{s})\,\underline{\lambda}_{(\underline{b})}{}^{\underline{\mu}}(\underline{s}), \tag{88}$$

where the inertial accelerations are represented by a second rank antisymmetric tensor $\underline{\Phi}_{(\underline{a})}{}^{(\underline{b})}(\underline{s})$ under global $SO(1,1)$ transformations. The $\underline{\Phi}_{(\underline{a})(\underline{b})}$ can be interpreted as the inertial accelerations of the frame along the timelike curve $\underline{\mathcal{C}}$ (the translational acceleration and the frequency of rotation of the frame):

$$\begin{array}{l} \underline{\Phi}_{(\underline{1})}{}^{(0)}\underline{X}^{\underline{1}} = (\vec{a}\cdot\vec{X})^{\underline{1}} = |\vec{a}\cdot\vec{X}|, \\ \underline{\Phi}_{(\underline{1})}{}^{(\underline{1})}\underline{X}^{\underline{1}} = [\vec{\omega}\times\vec{X}]^{\underline{1}} = |\vec{\omega}\times\vec{X}|. \end{array} \tag{89}$$

According to the hypothesis of locality, at any time $(\underline{s})$ along the accelerated world line the spacelike $\underline{\mathcal{S}}(\underline{\mathcal{P}})$ hyperplane orthogonal to the world line is Euclidean space and we usually describe some event on this hyperplane at $\underline{x}^{\underline{\mu}}$ to be at $\underline{X}^{\underline{\mu}}$, where $\underline{x}^{\underline{\mu}}$ and $\underline{X}^{\underline{\mu}}$ are connected via $\underline{X}^{\underline{0}} = \underline{s}$ and

$$\underline{x}^{\underline{\mu}} = \overline{\underline{x}}^{\mu}(s) + \underline{X}^{\underline{1}}\,\underline{\lambda}_{(\underline{1})}{}^{\mu}(s). \tag{90}$$

This gives

$$d\underline{x}^{\underline{\mu}} = d\overline{\underline{x}}^{\underline{\mu}}(\underline{s}) + d\underline{X}^{\underline{1}}\,\underline{\lambda}_{(\underline{1})}{}^{\underline{\mu}}(\underline{s}) + \underline{X}^{\underline{1}}\,d\underline{\lambda}_{(\underline{1})}{}^{\underline{\mu}}(\underline{s}), \tag{91}$$

where the displacement vector from the origin reads $d\overline{\underline{x}}^{\mu}(s) = \underline{\lambda}_{(\underline{0})}{}^{\mu}(s)\,d\underline{X}^{\underline{0}}$. The (91) yields the metric

$$d\underline{s}^2 = g_{\underline{\mu}\underline{\nu}}\,d\underline{x}^{\underline{\mu}}d\underline{x}^{\underline{\nu}} = \underline{\vartheta}^{\underline{0}} \otimes \underline{\vartheta}^{\underline{0}} - \underline{\vartheta}^{\underline{1}} \otimes \underline{\vartheta}^{\underline{1}}. \tag{92}$$

In doing so, we calculated the orthonormal frame, $\underline{e}_{\hat{\underline{a}}}$, and corresponding coframe, $\vartheta^{\hat{b}}$ members, carried by an accelerated observer, which by virtue of (86) and (87) are equal to

$$\begin{array}{l} \underline{e}_{\hat{\underline{0}}} = \frac{1}{1+(\vec{a}\cdot\vec{X})^{\underline{1}}}\,\{\underline{e}_{\underline{0}} - [\vec{\omega}\times\vec{X}]^{\underline{1}}\,\underline{e}_{\underline{1}}\}, \\ \underline{e}_{\hat{\underline{1}}} = \underline{e}_{\underline{1}}, \end{array} \tag{93}$$

and

$$\begin{array}{l} \underline{\vartheta}^{\hat{\underline{0}}} = (1 + (\vec{a}\cdot\vec{X})^{\underline{1}})\,d\underline{X}^{\underline{0}}, \\ \underline{\vartheta}^{\hat{\underline{1}}} = d\underline{X}^{\underline{1}} + [\vec{\omega}\times\vec{X}]^{\underline{1}}\,d\underline{X}^{\underline{0}}, \end{array} \tag{94}$$

respectively. The metric (92) of 2D semi-Riemannian space, $V_2^{(0)}$, in noninertial system of the accelerating and rotating observer, computed on the basis of hypothesis of locality reads

$$\begin{array}{l} d\underline{s}^2 = (d\underline{X}^{\underline{0}})^2[(1 + (\vec{a}\cdot\vec{X})^{\underline{1}})^2 + (\vec{\omega}\times\vec{X})^{\underline{1}}(1- \\ (\vec{\omega}\times\vec{X})^{\underline{1}})] - (d\underline{X}^{\underline{1}})^2 - 2d\underline{X}^{\underline{0}}d\underline{X}^{\underline{1}}[(\vec{\omega}\times\vec{X})^{\underline{1}}(1- \\ (\vec{\omega}\times\vec{X})^{\underline{1}})]^{1/2}. \end{array} \tag{95}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206

Using the following embedding relations as a converting guide:

$$\begin{array}{l} g_{\tilde{0}\tilde{0}}(d\underline{X}^{\underline{0}})^2 = g_{\tilde{0}\tilde{0}}(dX^0)^2, \quad g_{\tilde{1}\tilde{1}}(d\underline{X}^{\underline{1}})^2 \\ = g_{\tilde{1}\tilde{1}}(d\vec{X}\cdot d\vec{X}), \quad g_{\tilde{1}\tilde{0}}d\underline{X}^{\underline{1}} = g_{\tilde{i}\tilde{0}}d\underline{X}^{\underline{i}}, \\ g_{\tilde{i}\tilde{0}} = g_{\tilde{0}\tilde{i}} = n_i g_{\tilde{0}\tilde{1}} = n_i g_{\tilde{1}\tilde{0}}, \quad b_{2i} = n_i b_2, \end{array} \tag{96}$$

from (30) and (31), we obtain

$$\begin{array}{l} \pi_{(\hat{0})}{}^{\hat{0}}(\varrho) = \frac{1+\vec{a}\cdot\vec{X}}{b_0}, \quad \pi_{(\hat{i})}{}^{\hat{0}}(\varrho) = 0, \\ \pi_{(\hat{i})}{}^{\hat{j}}(\varrho) = \frac{1}{b_1}\,\delta_i^j, \quad \pi_{(\hat{0})}{}^{\hat{i}}(\varrho) = \frac{b_{2i}+b_1(\vec{\omega}\times\vec{X})^i}{b_0 b_1}, \end{array} \tag{97}$$

and

$$\begin{array}{l} \pi^{(\hat{0})}{}_{\hat{0}}(\varrho) = \frac{b_0}{1+\vec{a}\cdot\vec{X}}, \quad \pi^{(\hat{0})}{}_{\hat{i}}(\varrho) = 0, \\ \pi^{(\hat{i})}{}_{\hat{0}}(\varrho) = -\frac{b_{2i}+b_1(\vec{\omega}\times\vec{X})^i}{1+\vec{a}\cdot\vec{X}}, \quad \pi^{(\hat{i})}{}_{\hat{j}}(\varrho) = b_1\,\delta^i_j. \end{array} \tag{98}$$

By means of (93), (94), we obtain the generalized frame and coframe members referred to the 4D background space as follows:

$$\begin{array}{l} e_{(\hat{0})} = b_0^{-1}\left\{(1+\vec{a}\cdot\vec{X})\,e_{\hat{0}} + (\frac{b_{2i}}{b_1} + (\vec{\omega}\times\vec{X})^i)\,e_{\hat{i}}\right\}, \\ e_{(\hat{i})} = b_1^{-1}\,e_{\hat{i}}, \end{array} \tag{99}$$

and

$$\begin{array}{l} \vartheta^{(\hat{0})} = \frac{b_0}{1+\vec{a}\cdot\vec{X}}\,\vartheta^{\hat{0}}, \\ \vartheta^{(\hat{i})} = b_1\,\vartheta^{\hat{i}} - \frac{1}{1+\vec{a}\cdot\vec{X}}\,[b_{2i} + b_1(\vec{\omega}\times\vec{X})^i]\,\vartheta^{\hat{0}}, \end{array} \tag{100}$$

respectively. Then the complete metric in noninertial frame of arbitrary accelerating and rotating observer in Minkowski spacetime reads

$$\begin{array}{l} d\tilde{s}^2(\varrho) = g_{\mu\nu}(\varrho)dX^\mu dX^\nu = \vartheta^{(\hat{0})}\otimes\vartheta^{(\hat{0})} - \\ \vartheta^{(\hat{i})}\otimes\vartheta^{(\hat{i})} = [(\pi^{(\hat{0})}{}_{\hat{0}})^2 - (\pi^{(\hat{i})}{}_{\hat{0}})^2]\,\vartheta^{\hat{0}}\otimes\vartheta^{\hat{0}} \\ -(\pi^{(\hat{i})}{}_{\hat{k}})^2\,\vartheta^{\hat{j}}\otimes\vartheta^{\hat{j}} = -2\pi^{(\hat{i})}{}_{\hat{0}}\,\pi^{(\hat{i})}{}_{\hat{j}}\,\vartheta^{\hat{0}}\otimes\vartheta^{\hat{j}} \\ = (dX^0)^2\left[b_0^2 - b_2^2 - (2b_1^2+1)(\vec{\omega}\cdot\vec{X})^2\right. \\ \left.-(2b_1+1)\vec{b}_2\cdot(\vec{\omega}\cdot\vec{X})\right] - b_1^2\,d\vec{X}\cdot d\vec{X} \\ -dX^0\,d\vec{X}\cdot\left[b_1(2b_1+1)(\vec{\omega}\times\vec{X}) + \vec{b}_2\right]. \end{array} \tag{101}$$

doi:https://doi.org/10.52526/25792776-25.72.2-206