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The time characteristics of spectral line quanta diffusion in scattering and absorbing media

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Abstract

The paper presents new results on the patterns of evolution of spectral lines formed in absorbing and scattering media under the influence of non-stationary energy sources. When describing the time evolution of radiation diffusion, both possible causes of time consumption are taken into account: the time spent in the absorbed state and the time taken to travel the path between two successive scattering events. Mathematically, this is expressed as a convolution of the probability distributions of these two quantities. Proof is provided of the existence of a relationship between two time averages characterising the process of spectral lines radiation diffusion. It follows that the description of the evolution of a spectral line becomes a mathematically single-parameter problem, with the ratio of the mentioned two temporal values as a parameter. The value of the ratio for a given spectral line is determined by the number-density of emitting or absorbing atoms/ions (dependent on spectrum) and their thermal velocity. The fact that the dependence on the number-density far exceeds that on thermal velocity significantly simplifies the determination of the chemical composition of the object under study.

1. Introduction

The previous work (Nikoghossian, 2025) considered the temporal changes in spectral line profiles formed in an atmosphere of finite optical thickness in the presence of non-stationary primary energy sources. To provide a comprehensive picture of the evolution of the spectral lines under study, data were presented on changes in certain average statistical characteristics of the line radiation transfer in the medium, such as the average number of photon scattering acts and the average time taken during the diffusion process.

The observed evolution of the line spectrum depends, obviously, both on the nature of internal or external primary sources and on the response of the optically active medium, which is generally determined by the process of multiple scattering within it. The latter is characterised by the time photons spend in the medium, which, in turn, consists of the time spent by them in the absorbed state and the transit time between two successive scattering events. In cases where radiative equilibrium is established in the medium, only the second of these characteristic values is usually taken into account when solving time-dependent transfer problems (see e.g. Sobolev, 1950, 1956). It is assumed that the problem of determining the time the photon spends in an absorbed state is reduced to finding the average number of scattering events in the medium. This approach can be applied if the medium is in a state of equilibrium. However, in the non-stationary regime, one should take into account possible changes in the characteristic time taken by photons to travel after each scattering event due to possible changes in the physical conditions in the medium. Moreover, we shall see that there is some relationship between the two considered quantities that plays an important role in the observed changes in the line spectrum.

Thus, for an adequate description of the observed changes in the line spectra, there is a need to develop a proper theory of the evolution of spectral lines that takes into account the combined participation of the two characteristics considered in the photon scattering process.

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2. Basic equations

As we know (Nikoghossian, 2022, 2023, 2025), the probability density distribution of the time spent by a photon during a certain number of scattering acts is given by the so-called Erlang distribution, which is the sum of exponentially distributed random variables and is a particular case of two-parameter Gamma distribution. In our problem, the distributions of each of these two possible processes of time loss can be represented as

$$F_1(t,n,\lambda_1)=\frac{\Lambda_1^n t^{n-1}}{(n-1)!}\mathrm{e}^{-\Lambda_1 t},\qquad F_2(t,n,\Lambda_2)=\frac{\Lambda_2^{n+1}t^n}{n!}\mathrm{e}^{-\Lambda_2 t},\tag{1}$$

where n is the number of scattering events in the medium, the so-called 'rates' of distributions $\Lambda_1 = 1/t_1, \Lambda_2 = 1/t_2$ and t_1, t_2 represent the average time spent by a photon in a single scattering event, respectively, in the absorbed state and during the flight between two consecutive scattering events. The first of these is obviously determined by the value of Einstein's spontaneous transition coefficient A_{ji} corresponding to the spectral line under consideration $t_1 = 1/A_{ji}$ (for the sake of simplicity, we do not consider the effects of collision processes here).

The value of t_2 can be chosen (see Nikoghossian, 2025) the overflight time of the distance of unit optical thickness for a given spectral line. The latter can be presented in the form $t_2 = 1/n_a k_0 c$, where the following notations are used: c is the speed of light, n_a is the number density of the atom/ion participating in a given line formation, and k_0 is the coefficient of absorption of the atom/ion in the center of the line. Of importance for us here is the fact that a relationship can be established between the two time characteristics mentioned, provided that the normalization condition for the scattering coefficient in the spectral line is applied (see Sobolev, 1956)

$$\int\limits_{-\infty}^{\infty} k_\nu d\nu=\frac{h\nu_0}{c}B_{ij}\tag{2}$$

where ν_0 is the frequency in the centre of the line. Note that this formula is more accurate the sharper the peak of the absorption coefficient. Using known relationships between Einstein transition coefficients and given that the velocity distribution of absorbing atoms is Maxwellian, we find

$$k_0=\frac{c^3}{8\pi^{3/2}\nu_0^3 v}\frac{g_j}{g_i}A_{ji},\tag{3}$$

where $v=\sqrt{2kT/m}$ is the average thermal velocity of atoms/ions, k is the Boltzmann constant, m is the mass of an atom/ion, ν_0 is the central frequency, and g_j, g_i are the statistical weights of the upper and lower levels, respectively, between which transitions form a given spectral line. It follows that

$$\frac{t_1}{t_2}=0.0224\frac{g_j}{g_i}\frac{n_a\lambda_0^3}{\bar{v}},\tag{4}$$

where λ_0 is the central wavelength of the line, $\bar{v}$ is the thermal velocity of the medium in units of the speed of light. We see that the ratio between the two average values depends on the spectral range where the line is formed and, as expected, also on the physical characteristics of the medium itself, namely number density and the thermal velocity of the scattering centres. Both of these values must be taken into account jointly with the roles they play in the resulting evolution.

In this case, temporal changes are given by the convolution of the two Erlang distributions (Eq.1) considered above in the form

$$\Phi_n(z)=\int\limits_0^z F_1(x,n,\Lambda_1)F_2(z-x,n,\Lambda_2)\,dx,\tag{5}$$

where the dimensionless temporal variable here is determined as $z = \Lambda_1 t$. With regard of analytic forms of Erlang distributions we obtain

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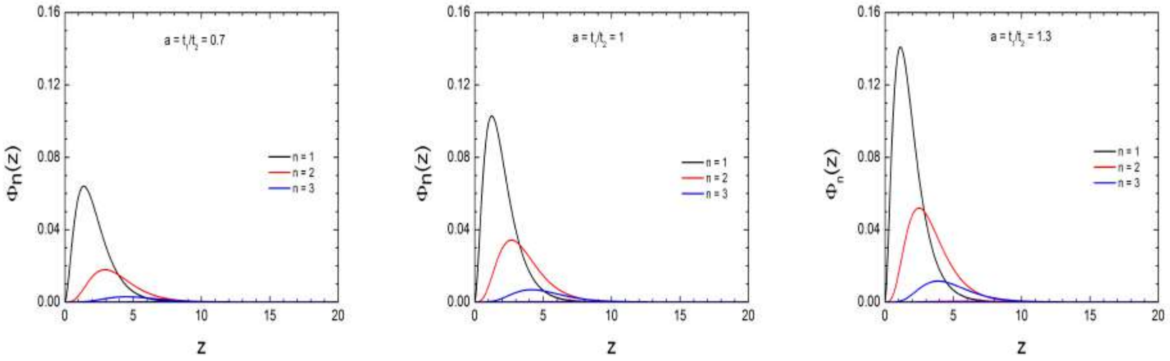

Figure 1. The first three functions Φ_n for different values of the ratio t_1/t_2.

$$\Phi_n(z) = \frac{\Lambda_2 a^n}{n!\,(n-1)!} \mathrm{e}^{-az} \int_0^z \mathrm{e}^{-(1-a)z'} z'^{n-1} \left(z - z'\right)^n dz' \tag{6}$$

where the parameter $a = \Lambda_2/\Lambda_1 = t_1/t_2$.

Thus, in general, the functions $\Phi_n(z)$ have a rather complex form, expressed through hypergeometric functions, and only in the special case of approximate equality of the mean values of time t_1 and t_2, we have $a = 1$ and arrive at the expression used in our previous works.

$$\Phi_n(z) = \frac{\Lambda z^{2n}}{(2n)!} \mathrm{e}^{-z}, \tag{7}$$

where $\Lambda = 1/t_1 = 1/t_2$.

Fig.1 shows graphs of the first few functions of the family functions $\Phi_n(z)$ for three different values of the parameter a chosen arbitrarily in such a way that its effect can be clearly distinguished. We conclude that the dependence is strong enough since even with relatively small changes in its value, the main characteristics of the functions under consideration, such as duration, the maximum value, the time at which this peak is reached, etc., undergo quite noticeable changes. The latter, in turn, obviously affects the behavior of the components of the Neumann series of the observed varying spectral lines over time.

For illustrative purposes, we construct the Neumann series for the reflection and transmission coefficients of a medium of optical thickness $\tau_0 = 3$ at the central frequency of the spectral line. It is assumed that the scattering in the medium occurs with complete redistribution of radiation over frequencies for a given value of the scattering coefficient λ and the value of β, which takes into account absorption in the continuous spectrum. As the nodes used to describe the frequency dependence within the spectral line are the zeros of the 25th-order Hermite polynomials. This model case has been considered in Nikoghossian (2022), where the Neumann series for both reflectance and transmittance of the medium were obtained. In contrast to the mentioned paper, here the evolution of the spectral lines will be determined by functions Eq.(6).

3. Evolution of the line spectra.

One way of describing the evolution of spectral lines is to study the temporal variations in the probability density functions (PDF) of radiation in the lines emerging from the medium. Fig.2 and Fig.3 demonstrate these functions for the above formulated transfer problem for the case of non-stationary energy sources of the Dirac δ-function form. The calculations performed for the lines formed with different values of the above-mentioned parameter a show that the temporal changes in the components of the time-dependent Neumann's series are largely determined by the dominance of the functions $\Phi_n(z)$ discussed above. Despite the relatively small interval covered by its values

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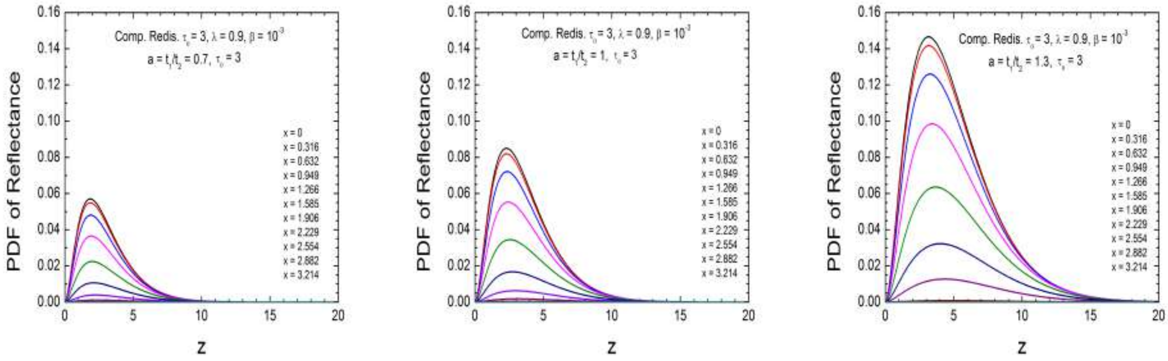

Figure 2. Typical changes in PDF for reflectance of the medium of optical thickness $\tau_0 = 3$ dependent on the value of the parameter a.

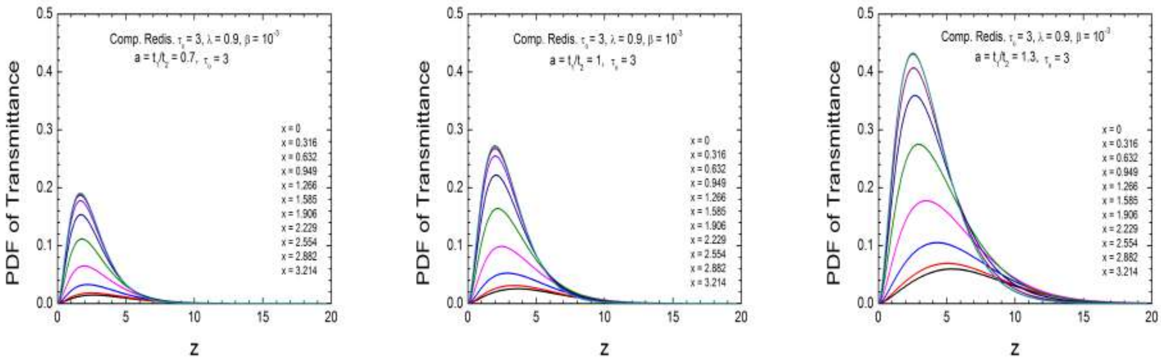

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Figure 3. Typical changes in PDF for transmittance of the medium of optical thickness $\tau_0 = 3$ dependent on the value of of the parameter a.

$0.7 \leq a \leq 1.3$, one sees significant changes in the form and values of the main characteristics, such as the moment when the distribution reaches its peak and the value of this maximum, as well as the total duration of the evolution.

Additional conclusions can be drawn from the temporal changes in spectral line profiles formed both before and after the peak is reached, which can be directly used for comparison with observational data. To obtain a complete picture of the temporal variations in spectral lines, it is important to reveal their dependence on the optical thickness of the medium and the role of multiple scattering in it.

To this end, we consider separately the emission and absorption spectra that arise as a result of reflection from the medium and transmission through it. As the main characteristics for comparison, we will choose the value of the maximum achieved and the time it takes to reach it.

It is easier to make such a comparison for the absorption spectrum since, as evidenced by calculations, both of the mentioned values depend very weakly on the optical thickness of the medium (right panels in Figs.4,5). Meanwhile, the dependence of these quantities on the scattering coefficient becomes more or less significant only in the case of media opaque enough for the lines (see right panels in Figs.6,7). Therefore, in the case of absorption spectra, the parameters relating to the maximum value of the PDF depend solely on the value of a. Such behavior of the PDF makes it possible to estimate its value, which, in turn, given the known characteristics of the spectral line under consideration, allows one to form an idea of the density of atoms/ions to which it belongs. The case of spectral lines formed by reflection from a medium is somewhat more complex. As might be expected, the dependence of the maximum value of the PDF is particularly pronounced for optically thin media. A similar study can be carried out by examining the evolution of theoretically constructed spectral line profiles before and after they reach their maximum values. Finally, it may be equally interesting to describe the changes in the equivalent widths of lines during their evolution. A typical example illustrating such changes

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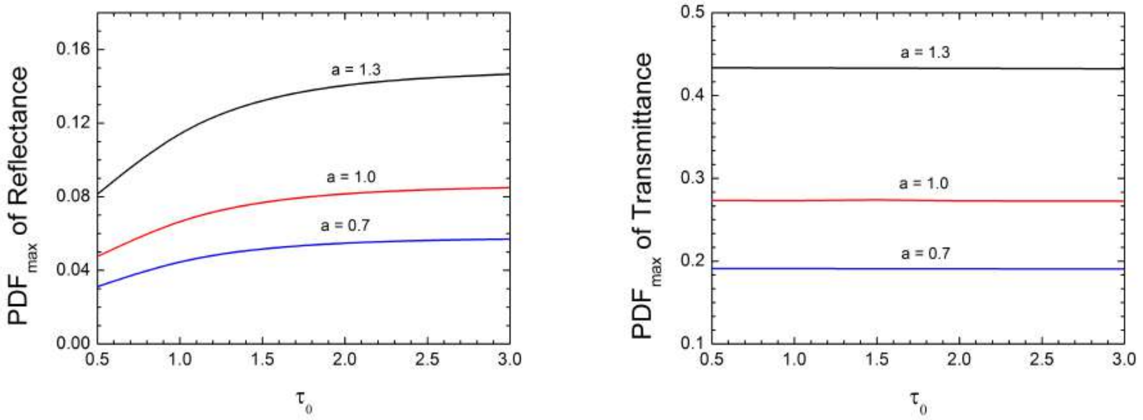

Figure 4. Dependence of Z_{max} (left panel) and maximum value of PDF (right panel) on optical thickness of the medium for different value of the parameter a

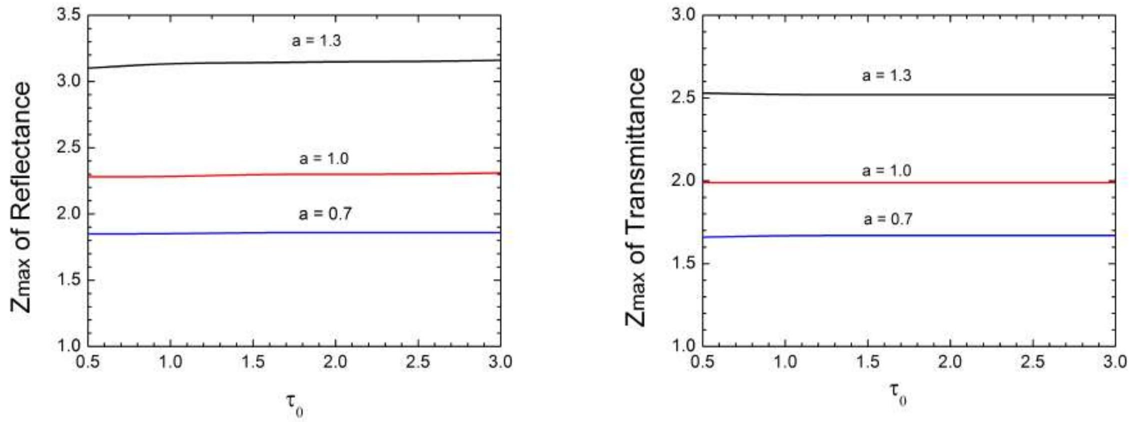

Figure 5. Dependence of Z_{max} (left panel) and maximum value of PDF (right panel) on the value of scattering coefficient λ for the line formed as a result of transmission.

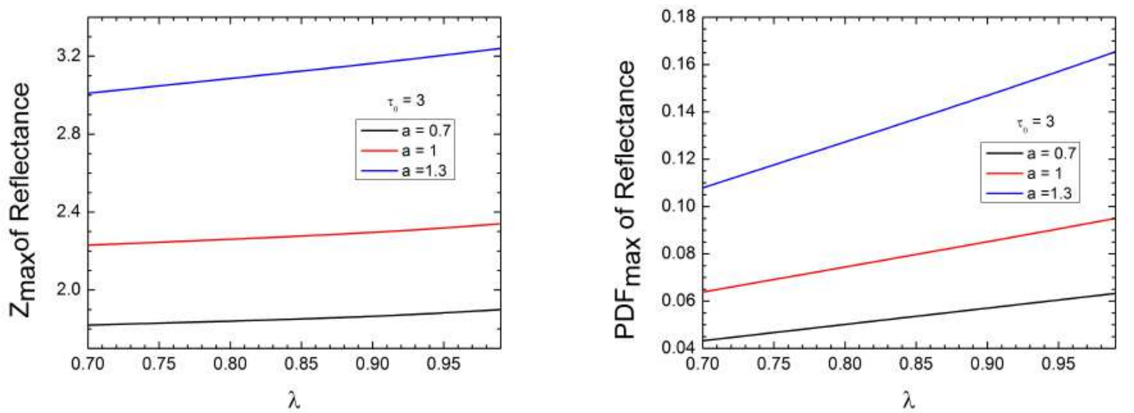

Figure 6. Dependence of Z_{max} (left panel) and maximum value of PDF (right panel) on optical thickness of the medium for different value of the parameter a

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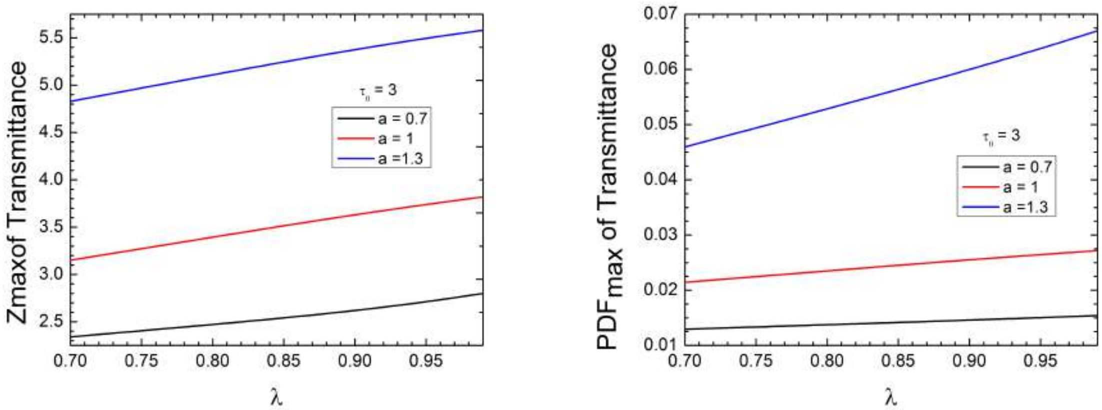

Figure 7. Dependence of Z_{max} (left panel) and maximum value of PDF (right panel) on the value of scattering coefficient λ for the line formed as a result of transmission.

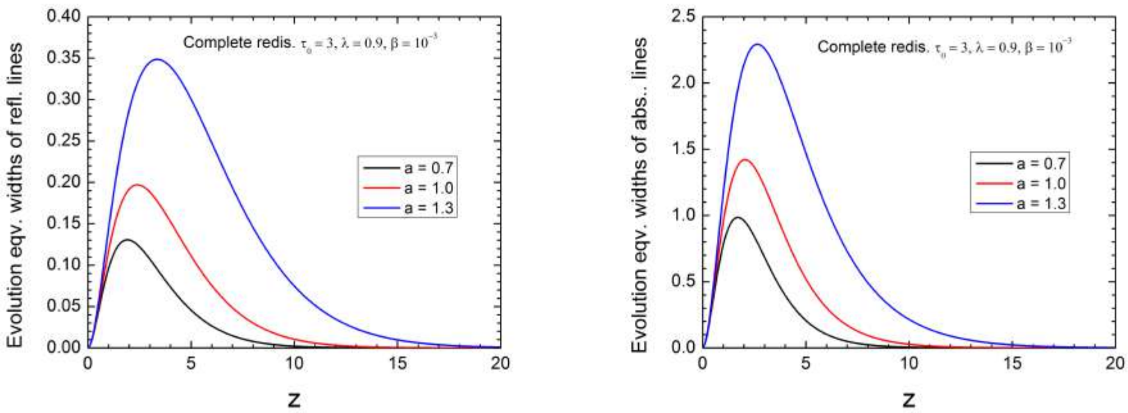

Figure 8. Evolution of equivalent widths of spectral lines for the case of external non-stationary illumination of the Dirac delta-function form.

is shown in Fig.8.

4. Concluding remarks

The relationship we have established between the two characteristic times t_1 and t_2 which describes the diffusion in a medium emitting in spectral line frequencies, now makes it possible to formulate the changes caused by non-stationary energy sources as a single-parameter problem that depends significantly on the value of the parameter a we have introduced. As the key parameters to be determined when monitoring the evolution of spectral lines, we have selected the time at which their peaks occur and the values they attain at that point. Note that a number of other parameters can, of course, be compared with theoretical results, including equivalent widths, rates of increase or decrease in intensity, and so on.

Estimation of the value of the parameter a from observations allows, in principle, to form an idea of the chemical composition of the non-stationary object after estimating the excitation temperature using other methods. It follows from Eq.(4) being written in the form

$$n_a = 44.64 \frac{g_i}{g_j} a \bar{v} \lambda_0^{-3}, \tag{8}$$

or

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$$n_a = 44.64 \frac{g_i}{g_j} \frac{a}{c\lambda_0^3} \sqrt{\frac{2kT}{m}} \tag{9}$$

It is also possible to use data from other lines produced by transitions from a given energy level, followed by the application of the Boltzmann formula.

In conclusion, let us consider the relationship between the two average time scales t_1 and t_2 of two phenomena occurring during the diffusion of radiation under different physical conditions, as found in various astrophysical objects frequently encountered in applications. For instance, in stellar photospheres and atmospheres, including those of dwarfs, the number density of hydrogen atoms varies in the interval between $6 \cdot 10^{15} sm^{-3} - 2 \cdot 10^{17} sm^{-3}$ while the dimensionless thermal velocity $\bar{v}$ is of the order $3 \cdot 10^{-4} - 4.3 \cdot 10^{-4}$ which implies that the inequality $a < 1$ becomes valid only in the extreme shortwave regions of wavelengths of the order of some angstroms and less. It follows that the average time spent by hydrogen photons in the optic domain on the flight between two consecutive acts of scattering is less compared to that spent by them in the absorbed state. However, the situation may be the opposite in the case of spectra of other atoms and ions, whose number densities are many orders of magnitude lower than that of hydrogen.

From this viewpoint, objects with lower density, such as the chromosphere and the solar corona, are of particular interest, as both the atomic number density and thermal velocities vary over a fairly wide range in these regions. Indeed, from the lower layers of the chromosphere to the corona, right up to its upper layers, the density of hydrogen atoms/ions decreases by 12 orders of magnitude, whilst the non-dimensional thermal velocity $\bar{v}$ decreases by approximately one order of magnitude (from $7 \cdot 10^{-5}$ to $4 \cdot 10^{-4}$), so that the inequality $t_1 \leq t_2$ depending on the height will hold if $\lambda \leq 70.6\dot{A}$ for lower optically thick layers of the chromosphere with $n \approx 10^{16} cm^{-3}$ and $\lambda \leq 7600\dot{A}$ for the upper, respectively thin layers. Finally, in the case of more rarefied media such as nebulae and the interstellar medium, the inequality in question for hydrogen atoms holds for all wavelengths shorter than those in the far-infrared region. As can be seen from the examples given, the value of the parameter a depends primarily on the density of the atoms (or ions) whose spectral lines are being considered.

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