Rearrangement of Vacuum State in Gravitation

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Abstract

We address the processes of rearrangement of vacuum state and spontaneous gauge symmetry breaking in gravity. This is achieved by an extension of 'standard gauge principle' to 'general gauge principle' (GGP). The GGP accounts for the gravity generated by hidden local internal symmetries implemented on 'maximally symmetric' (MS) auxiliary space. In doing this, we extend the curvature to a general 'distortion' of the space-time continuum as the theory of spontaneous breaking of 'distortion' symmetry. We have to implement the hidden gauge symmetry of two-parameter abelian group $U^{loc} = U(1)_Y \times \overline{U}(1)$ on the extended 12-dimensional MS-space, with the group elements of $exp\,[i\frac{Y}{2}\,\theta_Y(\zeta)]$ of $U(1)_Y$ and $exp\,[iT^3\,\theta_3(\zeta)]$ of $\overline{U}(1)$, respectively. This group has two generators, the hypercharge Y and the third component T^3 of isospin. The neutral complex Higgs scalar breaks the vacuum symmetry, leaving the 'gravitation' subgroup intact. The massive field component of the so-called 'inner distortion' (ID) causes the shift of zero point energy and, thus, simultaneously with the strong gravity, a significant change in the properties of the space-time continuum has occurred at huge energies. This leads to the 'matter-to-proto-matter' phase transition of second kind.

Keywords: *Gravity–General gauge principle–Spontaneous breaking of gauge symmetry–Inner distortion of spacetime*

1. Introduction

Although the geometrical interpretation of gravitation has advantages in solving the problems of cosmology, nevertheless such a distinction of the gravitational field among the fields gives rise to difficulties in the unified theories of all interactions of elementary particles and in the quantization of gravitation. Even though it being among the most significant advances in physics, in particular, General Relativity (GR) as a geometrized theory of gravitation clashes from the outset with the basic principles of field theory. It is rather not conducive to resolving the controversial problems of energy-momentum conservation laws, localization of energy of gravitational waves, etc. These stem from the fact that Riemannian geometry, in general, does not admit a group of isometries; i.e., Poincaré transformations no longer act as isometries, and, in particular, it is impossible to define energy-momentum as Noether local currents related to exact symmetries. This, in turn, posed severe problems in Riemannian space interacting quantum field theory, among which is the non-uniqueness of the 'physical' vacuum and the associated Fock space. It is perhaps worth remarking that one of the underlying principles of Minkowski space interacting quantum field theory is that the vacuum is unique, in particular, $|in>=|out>$ (up to a phase factor). The peculiar shortcoming of the interacting quantum field theory in curved space-time is the following two important questions to be addressed yet: a) an absence of the definitive concept of space-like separated points, particularly in the canonical approach, and the 'light-cone' structure at each space-time point; b) the separation of positive and negative-frequencies for completeness of the Hilbert-space description. Due to it, the definition of positive frequency modes cannot, in general, be unambiguously fixed in the past and future. This leads to $|in>\neq|out>$, because $|in>$ is unstable against decay into many particle $|out>$ states due to interaction processes allowed by lack of Poincaré invariance. Whereas, non-trivial Bogolubov transformation between past and future positive frequency modes implies that particles are created from the vacuum and this is one of the reasons for $|in>\neq|out>$. Riemannian space interacting quantum field theory based on GR, thus, cannot be a satisfactory ground for addressing the processes of rearrangement of vacuum state and spontaneous gauge symmetry breaking in gravity. As a remarkable extension of GR is an analogy revealed

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between the gravitons and $SU(2) \times SU(2)$ chiral dynamics based on non-linear realizations of chiral symmetry Borisov (1974). This based on the main fact that infinite-dimensional algebra of general covariance group is the closure of finite-dimensional algebra of conformal and linear affine groups Borisov (1973).

Our interest in Ter-Kazarian (2026) was to discuss the most controversial point that 'distortion' transformations are not uniquely specified, leaving the possibility to assign many physically inequivalent metrics to one and the same distortion gauge field. We therefore uniquely determined the 'distortion' transformations and, thus, the 'distortion' group. This was achieved by the non-linear realization of the Lie group G of 'distortion' of the G_{12} by employing the exponential parametrization of the metric tensor in the framework of the method of phenomenological Lagrangians Coleman & Zumino (1969), Curtis G. & Zumino (1969), Isham (1969), Weinberg (1970). Treating the 'distortion' group G and its stationary subgroup $H = SO(3)$, respectively, as the dynamical group and its algebraic subgroup, we identify the parameters of quotien space G/H of adjacent classes by the subgroup H with the Goldstone fields of spontaneous breaking of 'distortion' symmetry. We study the geometrical structure of the space of parameters in terms of Cartan forms introduced through the appropriate Maurer-Cartan's structure equations. This allows us to infer the group invariants and calculate the conserved currents as its inevitable corollaries. Conceptually and technique-wise this method is versatile and powerful. Continuing along this line, in the present article we have gained some insight into the proposed GGP formalism to clarify and rigorously present the results and formulations already given. We show that the theory of the gravitational field is, in fact, the theory of spontaneous breaking of the affine and conformal symmetries just as chiral dynamics is the theory of spontaneous breaking of chiral symmetry. In this manner, non-linear realizations of the group of general covariance, linearized on the Poincaré group, are constructed. Although such an unexpected treatment entails a significant extension beyond the geometrical interpretation of gravity, no single theory has been invented yet that successfully addresses the solution to the problems involved.

1.1. Rationale

We develop on the extension of standard gauge principle to the framework of GGP Ter-Kazarian (1997, 2001) (see Appendix), wherein general covariance and gauge invariance are maintained. In this, we discuss the gauge group of gravitation generated by hidden local internal symmetries implemented on the auxiliary MS-space. Involving the auxiliary MS-space, with the whole set of well-defined Killing's vectors, has a single aim as a guiding tool in dealing with an intricate'jungle' of curved geometry. We extend the curvature of the space-time continuum to general 'distortion' as the theory of spontaneous breaking of 'distortion' symmetry. This can be achieved by non-linear realizations of the 'distortion' group. In terms of Cartan's forms, implying the Maurer-Cartan's structure equations, we treat the modified 'distortion' fields as the Goldstone fields and calculate the conserved currents. The framework of GGP allows to address the processes of rearrangement of vacuum state and spontaneous gauge symmetry breaking in gravity. We have to implement the simplest hidden gauge symmetry of the abelian group $U^{loc} = U(1)_Y \times \overline{U}(1)$ on the extended 12-dimensional MS-space (sect.III). The introduced group entails two neutral gauge vector bosons of $\overline{U}(1) \equiv diag[SU(2)]$ or that coupled of third component T^3 of isospin $\vec{T}$, and of $U(1)_Y$, or hypercharge Y. Since in the MS-space interacting quantum field theory the vacuum is well-determined and unique, spontaneous symmetry breaking is achieved in the standard manner by introducing the neutral complex Higgs scalar. A Non-vanishing vacuum expectation value (VEV) leaves one Goldstone boson, which is gaged away from the scalar sector. However, it essentially reappears in the gage sector, providing the longitudinally polarized spin state of one of the gage bosons that acquires mass through its coupling to the Higgs scalar. Two neutral gauge bosons were mixed to form two physical orthogonal states: one is the massless component of the 'distortion' field responsible for gravitational interactions, and the other is its massive component responsible for the ID-regime. At this level, the theory is renormalizable, and thus one has a self-contained theory because gauge invariance ensures conservation of charge and the cancelation of quantum corrections that would otherwise result in infinitely large amplitudes. Without careful thought, we expect that in the framework of GGP the renormalizability of the theory will not be spoiled in curved space-time too because the infinities arise from the ultra-violet properties of Feynman integrals in momentum space which, in coordinate space, are short distance properties, and locally (over short distances) all curved space-time looks like MS-space. A more detailed analysis and calculations on this will be presented in another paper to follow at a later date. In the resulting theory, simultaneously with the strong gravity, the massive ID-field component causes the shift of zero point energy and, thus, a significant change in the properties of the space-time continuum occurs. This leads to the 'matter-to-proto-matter' phase transition occurring at huge energies.

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1.2. Result

The paper is organized as follows: In the sect.II we briefly outline the framework of GGP and relate it to non-linear realization of the group of general covariance. We will refrain from providing lengthy details of the formalism of GGP and unitary mapping. For this, the reader is referred to the Appendix. In the sect.III we extend the space-time continuum to the extra-dimensions, and show that the curvature is a particular regime of general 'distortion' of the space-time continuum. The latter is due to spontaneous breaking of 'distortion' symmetry. We supplement this by addressing the processes of rearrangement of vacuum state and spontaneous gauge symmetry breaking in gravity (sect.IV), and, further, study the resulting process of 'matter-to-proto-matter' phase transition (sect.V). In the Appendix, we complete the formalism of GGP; in particular, much more can be done to clarify and rigorize the results and formulations already given in Ter-Kazarian (1997, 2001). We will be brief and often suppress the indices without notice; also, unless otherwise stated, we take geometrized units throughout this paper.

2. GGP and Non-Linear Realization of The Group of General Covariance in R_4

In the framework of GGP Ter-Kazarian (1997, 2001) the most general trend emerges as the formalism of unitary mapping of the standard gauge field dynamics from auxiliary MS-space M_4 to curved Riemannian R_4 space. The quantities denoted by wiggles hereafter refer to 'distorted' space; in particular, in this section, they refer to curved space, while the quantities referring to MS- space are left without wiggles.

2.1. General prescription

• Let $a(x)\,(\equiv a_\mu(x))$ be the massless gauge field, taking values in the Lie algebra $\hat{g}$ of abelian group U^{loc}. It is a local form of expression of connection in the principal bundle $p: E \to M_4$ with a structure group U^{loc}. The coordinates (x) exist in the whole region $\mathcal{U}$ of M_4, where the metric $(\hat{\eta})$ is $S^2\tau^*$: $\hat{\eta}: \tau\times\tau \to C^\infty(M_4)$, with symmetric components (η_{lk}) in the basis e_l. A collection Φ of matter fields of arbitrary spin are the sections of vector bundles associated with U^{loc} by mapping $\Phi: M_4 \to E$ such that $p\,\Phi(x) = x$. The various suffixes of Φ are left implicit. They take values in the standard fiber $\mathcal{F}_x$ upon $x: p^{-1}(\mathcal{U}) = \mathcal{U}\times\mathcal{F}_x$, where an expansion into a direct product $p^{-1}(\mathcal{U}) = \mathcal{U}\times M_4$ is defined upon $\mathcal{U}$. The fiber is a Hilbert vector space on which a linear representation $U(x)$ of the group U^{loc} is given. This space can be regarded as the Lie algebra of the group U^{loc} upon which the Lie algebra acts according to the law of adjoint representation: $a \leftrightarrow ad\,a\;\Phi \to [a\,,\Phi]$.
• Furthermore, let $(\widetilde{x})$ be the coordinates existing in the whole region $\widetilde{\mathcal{U}}$ of curved Riemannian space R_4, where the metric is the section of conjugate vector bundle $S^2\widetilde{\tau}^*$ (symmetric part of tensor degree): $\hat{g}: \widetilde{\tau}\times\widetilde{\tau} \to C^\infty(R_4)$, with symmetric components given in the basis $\widetilde{e}_\mu$. That $\widetilde{e}_\mu = g_{\mu\nu}\,\widetilde{e}^\nu$. In holonomic basis, one has $\hat{g} = g_{\mu\nu}\,d\,\widetilde{x}^\mu \times d\,\widetilde{x}^\nu$. The general coordinate transformations $\delta\widetilde{x}_\mu = f_\mu(\widetilde{x})$, where $f_\mu(\widetilde{x})$ is an arbitrary function of coordinates $\widetilde{x}_\mu$, give rise to the infinite-parameter group of general covariance in R_4 if only the functions $f_\mu(\widetilde{x})$ can be expanded in power series of $\widetilde{x}_\mu$. While the expansion coefficients can be considered as the group-parameters, and the group-algebra includes an infinite number of generators $L_\mu^{n_1\,n_2\,n_3\,n_4} = -i\widetilde{x}_1^{n_1}\,\widetilde{x}_2^{n_2}\,\widetilde{x}_3^{n_3}\,\widetilde{x}_4^{n_4}\,\widetilde{\partial}_\mu \qquad (\widetilde{\partial}_\mu = \partial/\partial\widetilde{x}^\mu)$.

Remark: The invariance of Lagrangian of the given field $\widetilde{\Phi}(\widetilde{x})$ of arbitrary spin under the infinite-parameter group of general covariance in R_4 entails the invariance of Lagrangian under the finite local gauge transformations $U_R\,(\widetilde{x})$ of the unitary Lie group of 'gravitation' G_R and vice versa, if only the local gauge transformations of the field $\widetilde{\Phi}(\widetilde{x})$ and covariant derivative $\nabla_\mu\widetilde{\Phi}(\widetilde{x})$ hold as

$$
\begin{aligned}
&\widetilde{\Phi}'(\widetilde{x}) = U_R\,(\widetilde{x})\,\widetilde{\Phi}(\widetilde{x}),\\
&\left[g^\mu(\widetilde{x})\,\nabla_\mu\widetilde{\Phi}(\widetilde{x})\right]' = U_R\,(\widetilde{x})\left[g^\mu(\widetilde{x})\,\nabla_\mu\widetilde{\Phi}(\widetilde{x})\right].
\end{aligned}
\tag{1}
$$

The ∇_μ is the covariant derivative agreed with the metric: $\nabla_\mu = \partial_\mu + \Gamma_\mu$, where $\Gamma_\mu(\widetilde{x}) = \frac{1}{2}\,\Sigma^{\alpha\beta}\,V^\nu_\alpha(\widetilde{x})\widetilde{\partial}_\mu\,V_{\beta\nu}(\widetilde{x})$ is the connection Birrell & Davies (1982), $\Sigma_{\alpha\beta}$ are the generators of Lorentz group Λ. The components $V^\mu_\alpha(\widetilde{x}) =< \widetilde{e}^\mu, \widetilde{e}_\alpha >$ ($V_{\beta\nu} = g_{\mu\nu}V^\mu_\beta$) of affine tetrad vectors $\widetilde{e}^\alpha$ in used coordinate net $\widetilde{x}^\mu$ associate with the chosen representation $D(\Lambda)$ by which the many-component filed $\Phi(x)$ is transformed as $[D(\Lambda)]^{l'\cdots k'}_{l\cdots k}\,\Phi(x)$, $\quad D(\Lambda) = I + \frac{1}{2}\omega^{\alpha\beta}\Sigma_{\alpha\beta}$, where $\omega_{\alpha\beta} = -\omega_{\beta\alpha}$ are the parameters of the Lorentz group.

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One has, for example, to set $g^{\mu}(\widetilde{x}) \to \widetilde{e}^{\mu}(\widetilde{x})$ and $\gamma^{l} \to e^{l}$ for the fields of spin $(j = 0, 1)$; for vector field $[\Sigma_{\alpha\beta}]^{l}_{k} = \delta^{l}_{\alpha}\eta_{\beta k} - \delta^{l}_{\beta}\eta_{\alpha k}$; but $g^{\mu}(\widetilde{x}) = V^{\mu}_{\alpha}(\widetilde{x})\,\gamma^{\alpha}$ and $\Sigma_{\alpha\beta} = -(1/4)[\gamma_{\alpha}, \gamma_{\beta}]$ for the spinor field $(j = \frac{1}{2})$, where γ^{α} are Dirac's matrices. We shall for the moment write the unitary matrix $U_R\,(\widetilde{x})$ in symbolic form which will be determined below as a function of U^{loc}. We can state:

Theorem 1: There always exists the gauge field dynamics defined on M_4 underlying the given gauge dynamics of Eqs. (1) defined on R_4.

Proof: One has to construct the diffeomorphism $\widetilde{x}^{\mu}(x^{l}) : M_4 \to R_4$ (App.1) and then show that the following smooth unitary mapping of the fields and their covariant derivatives holds:

$$
\begin{aligned}
&R(a) : \Phi \to \widetilde{\Phi}(\Phi), \quad (\mathcal{F}_x \to \widetilde{\mathcal{F}}_{\widetilde{x}}), \\
&S(a)\,R(a) : \left(\gamma^{k} D_k \Phi\right) \to \left(g^{\nu}(x)\nabla_{\nu}\widetilde{\Phi}\right),
\end{aligned}
\tag{2}
$$

where $\Phi \subset \mathcal{F}_x$, $\widetilde{\Phi} \subset \widetilde{\mathcal{F}}_{\widetilde{x}}$, $\widetilde{\mathcal{F}}_{\widetilde{x}}$ is the fiber upon $\widetilde{x} : p^{-1}(\widetilde{\mathcal{U}}) = \widetilde{\mathcal{U}} \times \widetilde{\mathcal{F}}_{\widetilde{x}}$, $\widetilde{\mathcal{U}}$ is the region of base R_4, $R(a)$ denotes the unitary mapping matrix, $D_k = \partial_k - i\text{æ}\,a_k(x)$, æ is the gauge coupling constant which relates to the Newton gravitational constant as in Eq. (51), $S(a)$ is the gauge invariant scalar function given on M_4 (see Eq. (44)). We use the following notational conventions throughout this section: Greek indices stand for variables in R_4, but Latin indices refer to M_4, also $\psi^{\mu}_{l} \equiv \partial_l\,\widetilde{x}^{\mu}$ where $\partial_l = \frac{\partial}{\partial\,x^{l}}$. Putting forth the tensor suffixes in illustration of the point at issue, the Eqs. (2) explicitly read

$$
\widetilde{\Phi}^{\mu\cdots\delta}(\widetilde{x}) = \psi^{\mu}_{l}\cdots\psi^{\delta}_{m}\,R(a)\,\Phi^{l\cdots m}(x) \equiv (R_{\psi})^{\mu\cdots\delta}_{l\cdots m}\,\Phi^{l\cdots m}(x),
\tag{3}
$$

and

$$
g^{\nu}(x)\nabla_{\nu}\widetilde{\Phi}^{\mu\cdots\delta}(\widetilde{x}) = S(a)\,\psi^{\mu}_{l}\cdots\psi^{\delta}_{m}\,R(a)\,\gamma^{k} D_k\,\Phi^{l\cdots m}(x).
\tag{4}
$$

It is a straightforward to determine the transformation matrix $U_R\,(\widetilde{x})$ from the Eq. (3) or Eq. (4) and gauge transformations on M_4, such that

$$
\widetilde{\Phi}'(\widetilde{x}) = U_R\,(\widetilde{x})\,\widetilde{\Phi}(\widetilde{x}) = U_R\,R_{\psi}(a)\,\Phi = R'_{\psi}\,\Phi' = R'_{\psi}\,U\,\Phi,
$$

or

$$
\begin{aligned}
&\left(g^{\nu}(x)\nabla_{\nu}\widetilde{\Phi}(\widetilde{x})\right)' = U_R\,(\widetilde{x})\,\left(g^{\nu}(x)\nabla_{\nu}\widetilde{\Phi}(\widetilde{x})\right) = U_R\,S(a)\,R_{\psi}(a)\,\left(\gamma^{k} D_k \Phi\right) = \\
&S(a')\,R'_{\psi}\,\left(\gamma^{k} D_k\,\Phi\right)' = S(a')\,R'_{\psi}\,U\,\left(\gamma^{k} D_k\,\Phi\right).
\end{aligned}
$$

Hence $U_R = R'_{\psi}\,U\,R^{-1}_{\psi}$, where $R'_{\psi} \equiv R_{\psi}(a')$ and a' is the gauge field transformed under the gauge group U^{loc}. The Theorem 1 leads to the extension of standard gauge scheme to GGP, the principle idea of which is framed into requirement of *invariance of physical system of fields $\widetilde{\Phi}(\widetilde{x})$ under the finite local gauge transformations Eq. (1) of the Lie group of gravitation U_R. The latter is generated by hidden local internal symmetry U^{loc} implemented on the MS-space*:

$$
\begin{tikzcd}
\widetilde{\Phi}'(\widetilde{x}) = U_R\,\widetilde{\Phi}(\widetilde{x}) & & \widetilde{\Phi}(\widetilde{x}) \arrow[ll, "U_R = R'_{\psi}\,U\,R^{-1}_{\psi}"'] \\
\Phi'(x) = U\,\Phi(x) \arrow[u, "R'_{\psi}(\widetilde{x},\,x)"] & & \Phi(x) \arrow[ll, "\mathrm{U}"'] \arrow[u, "R_{\psi}(\widetilde{x},\,x)"']
\end{tikzcd}
$$

A scheme of GGP.

In this paper we employ a simplest abelian symmetry U^{loc} as the hidden symmetry, but this can be easily generalized to non-abelian symmetries.

• Following Ter-Kazarian (2026), the infinite-parameter group of general covariance can be substituted by the joint invariance with respect to affine and conformal groups simultaneously. Therefore, the GR is the theory of spontaneous breaking of the affine and conformal symmetries, just as chiral dynamics is the theory of spontaneous breaking of the chiral symmetry. This is achieved in the framework of the method of phenomenological Lagrangians Coleman & Zumino (1969), Curtis G. & Zumino (1969), Isham (1969), Weinberg (1970). The key idea of this powerful approach based on the identification of Goldstone fields with those parameters of the dynamical group corresponding to the generators which do not relate to the conservation laws. A study of geometrical structure of the space of these parameters allows to infer the

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explicit form of the group invariants and calculate the conserved currents. The affine group $A(4)$ is the group of all the linear transformations in the 4-dimensional space-time, having Poincaré group as subgroup. The algebra of $A(4)$ consists of 4-generators of translations P_μ, 6-generators of Lorentz subgroup $L_{\mu\nu}$, and 10-generators of special linear (affine) transformations $R_{\mu\nu}$ containing the strains. The Poincaré subgroup relates to conservation of total energy-momentum and angular momentum. But a non-linear realization of the affine group, linearized on the Poincaré group, leads to 10-component symmetric tensor of Goldstone fields $h_{\mu\nu}$. The $h_{\mu\nu}$ correspond to the rest of 10-parameters of special affine transformations of the quotien space of adjacent classes by the Lorentz stationary subgroup. The abelian group of translations has an essential role because of the coordinates, now being the parameters of the group they are figured as Goldstone fields. In the framework of GGP formalism, it is straightforward to re-write the $h_{\mu\nu}$ in terms of 'distortion' transformations matrix $D(a)$: $h_{\mu\nu} = (\ln D(a))_{\mu\nu}$, where the basis vectors e at point $P \in M_4$ transformed into the basis vectors $\widetilde{e}(a)$ (Eq. (41)) at point $\widetilde{P} \in R_4$ under the action of the gauge field (a). The action of element of the group $A(4)$ in exponential parametrization is given as

$$\begin{array}{l} g\ \exp(i\widetilde{x}_\mu P_\mu)\exp(ih_{\mu\nu}R_{\mu\nu}/2) = \\ \exp(i\widetilde{x}'_\mu P_\mu)\ \exp[ih'_{\mu\nu}(\widetilde{x}')R_{\mu\nu}/2]\exp[iU_{\mu\nu}L_{\mu\nu}/2], \end{array} \tag{5}$$

where the $\widetilde{x}'_\mu$, $h'_{\mu\nu}(\widetilde{x}')$ and $U_{\mu\nu}$ are dependent of the parameters of the transformation g and the field $h_{\mu\nu}$. The Cartan's forms are determined from the equation

$$\begin{array}{l} \exp(-\frac{i}{2}h_{\alpha\beta}R_{\alpha\beta})\exp(-i\widetilde{x}_\mu P_\mu)d\ \exp(i\widetilde{x}_\mu P_\mu)\ \exp(\frac{i}{2}h_{\alpha\beta}R_{\alpha\beta}) = \\ i[\omega^p_\mu(d)P_\mu + \omega^R_{\mu\nu}(d)R_{\mu\nu}/2 + \omega^L_{\mu\nu}(d)L_{\mu\nu}/2]. \end{array} \tag{6}$$

The latter allows to rewrite explicitly the Cartan's forms in terms of distortion matrix $D(a)$ (see Eq. (41)-Eq. (42)):

$$\begin{array}{l} \omega^p_\mu(d) = D_{\mu\nu}(a)d\,\widetilde{x}^\nu = (\delta_{\mu l}+ < e_\mu\chi_l >)d\,x^l = \\ d\,x_\mu - \frac{1}{2}D_{\mu\nu}\int_0^x(\partial_k D^\nu_l - \partial_l D^\nu_k)d\,x^k d\,x^l, \\ \omega^R_{\mu\nu}(d) = \left(D^{-1}_{\mu\sigma}(a)d\,D_{\sigma\nu}(a) + D^{-1}_{\nu\sigma}(a)d\,D_{\sigma\mu}(a)\right)/2, \\ \omega^L_{\mu\nu}(d) = \left(D^{-1}_{\mu\sigma}(a)d\,D_{\sigma\nu}(a) - D^{-1}_{\nu\sigma}(a)d\,D_{\sigma\mu}(a)\right)/2, \end{array} \tag{7}$$

where the invariant bilinear form reads

$$\begin{array}{l} (ds^2) = \omega^p_\mu(d)\,\omega^p_\mu(d) = g_{\mu\nu}\,d\,\widetilde{x}^\mu\,d\,\widetilde{x}^\nu = \\ r_{\mu\sigma}(\widetilde{x})\,r_{\sigma\nu}(\widetilde{x}) = D^\sigma_\mu(a)\,D^\nu_\sigma(a), \quad D^{-1}_{\mu\nu}(a) = (\exp^{-h})_{\mu\nu}. \end{array} \tag{8}$$

The forms ω^p and ω^R determine the covariant differentials of coordinate and Goldstone field h, but the form ω^L determines the covariant differential of the matter field $\widetilde{\Phi}(\widetilde{x})$. However, in general, the theory cannot be unambiguously fixed just by requirement of introducing the minimal number derivatives. As it is shown in Borisov (1974), the requirement that the theory will correspond at the same time to the realization of the conformal group, uniquely leads to the Einstein 's Lagrangian of gravitational field. Algebra of the conformal group contains the generators of the Poincaré group $L_{\mu\nu}$ and P_μ, and the generators of scaling and special conformal transformations. A determination of covariant derivative of the spinor field is identical to that as in tetrad formalism Eq. (1). Utilizing then Einstein 's Lagrangian of gravitational field as $\widetilde{L}_a(\widetilde{x}) = R/16\pi G$, where R denotes the scalar curvature on R_4, and we readily get the 10 field equations (see Eq. (50))

$$\left(R^\mu_\lambda - \frac{1}{2}\delta^\mu_\lambda R\right)\ D^l_\mu = -8\pi G\widetilde{T}_{\lambda\nu}D^{\nu l}, \tag{9}$$

obviously leading to Einsten 's 10 equations for 10 metric components of $g_{\mu\nu}(a)$. But the principle difference is now that: one is allowed to determine the energy-momentum tensor of gravitational field Ter-Kazarian (1997) and formulate local energy-momentum conservation laws for the whole gravitational system quite naturally by exploiting the advantages of auxiliary 'shadow' field dynamics on M_4, and transferring it further to R_4 in accordance to unitary mapping laws.

3. Static spherical-symmetric gravitational field

According to general prescription, the field equations are derived from an invariant action $S = S_a + \widetilde{S}_{\widetilde{\Phi}}$, being set up in the similar form of Eq. (48). The action of gauge field of distortion S_a, defined on G_{12}, is invariant under the coordinate Λ and gauge U^{loc} groups, but the action of matter fields $\widetilde{S}_{\widetilde{\Phi}}$, defined on $\widetilde{G}_{12}$,

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is invariant under the gauge group of gravitation G_R. Field equations follow in terms of Euler-Lagrange variations respectively on the G_{12} and $\widetilde{G}_{12}$. Using the Lagrangian of undegenerated Killing form on the Lie algebra of the group U^{loc} in adjoint representation

$$L_a(\zeta) = -\tfrac{1}{4} < F_{AB}(\zeta),\, F^{AB}(\zeta) >_K, \tag{10}$$

defined on G_{12}, the equation of distortion field then reads

$$\partial^B \partial_B\, a_A - (1-\zeta_0^{-1})\partial_A \partial^B\, a_B = -\tfrac{1}{2}\sqrt{g}\,\tfrac{\partial g^{BC}}{\partial a_A}\widetilde{T}_{BC}, \tag{11}$$

where $\widetilde{T}_{BC}$ denotes the energy-momentum tensor, ζ_0 is the gauge fixing parameter. The curvature of manifold $G_6 \to \widetilde{G_6}$, which leads to Riemannian space R_4 (Eq. (69)), is the particular distortion at

$$a_{(1,1,\alpha)} = a_{(2,1,\alpha)} \equiv \tfrac{1}{\sqrt{2}}\, a_{(+\alpha)}, \quad a_{(1,2,\alpha)} = a_{(2,2,\alpha)} \equiv \tfrac{1}{\sqrt{2}} a_{(-\alpha)}. \tag{12}$$

We determine, for example, the line element in case of static spherical-symmetric gravitational field $a_0(r)$ in outside of the configuration with mass M. Passing back $\widetilde{G}_6 \to R_4$, there is the group of motions $SO(3)$ with two-dimensional space-like orbits S^2 where the standard coordinates are $\widetilde{\theta}$ and $\widetilde{\varphi}$. The stationary subgroup of $SO(3)$ acts isotropically upon the tangent space at the point of sphere S^2 of radius $\widetilde{r}$. So, the bundle $p: R_4 \to \widetilde{R}^2$ has the fiber $S^2 = p^{-1}(\widetilde{x})$, $\quad \widetilde{x} \in R_4$ with a trivial connection on it, where $\widetilde{R}^2$ is the quotient-space $R_4/SO(3)$. The coordinates $\widetilde{x}^\mu(\widetilde{t}, \widetilde{r}, \widetilde{\theta}, \widetilde{\varphi})$ implying the diffeomorphism $\widetilde{x}^\mu(x)^l : M_4 \to R_4$ exist in the whole region $p^{-1}(\widetilde{\mathcal{U}}) \in R_4$, where $\widetilde{x}^{0r} \equiv \widetilde{t}$, $\quad \widetilde{x}^{0\theta} = \widetilde{x}^{0\varphi} = 0$. The field equation Eq. (11) given in Feynman gauge is reduced to $\nabla^2 a_0 = 0$, which has the solution $x_0 \equiv æ\, a_0 = -\frac{r_g}{2r}$, where the diffeomorphism $\widetilde{r}(r) : M_4 \to R_4$ is given $r = \widetilde{r} - \frac{r_g}{4}$, $\quad r_g$ is the gravitational radius. A straightforward calculations show that the relations $\frac{\partial \psi^l_\mu}{\partial \widetilde{x}^\nu} \neq \Gamma^\lambda_{\mu\nu}\, \psi^l_\lambda$ hold for the non-vanishing Christoffel symbols, where

$$\frac{\partial \widetilde{x}^\mu}{\partial x^l} \equiv \psi^\mu_l = \frac{1}{2}\left(D^\mu_l + \omega^m_l D^\mu_m\right), \tag{13}$$

provided by $D^0_0 = 1 + x_0$, $\quad D^r_r = 1 - x_0$, $\quad \omega^1_1 = \omega^2_2 = \omega^3_3 = 1$, $\quad \omega^0_1 = t\, \partial_r D^0_0$. Hence, the curvature is not zero. The line element can be written as

$$d\,s^2 = (1-x_0)^2\, d\widetilde{t}^2 - (1+x_0)^2\, d\widetilde{r}^2 - \widetilde{r}^2(\sin^2\widetilde{\theta}\, d\,\widetilde{\varphi}^2 + d\,\widetilde{\theta}^2), \tag{14}$$

where $x_0 = \frac{r_g}{2\,r}$. At $\frac{r_g}{r} << 1$, one has $x_0 \simeq \frac{r_g}{2\,\widetilde{r}}\left(1 + \frac{r_g}{4\,\widetilde{r}}\right)$, such that

$$\begin{aligned} &g_{00} = 1 - \tfrac{r_g}{\widetilde{r}}, \quad g_{11} = -\left(1 + \tfrac{r_g}{\widetilde{r}} + \tfrac{r_g^2}{2\,\widetilde{r}^2}\right), \\ &g_{22} = -\widetilde{r}^2, \quad g_{33} = -\widetilde{r}^2\, \sin^2\widetilde{\theta}, \end{aligned} \tag{15}$$

while all the other components are zero. Therefore, from the view point of post-Newton experiments, the discussed gravitational theory is indiscernible from the general relativity . But, the milestone of difference will be arisen at strong gravity.

4. Rearrangement of Vacuum State

Collecting together the results just established in previous sections we finally arrive at the quite promising discussion of the rearrangement of vacuum state in gravitation, while the renormalizability of the theory remains intact because of spontaneously broken gauge symmetry. Notice that abelian U^{loc}-gauge invariant theory is renormalizable because gauge invariance gives conservation of charge, also ensures the cancelation of quantum corrections that would otherwise result in infinitely large amplitudes. To trace this line we implement the simple abelian gauge group

$$U^{loc} = U(1)_Y \times \overline{U}(1) \equiv U(1)_Y \times diag[SU(2)], \tag{16}$$

on the G_{12}, with the group elements of $exp\,[i\frac{Y}{2}\,\theta_Y(\zeta)]$ of $U(1)_Y$ and $exp\,[iT^3\,\theta_3(\zeta)]$ of $\overline{U}(1)$. The group Eq. (16) leads to the renormalizable theory on G_{12}. This has two generators, the third component T^3 of isospin $\overrightarrow{T}$ related to the Pauli spin matrix $\frac{\overrightarrow{\tau}}{2}$, and hypercharge Y implying $Q^{gr} = T^3 + \frac{Y}{2}$, where Q^{gr} is the "gravitational charge" operator assigning the number -1 to particles, but +1 to anti-particles. The

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group Eq. (16) entails two neutral gauge bosons W_A^3 of $\overline{U}(1)$ or that coupled of T^3, and B_A of $U(1)_Y$, or hypercharge Y. Gauge invariant Lagrangian of fermion field is given in standard form $\mathcal{L} = \overline{\psi}(\zeta) i\gamma^A D_A \psi(\zeta)$, provided by covariant derivative

$$D_A \psi(\zeta) = \left(\partial_A - ig T^3 W_A^3 - ig' \tfrac{Y}{2} B_A\right) \psi(\zeta), \tag{17}$$

and g, g' being the $\overline{U}(1)$, $U(1)_Y$ coupling strength, respectively. Spontaneous symmetry breaking can be achieved by the introduction of neutral complex scalar Higgs field

$$\phi = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}, \quad Y(\phi) = 1, \quad \phi^0 = \tfrac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \tag{18}$$

with the standard potential energy density function $V(\phi) = -\mu^2 \phi^+ \phi + \lambda \left(\phi^+ \phi\right)^2$, where $\mu^2 > 0$, $\lambda > 0$. This is ingredient of the gauge invariant Lagrangian of Higgs field $\mathcal{L}_H = (D_A \phi)^+ \left(D^A \phi\right) - V(\phi)$, where

$$D_A \phi(\zeta) = \left(\partial_A - ig T^3 W_A^3 - ig' \tfrac{Y}{2} B_A\right) \phi(\zeta). \tag{19}$$

Minimization of the vacuum energy fixes non-vanishing VEV:

$$< \phi >_0 \equiv < 0|\phi|0 >= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = \left(\frac{\mu^2}{\lambda}\right)^{1/2}, \tag{20}$$

leaving one Goldstone boson. The VEV of Eq. (20) spontaneously breaks the theory, leaving the $U(1)_{gr}$ subgroup intact. The unitary gauge

$$\phi(\zeta) = U^{-1}(\xi_3) \begin{pmatrix} 0 \\ \frac{v+\eta(\zeta)}{\sqrt{2}} \end{pmatrix}, \quad U(\xi_3) = \exp\left[\frac{i\xi_3 \cdot \tau^3}{v}\right], \tag{21}$$

is parameterized by two real shifted fields ξ_3 and η, such that $< 0|\xi_3|0 >=< 0|\eta|0 >= 0$. The gauge transformation

$$\phi' = U(\xi_3)\,\phi = \frac{v+\eta}{\sqrt{2}}\,\chi, \quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{22}$$

then leads to $V(\phi') = \mu^2\eta^2 + \lambda v \eta^3 + \frac{\lambda}{4}\,\eta^4$, which gives the mass of Higgs boson $M_H = \sqrt{2}\,\mu$. An examination of the v^2-terms in the kinetic piece of the Lagrangian $\mathcal{L}_H = (D_A \phi')^+ \left(D^A \phi'\right) - V(\phi')$, reveals the mass terms for the physical gauge bosons:

$$\begin{aligned} &\tfrac{v^2}{2}\, \left|\left(i\tfrac{g}{2}\,\tau^3\, {W'}_A^3 + ig'\,\tfrac{Y}{2}\, B'_A\right)\chi\right|^2 = \\ &\tfrac{1}{2}\,(Z_A,\, A_A) \begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^A \\ A^A \end{pmatrix}. \end{aligned} \tag{23}$$

The mass matrix is diagonalized by the standard orthogonal transformations:

$$\begin{aligned} &Z_A = \cos\theta_W\, {W'}_A^3 + \sin\theta_W\, B'_A, \quad M_Z = \tfrac{v}{2}\sqrt{g^2 + g'^2}, \\ &A_A = \sin\theta_W\, {W'}_A^3 + \cos\theta_W\, B'_A, \quad M_A = 0, \end{aligned} \tag{24}$$

where $\tan\theta_W = g'/g$. Namely, the neutral gauge field ${W'}_A^3$ mixes with the abelian gauge field B'_A to form the physical states Z_A and A_A, with the masses M_Z and M_A, respectively. For neutral current we get

$$\mathcal{L}_{int} = æ\,\left(\mathcal{J}_A^{gr}\, A^A + \mathcal{J}_A^{in}\, Z^A\right) \equiv æ\,\left(\mathcal{J}_A^{dis}\, a^A\right), \tag{25}$$

where $æ = g\,\sin\theta$, and

$$\begin{aligned} &\mathcal{J}_A^{gr} = \overline{\psi}(\zeta) i\gamma_A\, Q^{gr}\, \psi(\zeta), \quad \mathcal{J}_A^{in} = \overline{\psi}(\zeta) i\gamma_A\, Q^{in}\, \psi(\zeta), \\ &\mathcal{J}_A^{dis} = \overline{\psi}(\zeta) i\gamma_A\, \psi(\zeta), \quad Q^{in} = \frac{T^3 - \sin^2\theta_W,\, Q^{gr}}{\sin\theta_W\, \cos\theta_W},. \end{aligned} \tag{26}$$

To obtain some feeling for the distortion field $a_A(\zeta)$ defined on G_{12}, the massless field $Q^{gr} \cdot A_A$ will be identified with the gravitational field component a_a $(a \equiv (\lambda\alpha),\ \lambda = \pm)$ of $a_A \equiv (a_a,\, \overline{a}_a)$, but the massive field $Q^{in} \cdot Z_A$ will be identified with the ID-component $\overline{a}_a$, that

$$Q^{gr} \cdot A_A(\zeta) \equiv (a_a(\zeta),\, 0)\,, \quad Q^{in} \cdot Z_A(\zeta) \equiv (0,\, \overline{a}_a(\zeta))\,. \tag{27}$$

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Hence

$$\begin{array}{l} A_a = \frac{a_a}{Q^{gr}} = \frac{1}{\cos\theta_W} B'_a = \frac{1}{\sin\theta_W} W'^3_a, \\ Z_a = \frac{\overline{a}_a}{Q^{in}} = -\frac{1}{\sin\theta_W} \overline{B}'_a = \frac{1}{\cos\theta_W} \overline{W}'^3_a, \end{array} \tag{28}$$

where $B'_A \equiv \left(B'_a, \overline{B'}_a\right)$, $W^{3\prime}_A \equiv \left(W^{3\prime}_a, \overline{W^{3\prime}}_a\right)$. Therefore,

$$\begin{array}{l} M_{B'} = M_{W'} = M_a = 0, \quad M_Z = \sin\theta_W M_{\overline{B}'} = \cos\theta_W M_{\overline{W}'}, \\ M_{\overline{a}} = M_Z/Q^{in} = \tan\theta_W M_Z, \end{array} \tag{29}$$

where we have taken into account that $Y(a) = Q(a) = T^3(a) = 0$. Corresponding coupling constants are: $æ_A = æ Q^{gr}$ and $æ_Z = æ Q^{in} = æ \cot\theta_W$, respectively for the components A_a and Z_a. The Compton wave-length of the massless gravitational component A_a is infinite $\lambda_A = \infty$, but the Compton wave-length of the massive ID-component Z_a is finite due to spontaneous symmetry breaking. This may be of vital interest for the physics of superdense matter in very compact astrophysical sources if (Ter-Kazarian (2001))

$$\lambda_Z = \frac{2\hbar}{cv\sqrt{g^2 + g'^2}} \leq 0.4\, fm, \tag{30}$$

where $\simeq 0.4\, fm$ is the distance between the particles at nuclear density (we re-instate c).

Remark: The principle assumption went into the building of the suggested approach that the Higgs boson is coupled only with the distortion field $a_A(A_a, Z_a)$, but not with other auxiliary fields defined on G_{12}. This means that all the coupling constants of Yukawa's interaction of Higgs particle with auxiliary "shadow" fields are zero. Therefore, the Higgs boson interacts with the real matter fields defined on the distorted space $\widetilde{G}_{12}$ only via the metric tensor.

5. The 'matter-to-proto-matter' phase transition

For the reminder of this article, we discuss the 'matter $\to$ proto-matter' phase transition processes occurred in the ID-regime at

$$a_{(1,1,\alpha)} = -a_{(2,1,\alpha)} \equiv \frac{1}{\sqrt{2}}\, \overline{a}_{(+\alpha)}, \quad a_{(1,2,\alpha)} = -a_{(2,2,\alpha)} \equiv \frac{1}{\sqrt{2}}\, \overline{a}_{(-\alpha)}. \tag{31}$$

This yields the 12 distorted basis vectors $(\widetilde{e}^\mu, \widetilde{\overline{e}}^\mu)$ transformed by the matrices $(D^\mu_\nu(æ_Z Z), \overline{D}^\mu_\nu(æ_Z Z))$, where $\widetilde{e}^\mu$ and $\widetilde{\overline{e}}^\mu$ are the basis vectors given in distorted spaces $\widetilde{G}_6 = \widetilde{R}^3 \oplus \widetilde{T}^3$ and $\widetilde{G}_6 = \widetilde{\overline{R}}^3 \oplus \widetilde{\overline{T}}^3$, respectively. The transformation matrices read

$$D^\mu_\nu(æ_Z Z) = \delta^\mu_\nu + d^\mu_\nu(æ_Z Z), \quad \overline{D}^\mu_\nu(æ_Z Z) = \delta^\mu_\nu + \overline{d}^\mu_\nu(æ_Z Z), \tag{32}$$

where we leave the matrices $d^\mu_\nu(æ_Z Z)$ and $\overline{d}^\mu_\nu(æ_Z Z)$ implicit, and $\tan\theta_a = -æ_Z Z_a$. Given the distorted basis vectors, it is straightforward to derive the laws of phase transition of individual particle found in the ID-region. Actually, the 12-momentum $\widetilde{P}_A$ of the particle, as the Poincaré generator of translation in the $\widetilde{G}_{12}$, can be written

$$\widetilde{\overline{P}} = \widetilde{e}^A\, \widetilde{P}_A = \widetilde{e}^\mu\, p_\mu - \widetilde{\overline{e}}^\mu\, \overline{p}_\mu \equiv e^\mu\, p^f_\mu - \overline{e}^\mu\, \overline{p}^f_\mu. \tag{33}$$

Accordingly,

$$p^f_\mu \equiv p_\mu - \overline{d}^\nu_\mu\, \overline{p}_\nu, \quad \overline{p}^f_\mu \equiv \overline{p}_\mu - d^\nu_\mu\, p_\nu. \tag{34}$$

Due to condition $\widetilde{P}^2 = P^2 = 0$ held for any field defined on $\widetilde{G}_{12}$ or G_{12}, one has

$$p^{f\,2}_\mu = \overline{p}^{f\,2}_\mu \equiv m^2_f, \quad p^2_\mu = \overline{p}_\mu^{\,2} \equiv m^2, \tag{35}$$

where m and m_f are taken to denote the ordinary and distorted masses at rest of a particle, respectively. According to the laws described in section III of the passage, we may then proceed to the 4-dimensional space-time continuum. Thus, the matter found in the ID-region of the 4-dimensional space-time continuum undergoes a phase transition of the second kind, i.e., the new phase of 'proto-matter' comes into being, and the particles of which are released from the mass shell as the shift of mass $(m \to m_f)$ and energy-momentum $(p_\mu \to p^f_\mu)$ occurs upwards along the energy scale.

In the particular case, for example, of a static spherically symmetric distortion field a_A, with two components

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of a one-dimensional massless gravitational field $a(r) \equiv a_{0\,3}(r)$ and a one-dimensional space-like massive ID-field $\overline{a}(r) \equiv \overline{a}_3(r)$, we have

$$
\begin{array}{l}
a_{(1,1,3)} = a_{(2,2,3)} = \frac{1}{2}\left(-a + \overline{a}\right), \quad a_{(1,2,3)} = a_{(2,1,3)} = \frac{1}{2}\left(-a - \overline{a}\right), \\
a_{(\lambda,\mu,1)} = a_{(\lambda,\mu,2)} = 0, \quad (\lambda, \mu = 1, 2).
\end{array} \tag{36}
$$

In this case, the metric Eq. (14), given in a holonomic coordinate basis, is generalized as

$$
\begin{array}{l}
g_{00} = (1 - x_0)^2 + \overline{x}^2, \quad g_{11} = -\widetilde{r}^2, \quad g_{22} = -\widetilde{r}^2 \sin^2 \widetilde{\theta}, \\
g_{33} = -\left[(1 + x_0)^2 + \overline{x}^2\right], \quad g_{\mu\nu} = 0 \quad (\mu \neq \nu),
\end{array} \tag{37}
$$

where $x_0 \equiv æ_A\, a$ and $\overline{x} \equiv æ_Z\, \overline{a}$.

6. Concluding remarks

The missing ingredient of our approach is the Higgs boson. The four parameters g, g', μ, λ are inserted by hand, which, in turn, determine the coupling constants $æ_A$ and $æ_Z$ of the massless gravitational (A) field and the massive (Z) ID-component, respectively. Two relations can be imposed upon these parameters:

$$
2\sqrt{\pi\, G_N} = \frac{g\, g'}{\sqrt{g^2 + g'^2}}, \quad \frac{2\hbar}{cv\, \sqrt{g^2 + g'^2}} \leq 0.4\, fm.
$$

It is worth to note that the quanta of the gravitational gauge field (a) possess one unit of spin, and the gauge coupling constant ($æ_A$) has dimension -1. Though the properties of suggested theory are far better: some divergences are got rid off. One could hope that the problems above will disappear and the quantizing gravity will be straightforward if one proceeds heuristically, from the stand point of particle physicist. That is, one may start with building the quantized theory of interacting distortion gauge field a on the MS-space, at which the theory is renormalizable. Then, in the framework of GGP, to transform this, further, into the theory of quantum gravity defined on R_4. Given a particular choice of a regularization scheme, a theory of interest is said to be renormalizable if only a finite number of constants require normalization. Without careful thought, we expect that the renormalizability of the theory will not be spoiled in curved space-time too, because, the infinities arise from ultra-violet properties of Feynman integrals in momentum space which, in coordinate space, are short distance properties, and locally (over short distances) all curved space-time look like MS-space. A more detailed analysis and calculations on this will be presented in another paper to follow at a later date. The advantages of suggested approach may summary as:

- Unitary.
- Renormalizability at least on the MS-space.
- The phase transition of "matter $\rightarrow$ proto-matter" in ID-regime ($Z \neq 0$) simultaneously with strong gravity($A \neq 0$) at density range above nuclear density.
- The prospect for the future theory of quantum gravity.

Despite these success, a number of important questions, which may have rather interesting implications for the physics of superdense astrophysical sources, should be addressed yet.

Appendices

Appendix A The GGP

The following steps went into the building of the framework of GGP:

A.1 Diffeomorphism $\widetilde{x}^{\mu}(x^l) : M_4 \rightarrow R_4$

The basis vectors e at point $P \in M_4$ transformed into the basis vectors $\widetilde{e}(a)$ at point $\widetilde{P} \in R_4$ under the action of the gauge field a:

$$
\widetilde{e}^{\mu}(a) = D^{\mu}_{l}(a)\, e^{l}. \tag{38}
$$

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The explicit form of transformation matrix $D(a)$ is uniquely specified in the sect. III. Constructing the diffeomorphism $\widetilde{x}^\mu(x^l): M_4 \to R_4$, the holonomic functions $\widetilde{x}^\mu(x^l)$ are the solution of following equation:

$$\widetilde{e}_\mu(a)\,\psi^\mu_l = e_l + \chi_l(a), \tag{39}$$

where

$$\chi_l(a) = \widetilde{e}_\mu\,\chi^\mu_l = -\frac{1}{2}\,\widetilde{e}_\mu \int_0^x (\partial_k\,D^\mu_l - \partial_l\,D^\mu_k)\,dx^k. \tag{40}$$

Proposition: The covector $\chi_l(a)$ realizes coordinates $\widetilde{x}^\mu$ by providing the conditions of integrability

$$\partial_k\,\psi^\mu_l = \partial_l\,\psi^\mu_k \tag{41}$$

and undegeneration $\|\psi\| \neq 0$ Dubrovin & et al. (1986), Pontryagin (1984).

Proof: It is straightforward to show that $\psi^\mu_l = D^\mu_l + \chi^\mu_l$, where

$$\chi^\mu_l = \frac{1}{2}\int_0^x \chi^\mu_{kl}\,dx^k, \quad \chi^\mu_{kl} \equiv -(\partial_k\,D^\mu_l - \partial_l\,D^\mu_k).$$

Therefore $\partial_k\,\chi^\mu_l - \partial_l\,\chi^\mu_k = \chi^\mu_{kl}$, and Eq. (42) holds.

Remark: Out of all the arbitrary coordinates in R_4 the *real -curvilinear* coordinates $\widetilde{x}$ can be distinguished, which are inferred from Eq. (39) under all possible coordinate (Λ) and gauge (U^{loc}) transformations. Therefore, there exist preferred systems and group of transformations of real -curvilinear coordinates. The wider group of arbitrary coordinate transformations in R_4 no longer is of importance for field dynamics. If an inverse function ψ^l_μ meets condition

$$\frac{\partial\psi^l_\mu}{d\widetilde{x}^\nu} \neq \Gamma^\lambda_{\mu\nu}\,\psi^l_\lambda, \tag{42}$$

where $\Gamma^\lambda_{\mu\nu}$ denotes the usual Christoffel symbols agreed with the metric $g_{\mu\nu}$, then the curvature on R_4 is not zero Weinberg (1972). The equation (39) yields an invariant bilinear form $d\,s^2$ on R_4, as well as the gauge invariant scalar functions χ and S as follows:

$$\begin{array}{l} d\,s^2 = g_{\mu\nu}\,d\,\widetilde{x}^\mu d\,\widetilde{x}^\nu = \left[(e_l + \chi_l)\,d\,x^l\right]^2 = inv, \\ \chi = <e^l, \chi_l> = \frac{\omega}{2} - 2, \quad S = \frac{1}{4}\,\psi^l_\mu\,D^\mu_l = \frac{1}{2}(1 + \frac{\omega}{4}), \end{array} \tag{43}$$

where the tensor ω implies

$$\partial_l D^\mu_k(a) = \partial_k\left[\omega^m_l(x)\,D^\mu_m(a)\right]. \tag{44}$$

The trace $\omega = tr\,\omega^m_l$ should be invariant under U^{loc}-gauge group. This can be satisfied if ω is a function of the trace of curvature form Ω of connection $\mathbf{a}$ with the values in Lie algebra of group U^{loc}: $\omega = \omega(tr\,\Omega) = \omega(d\,tr\,\mathbf{a})$, provided by

$$\Omega = \sum_{l<k} F_{lk}\,dx^l \wedge dx^k, \quad \mathbf{a} = a_l\,dx^l,$$

where $F_{lk} = \partial_l\,a_k - \partial_k\,a_l$. Hence, a functional ω has a null variational derivative $\frac{\delta\omega(tr\,\Omega)}{\delta a_l} = 0$ at local variations of connection $a_l \to a_l + \delta a_l$. Thus, the ω is an invariant with respect to coordinate $(\Lambda^{l'}_k = \partial_k\,x'^l)$ and U^{loc}-gauge transformations: $\omega = inv(\Lambda, U^{loc})$.

There is a conformity between corresponding tensors with various suffixes on R_4 and M_4. While, each co- or contra -variant index transformed incorporating the functions ψ^μ_l or ψ^l_μ, respectively. Then, some coordinate or gauge transformations underly the given transformation of real -curvilinear coordinates $\widetilde{x} \to \widetilde{x}'$: $\frac{\partial\,\widetilde{x}'^{\mu'}}{\partial\,\widetilde{x}^\mu} = \psi^{\mu'}_{l'}(a')\psi^l_\mu(a)\Lambda^{l'}_l$. As we are presumably living in curved Riemannian geometry, therefore, the fields $\Phi(x)$ no longer hold and $\widetilde{\Phi}(\widetilde{x})$ should be regarded as the real physical fields. But a conformity mentioned above enables the $\Phi(x)$ to serve as an auxiliary 'shadow' fields, given on the MS-space M_4, as a guiding tool in dealing with an intricate curved geometry. These notions arise basically from the most important fact that the Lagrangian $\widetilde{L}(\widetilde{x})$ of the fields $\widetilde{\Phi}(\widetilde{x})$ can be obtained under the unitary mapping from the Lagrangian $L(x)$ of corresponding $\Phi(x)$ fields and vice versa:

$$J_\psi\widetilde{L}(\widetilde{x})\Big|_{inv(G_R;\,\widetilde{x}\to\widetilde{x}')} = L(x)|_{inv(\Lambda;\,U^{loc})}\,, \tag{45}$$

where $J_\psi \equiv \|\psi\|\,\sqrt{-g}$, g denotes the determinant of metric tensor. The resulting Lagrangian $\widetilde{L}(\widetilde{x})$ is also an invariant under the wider group of arbitrary curvilinear transformations. The local internal gauge

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symmetry U^{loc} is now hidden, as it was replaced by gauge group of gravitation G_R. The tetrad $\widetilde{e}^\alpha \in R_4$ and basis $e^l \in M_4$ vectors meet the condition

$$< \widetilde{e}^\alpha,\, e_l >= V^\alpha_\mu(\widetilde{x})\, D^\mu_l(a), \quad < e^\alpha,\, e_\beta >=< \widetilde{e}^\alpha,\, \widetilde{e}_\beta >= \delta^\alpha_\beta, \tag{46}$$

while the Minkowski metric reads $< \widetilde{e}^\alpha,\, \widetilde{e}^\beta >= diag(1,-1,-1,-1)$. In case of zero curvature $R^\lambda_{\mu\nu\tau} = 0$, the Eq. (39) can be satisfied globally in M_4 by setting

$$\psi^\mu_l = D^\mu_l = V^\mu_l = \frac{\partial x^\mu}{\partial \xi^l}, \quad \|D\| \neq 0, \quad \chi_l = 0, \tag{47}$$

where ξ^l are inertial coordinates. Namely in this case $S = J_\psi = 1$, we have gauge theory given on the M_4 in both the curvilinear and inertial coordinates.

A.2 Field equations

Field equations can be inferred from an invariant action as

$$\begin{aligned} &S = S_a + \widetilde{S}_{\widetilde{\Phi}} = \int \sqrt{-\eta}\, L_a(x)\, d^4x + \int \sqrt{-g}\, \widetilde{L}_{\widetilde{\Phi}}(\widetilde{x})\, d^4\widetilde{x} = \\ &\widetilde{S}_{\widetilde{a}} + \widetilde{S}_{\widetilde{\Phi}} = \int \sqrt{-g}\, \Big(\widetilde{L}_a(\widetilde{x}) + \widetilde{L}_{\widetilde{\Phi}}(\widetilde{x})\Big)\, d^4\widetilde{x}, \end{aligned} \tag{48}$$

where L_a and $\widetilde{L}_{\widetilde{a}}$ denote the Lagarangians of the gauge field (a) given on the M_4 and R_4, respectively. The Lagrangian $L_a(x)$ is invariant under Λ-coordinate and U^{loc}-gauge groups. But the Lagrangians $\widetilde{L}_{\widetilde{\Phi}}(\widetilde{x})$ and $\widetilde{L}_{\widetilde{a}}(\widetilde{x})$ of matter fields $\widetilde{\Phi}(\widetilde{x})$ and gauge field $\widetilde{a}$, respectively, given on the R_4, are invariant under the gauge group of gravitation G_R. In terms of Euler-Lagrange variations in M_4 and R_4, we readily obtain

$$\begin{aligned} &\frac{\delta(\sqrt{-\eta}\, L_a)}{\delta\, a_l} = J^l = -\frac{\delta(\sqrt{-\eta}\, L_\Phi)}{\delta\, a_l} = -\frac{\partial\, g^{\mu\nu}}{\partial\, a_l}\frac{\delta(\sqrt{-g}\, \widetilde{L}_{\widetilde{\Phi}})}{\delta\, g^{\mu\nu}} = - \\ &\frac{1}{2}\frac{\partial\, g^{\mu\nu}}{\partial\, a_l}\frac{\sqrt{-g}}{\|D\|}\frac{D_{k\mu}\,\delta(\sqrt{-g}\widetilde{L}_{\widetilde{\Phi}})}{\delta\, D^\nu_k} = -\frac{\sqrt{-g}}{2}\frac{\partial\, g^{\mu\nu}}{\partial\, a_l}\widetilde{T}_{\mu\nu}, \\ &\frac{\delta \widetilde{L}_{\widetilde{\Phi}}}{\delta\,\widetilde{\Phi}} = 0, \quad \frac{\delta\, \widetilde{L}_{\widetilde{\Phi}}}{\delta \overline{\widetilde{\Phi}}} = 0, \end{aligned} \tag{49}$$

where $\widetilde{T}_{\mu\nu}$ is the energy-momentum tensor of the matter fields $\widetilde{\Phi}(\widetilde{x})$. It is straightforward to choose the 10 components of symmetric tensor $D^{\mu\nu}(a)$ as the characteristic of gravitational field defined on M_4 without invoking the gauge field a. Actually, from Eq. (49) we get

$$\frac{D_{k\mu}\,\delta\, L_a}{\|D\|\,\delta\, D^\nu_k} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,\widetilde{L}_a)}{\delta\, g^{\mu\nu}} = -\widetilde{T}_{\mu\nu}. \tag{50}$$

Therefore, we must fix either the Lagrangians L_a or $\widetilde{L}_a$. In section III we would uniquely determine the transformation matrix $D(a)$ and Lagrangian L_a.

Remark: To re-define the gauge coupling constant æ in terms of Newton gravitational constant G_N, we ought to consider the theory at the limit of Newton's non-relativistic law of weak gravitation for the static weak gravitational field given by Poisson equation $\nabla^2 g_{00} = -8\,\pi\, G_N\, T_{00}$. First equation in (49), for the static field $x_0 = æ\, a_0$, gives: $\nabla^2\, x_0 = \frac{æ^2}{2}\frac{\partial\, g_{00}}{\partial\, x_0}T_{00}$, where in linear approximation fitting our interest $g_{00} \approx (1-2x_0)$. Hence $\nabla^2\, g_{00} = -2\,\nabla^2\, x_0 = -2æ^2\, T_{00}$ and

$$G_N = æ^2/4\pi, \tag{51}$$

which shown that only the gravitational attraction exists.

A.3 Unitary mapping matrix for the fields of spin $j = 0, 1\,\frac{1}{2}$

In this section we determine the unitary mapping matrix $R(a)$ and gauge invariant scalar function $S(a)$ in case of the most simple fields of spin 0, 1, and $\frac{1}{2}$. But, this can be readily extended to any spin (see next section).

Remark: A general recipe for determination of the functions $R(a)$ and $S(a)$ for given field is to insert Eq. (3) into Eq. (4) to obtain the identity, in which then one may equate the coefficients in front of either functions $\partial\,\Phi$ and Φ to zero. In this manner one obtains two equations from which one determines the functions $R(a)$ and $S(a)$:

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1) A straightforward calculation for the fields of spin $j = 0, 1$ gives the unitary matrix

$$R(\widetilde{x}, x) = R(x)\, R_g(\widetilde{x}) = \exp\left(-i\Theta(x) - \Theta_g(\widetilde{x})\right), \tag{52}$$

provided by

$$\Theta(x) = æ \int_0^x a_l(x)\, d\, x^l, \quad \Theta_g(\widetilde{x}) = \int_0^x \left[R^+\, \Gamma_\mu\, R + \psi^{-1} \widetilde{\partial}_\mu\, \psi\right] d\, \widetilde{x}^\mu, \tag{53}$$

where $\psi \equiv \left(\psi_l^\mu\right)$, $\Theta_g = 0$ for scalar field and $\Theta_g + \Theta_g^+ = 0$ for vector field, because of $\Gamma_\mu + \Gamma_\mu^+ = 0$. The scalar function $S(a)$ has the form of Eq. (44) : $S(B_f) = \frac{1}{4}\, R^+(\psi_\mu^l\, D_l^\mu)\, R = \frac{1}{4}\, \psi_\mu^l\, D_l^\mu$.

2) Unitary mapping of spinor field $\Psi(x)$ $(j = \frac{1}{2})$ can be written:

$$\begin{array}{l} \widetilde{\Psi}(\widetilde{x}) = R(a)\, \Psi(x), \quad \widetilde{\overline{\Psi}}(\widetilde{x}) = \overline{\Psi}(x)\, \widetilde{R}^+(a), \\ g^\mu(\widetilde{x}) \nabla_\mu\, \widetilde{\Psi}(\widetilde{x}) = S(a)\, R(a)\, \gamma^l D_l\, \Psi(x), \\ \left(\nabla_\mu\, \widetilde{\overline{\Psi}}(\widetilde{x})\right) g^\mu(\widetilde{x}) = S(a)\, \left(D_l\, \overline{\Psi}(x)\right)\, \gamma^l\, \widetilde{R}^+(a), \end{array} \tag{54}$$

where

$$\widetilde{R} = \gamma^0 R \gamma^0, \quad \Sigma^{\alpha\beta} = \tfrac{1}{4}[\gamma^\alpha, \gamma^\beta], \quad \Gamma_\mu(x) = \tfrac{1}{4} \Delta_{\mu,\alpha\beta}(x) \gamma^\alpha \gamma^\beta, \tag{55}$$

$\Delta_{\mu,\alpha\beta}(x)$ denote the Ricci rotation coefficients. The unitary mapping matrix $R(a)$ is in the form of Eq. (52), provided we make single change

$$\Theta_g(x) = \tfrac{1}{2} \int_0^x R^+ \left\{g^\mu \Gamma_\mu R,\, g_\nu\, d\, x^\nu\right\}. \tag{56}$$

A calculation now gives

$$S(a) = \tfrac{1}{8K} \psi_\mu^l \left\{\widetilde{R}^+ g^\mu R,\, \gamma_l\right\} = inv, \tag{57}$$

where $K = \widetilde{R}^+\, R = \widetilde{R}_g^+\, R_g$, and

$$\begin{array}{l} K = \exp\left(-\tfrac{1}{2} \int_0^x \left(\left\{R^+ \widetilde{\Gamma}_\mu^+ g^\mu,\, g_\nu dx^\nu\right\} R +\, R^+ \left\{g^\mu \Gamma_\mu R,\, g_\nu dx^\nu\right\}\right)\right), \\ \widetilde{\Gamma}_\mu^+ = \gamma^0 \Gamma_\mu^+ \gamma^0. \end{array} \tag{58}$$

Taking into account a commutation relation $[R, g_\nu] = 0$, and substituting (Weinberg (rf1)):

$$\widetilde{\Gamma}_\mu^+ g^\nu + g^\nu \Gamma_\mu = -\nabla_\mu g^\nu = 0, \tag{59}$$

we get $K = 1$. Note that $\widetilde{U}_R^+ U_R = \widetilde{R} U^+ \widetilde{R'}^+ R' U R^+ = \widetilde{R} U^+ \widetilde{R}^+ R U R^+$, and $\widetilde{R'}^+ R' = \widetilde{R}^+ R = 1$, therefore

$$\widetilde{U}_R^+ U_R = \gamma^0 U_R^+ \gamma^0 U_R = 1. \tag{60}$$

A.4 The GGP for any spin

The higher-spin equations in given curved space were studied earlier by many authors Belifante (1940), Buchdhal (1958, 1962), Christensen & Duff (1978), M.M. (1954), Penrose (1962), which have led to a number of insights, but also obvious limitations. For instance, it is well known inconsistencies emerged in the standard approach Buchdhal (1958, 1962), Dowker (1972) to derive these equations, except for spins half and one, by the formal prescription of replacing ordinary derivatives by covariant ones. It is necessary to introduce modifications which would provide compatibility condition in curved space Buchdhal (1962), but it is open question how this can be achieved without the rather artificial implication of auxiliary fields. Some authors prefer to describe a spin-j particle by a $(2j+1)$-component field ϕ_m^j Cap (1953), Dowker (1972), Weinberg (1964) in order to avoid this inconsistency problem. A new framework that we shall implement here has the following features. We employ an unitary mapping formalism developed in section II incorporated with the most convenient Bargman-Wigner's wave functions for higher-spin in flat space Bargman & Wigner (1948). The formalism of GGP will then hold for the field of arbitrary spin j, because the latter is treated as a system of $2j$ fermions of half-integer spin. This approach is obviously free of inconsistency problem mentioned above. Actually, note that the latter emerges under unwanted subsidiary conditions occurring only if the original spin-j field and the pure spin-2 object Ω formed from the Weyl conformal tensor can combine to produce a nonzero spin-$(j-1)$ object, which becomes meaningless for $j = 1/2$ Ter-Kazarian (2026).

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The wave function of spin-$j=\frac{n}{2}$ particle with momentum $\vec{p}$ defined on the M_4 can be obtained by Lorentz transformation from the symmetric Dirac spinor of rank n corresponding to the particle in the rest $U_{\alpha_1\ldots\alpha_n}(0)$ implying $(\gamma_4-1)_\beta^{\beta'}U_{\beta'\beta_2\ldots\beta_n}(0)$ for each index

$$U_{\beta_1\ldots\beta_n}(\vec{p})=S_{\beta_1}^{\beta_1'}(\alpha(\vec{p}))\ldots S_{\beta_n}^{\beta_n'}(\alpha(\vec{p}))U_{\beta_1'\ldots\beta_n'}(0), \tag{61}$$

where $\overline{U}'(\vec{p})\,U'(\vec{p})=\overline{U}(\Lambda^{-1}\vec{p})\,U(\Lambda^{-1}\vec{p})$. A spin part is written $\Sigma_{\mu\nu}=\frac{1}{2}\sum_{r=1}^{n}\Sigma_{\mu\nu}^{(r)}$, where a matrix $\Sigma_{\mu\nu}^{(r)}$ acts only on the r-th index

$$\left(\Sigma_{\mu\nu}^{(r)}\right)_{\alpha_1\ldots\alpha_n}^{\beta_1\ldots\beta_n}=\delta_{\alpha_1}^{\beta_1}\cdots\delta_{\alpha_{r-1}}^{\beta_{r-1}}\left(\Sigma_{\mu\nu}^{(r)}\right)_{\alpha_r}^{\beta_r}\delta_{\alpha_{r+1}}^{\beta_{r+1}}\cdots\delta_{\alpha_n}^{\beta_n},$$

that $\Sigma_{\mu\nu}^{(r)}=\frac{1}{4}[\gamma_\mu^r,\,\gamma_\nu^r]$, and

$$\left(\gamma_\mu^r\right)_{\alpha_1\ldots\alpha_n}^{\beta_1\ldots\beta_n}=\delta_{\alpha_1}^{\beta_1}\cdots\delta_{\alpha_{r-1}}^{\beta_{r-1}}\left(\gamma_\mu^r\right)_{\alpha_r}^{\beta_r}\delta_{\alpha_{r+1}}^{\beta_{r+1}}\cdots\delta_{\alpha_n}^{\beta_n}.$$

The spin-j field $\Phi_{\beta_1\ldots\beta_n}(x)$ (Eq. (61)) takes values in standard fiber $\mathcal{F}_x$ upon $x:\,p^{-1}(\mathcal{U})=\mathcal{U}\times\mathcal{F}_x$. In the framework of GGP, the mapped spin-j field $\widetilde{\Phi}_{\beta_1\ldots\beta_n}^{(r)}(\widetilde{x})$, where $\widetilde{\Phi}^{(r)}=R^{(r)}\,\Phi$, takes values in the fiber $\widetilde{F}_{\widetilde{x}}\quad(\widetilde{x}\in\widetilde{\mathcal{U}},\quad\widetilde{\mathcal{U}}$ denotes the region of the base R_4. The unitary mapping matrix $R^{(r)}$ is given in the form of Eq. (52) and Eq. (56), but now refers to the r-th index. The Lagrangian of this field will be invariant under the local gauge transformations

$$\begin{aligned}&\widetilde{\Phi}'(\widetilde{x})=U_R^{(r)}\,\widetilde{\Phi}(\widetilde{x}),\\&\left(g_{(r)}^{\mu}(\widetilde{x})\,\nabla_\mu^{(r)}\widetilde{\Phi}(\widetilde{x})\right)'=U_R^{(r)}\left(g_{(r)}^{\mu}(\widetilde{x})\,\nabla_\mu^{(r)}\widetilde{\Phi}(\widetilde{x})\right),\end{aligned} \tag{62}$$

where $g_{(r)}^{\mu}(\widetilde{x})=V_\alpha^\mu(\widetilde{x})\gamma_{(r)}^{\alpha}$, $\nabla_\mu^{(r)}$ denotes the covariant derivative on R_4 defined only for the r-th index by the conventional substitution Bargman & Wigner (1948):

$$\nabla_\mu^{(r)}\widetilde{U}_{\beta_1\ldots\beta_n}\to\Lambda_\alpha^{\alpha'}(\widetilde{x})S_{\beta_1}^{\beta_1'}(\alpha(\Lambda))\ldots S_{\beta_n}^{\beta_n'}(\alpha(\Lambda))\nabla_\mu^{(r)}\widetilde{U}_{\beta_1\ldots\beta_n}. \tag{63}$$

Here, as usual, we denote $\nabla_\alpha^{(r)}=V_\alpha^\mu\,(\partial_\mu+\Gamma_\mu^{(r)})$ and

$$\Gamma_\mu^{(r)}=\frac{1}{2}\,\Sigma_{(r)}^{\alpha\beta}\,V_\alpha^\nu(\widetilde{x})\,\partial_\mu V_{\beta\nu}(\widetilde{x})=\frac{1}{4}\Delta_{\mu,\alpha\beta}^{(r)}\,\gamma_{(r)}^{\alpha}\,\gamma_{(r)}^{\beta}, \tag{64}$$

$\Delta_{\mu,\alpha\beta}^{(r)}(x)$ are the Ricci rotation coefficients. The Eqs. (62) hold if

$$g_{(r)}^{\mu}\,\nabla_\mu^{(r)}\widetilde{\Phi}^{(r)}=R^{(r)}\,S^{(r)}\,(\gamma^l\,D_l\,\Phi), \tag{65}$$

where $D_l=\partial_l-ig\,a_l(x)$, $S(a)$ is the gauge invariant Lorentz scalar of Eq. (57) but refers to r-th index. According to the results of subsection B, we get $\widetilde{U_R^{(r)}}^{+}\,U_R^{(r)}=\gamma^0\,{U_R^{(r)}}^{+}\,\gamma^0\,U_R^{(r)}=1$. The Lagrangian of the spin-j field can be set up as

$$\begin{aligned}&L(x)=J_\psi\widetilde{L}(\widetilde{x})=J_\psi\left\{\frac{i}{2}\left[\overline{\widetilde{\Phi}}^{(r)}(\widetilde{x})\,g_{(r)}^{\mu}(\widetilde{x})\nabla_\mu^{(r)}\widetilde{\Phi}^{(r)}(\widetilde{x})-\right.\right.\\&\left.\left.(\nabla_\mu^{(r)}\overline{\widetilde{\Phi}}^{(r)}(\widetilde{x}))\,g_{(r)}^{\mu}(\widetilde{x})\widetilde{\Phi}^{(r)}(\widetilde{x})\right]-m\overline{\widetilde{\Phi}}^{(r)}(\widetilde{x})\widetilde{\Phi}^{(r)}(\widetilde{x})\right\}=\\&J_\psi\left\{S^{(r)}(a)\frac{i}{2}\left[\overline{\Phi}\,\gamma_{(r)}^{l}D_l\,\Phi-(D_l\overline{\Phi})\,\gamma_{(r)}^{l}\Phi\right]-m\overline{\Phi}\,\Phi\right\}.\end{aligned} \tag{66}$$

Generalized Bargman-Wigner's equation for the spin-$j=\frac{n}{2}$ particle in curved space stems from the Lagrangian eq. (66):

$$\begin{aligned}&\left(g'^{\mu}\,\nabla_\mu'\widetilde{\Phi}^{(r)}-m\right)_\beta^{\beta'}\,\widetilde{\Phi}_{\beta'\beta_2\ldots\beta_n}(\widetilde{\vec{p}})=\\&\left[R'\,(S'\,D-m)\right]_\beta^{\beta'}\,\Phi_{\beta'\beta_2\ldots\beta_n}(\vec{p})=0,\end{aligned} \tag{67}$$

where $R', S', \ldots$ refer to index β'.

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A.5 The manifold G_{12}, and a passage to M_4 or R_4

Consider the curve $\lambda(t) : R^1 \to G_{12}$ passing through the point $p = \lambda(0) \in G_{12}$ with tangent vector $\mathbf{A}|_{\lambda(t)}$. The $\{\zeta\}$ are local coordinates in open neighborhood of $p \in \mathcal{U}$. The 12 dimensional smooth vector field $\mathbf{A}_p = \mathbf{A}(\zeta)$ belongs to the section of tangent bundle $\mathbf{T}_p$ at the point $p(\zeta)$. The one parameter group of diffeomorphisms A^t is given for the curve $\zeta(t)$ passing through point p and $\zeta(0) = \zeta_p, \quad \dot{\zeta}(0) = \mathbf{A}_p$: $d\,A^t_p(\mathbf{A}) = \frac{d}{d\,t}\big|_{t=0} A^t\,(\zeta(t)) = \mathbf{A}_p(\zeta)$, that

$$d\,A^t : \mathbf{T}(G_{12}) \to R\left(\mathbf{T}(G_{12}) = \bigcup_{p(\zeta)} \mathbf{T}_p\right).$$

The $e_{(\lambda,\mu,\alpha)} = O_{\lambda,\mu} \times \sigma_\alpha \quad (\lambda, \mu = 1, 2; \quad \alpha = 1, 2, 3)$ are linear independent 12 unit basis vectors at the point $p = \lambda(0) \in G_{12}$. The basis vectors σ_α refer to three dimensional ordinary space $R^3_{\lambda\mu}$ at given $(\lambda,\, \mu)$. The metric on G_{12} is $\hat{\mathbf{g}} : \mathbf{T}_p \times \mathbf{T}_p \to C^\infty(G_{12})$. The decompositions

$$G_6 = R^3 \oplus T^3, \quad \overline{G}_6 = \overline{R}^3 \oplus \overline{T}^3, \tag{68}$$

hold with $sgn(R^3) = (+++)$, $\quad sgn(T^3) = (---)$ and inverse signatures for $sgn(\overline{R}^3) = (---)$, $\quad sgn(\overline{T}^3) = (+++)$. Within this scheme, one is presumably allowed to perceive directly the G_6 related to space (R^3) and time (T^3), but not the $\overline{G}_6$ displayed as the space of inner degrees of freedom.

Remark: 1) To describe the dynamics of physical system on 4-dimensional Minkowski space, we use the fact that all directions in flat time space T^3 are equivalent. Therefore, reducing two extra dimensions of time vector and passing from G_6 to $M_4 = R^3 \oplus T^1$, under the notion 'time' one is free to imply its projection on fixed arbitrary universal direction in T^3. By this reduction, which clearly respects the physical ground, the goal pursued should be readily achieved.
2) In similar passage $\widetilde{G}_6 \to R_4$ made in case of curved space, we reduce the 'three time' components $\widetilde{e}_{0\alpha}$ of basis six-vector $\widetilde{e}\,(\widetilde{e}_\alpha,\, \widetilde{e}_{0\alpha})$, at given point, to single one $\widetilde{e}_0$ in given universal direction. By this we fixed the 'time' coordinate. Certainly, as far as the Lagrangian of given physical system defined on $\widetilde{G}_6$, is the function of scalars $A_{(\lambda\alpha)}B^{(\lambda\alpha)} = A_\alpha B^\alpha + A_{0\alpha}B^{0\alpha}$, then the following reduction $A_{0\alpha}B^{0\alpha} = A_{0\alpha} < \widetilde{e}^{0\alpha}, \widetilde{e}^{0\beta} > B_{0\beta} = A_0 < \widetilde{e}^0, \widetilde{e}^0 > B_0 = A_0 B^0$ leads to the 4-dimensional Riemannian space:

$$\begin{array}{l} d\,\widetilde{\zeta}^2 = d\,s_6^2 - d\,\overline{s}_6^2 = 0, \\ d\,s_6^2\big|_{6\to 4} = d\,s_4^2 = g_{\mu\nu} d\,\widetilde{x}^\mu d\,\widetilde{x}^\nu = d\,\overline{s}_6^2 = inv. \end{array} \tag{69}$$

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