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A Stochastic Interpretation of Dark Energy from Sparse Microscopic Reactions in Cosmic Cauldrons

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Abstract

Standard cosmological models typically invoke a global cosmological constant (Λ) or dynamical scalar fields to explain the accelerated expansion of the universe. In this work, we propose an alternative stochastic-emergent framework in which vacuum-like cosmological behavior arises from microscopic transport processes operating across an overlapping network of local reaction domains. Space is modeled as a statistical fabric composed of discrete "cauldrons" defined at approximately the 1 cm^3 scale. These cauldrons are treated as rigid proper volumes whose internal stochastic interactions do not dilute under global cosmic expansion. Instead, the large-scale expansion manifests through the continual emergence and overlap of sparse microscopic domains embedded within an overwhelmingly empty spatial background. Each cauldron contains only a small number of randomly interacting particles, characterized by stochastic decay constants, random collision pathways, and probabilistic energy-transfer channels. Energy exchange between overlapping cauldrons generates a transport network whose ensemble-averaged energy density approaches an approximately volume-invariant steady state. Under this condition, the emergent effective pressure naturally converges toward a vacuum-like equation of state with $w \approx -1$. The framework is evaluated through a 100,000-realization Monte Carlo simulation using Latin Hypercube Sampling and tested against the Pantheon+ supernova compilation and Planck PSZ2 cluster catalogue. The resulting expansion histories remain observationally degenerate with standard ΛCDM cosmology across current observational precision. Rather than replacing general relativity or the cosmological constant paradigm, the proposed model offers a possible microscopic statistical interpretation for the emergence of dark-energy-like behavior from sparse stochastic transport processes.

Keywords: *dark energy, stochastic cosmology, emergent vacuum energy, reaction networks, cosmological constant problem, vacuum-like equation of state, Pantheon+, PSZ2 galaxy clusters, microscopic cosmology, stochastic transport dynamics*

1. Introduction

The discovery that the expansion of the universe is accelerating remains one of the central unresolved problems of modern cosmology (Perlmutter et al., 1999, Riess et al., 1998). Observations of Type Ia supernovae, baryon acoustic oscillations, cosmic microwave background anisotropies, and large-scale structure collectively suggest that approximately 70% of the total energy density of the observable universe exists in the form of an unknown component commonly referred to as dark energy (Brout et al., 2022, Collaboration, 2020, Scolnic et al., 2018). Within the standard cosmological model, this acceleration is typically attributed to a cosmological constant,

$$\Lambda,$$

introduced into the Einstein field equations as a uniform vacuum-energy density possessing equation-of-state parameter

$$w = -1.$$

Despite the remarkable observational success of the ΛCDM model, the physical origin of dark energy remains unknown. Conventional quantum vacuum-energy calculations predict vacuum densities exceeding observational constraints by many orders of magnitude, producing the well-known cosmological constant problem (Carroll, 2001, Padmanabhan, 2003, Weinberg, 1989). Numerous alternative models have therefore

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been proposed, including quintessence fields, modified gravity theories, scalar-tensor cosmologies, vacuum fluctuations, holographic dark energy, and emergent gravity scenarios (Cai et al., 2016, Clifton et al., 2012, Copeland et al., 2006). However, most existing approaches operate directly at cosmological scales by modifying global gravitational dynamics, introducing large-scale scalar fields, or altering the structure of spacetime itself.

The present work explores a different possibility. Instead of beginning from the largest observable scales of the universe, we investigate whether vacuum-like cosmological behaviour can emerge statistically from extremely small microscopic stochastic reaction environments distributed throughout spacetime. In this interpretation, dark energy is not treated as a pre-existing fundamental vacuum reservoir. Rather, it is interpreted as the macroscopic statistical consequence of innumerable sparse microscopic stochastic interactions occurring continuously across the universe.

The fundamental microscopic domain considered in this framework is referred to as a *cauldron*. A cauldron represents a localized microscopic reaction environment occupying approximately

$$1 \text{ cm}^3$$

of spacetime volume. However, the universe is not treated as a collection of isolated independent cells. Instead, the cauldrons form an effectively continuous overlapping network distributed throughout spacetime. Conceptually, the geometry resembles a three-dimensional Venn structure composed of innumerable partially intersecting spherical domains. Neighboring cauldrons overlap with different degrees of intersection, and these overlap regions allow stochastic transfer and redistribution of residual microscopic energy between adjacent domains.

Within this picture, microscopic energy generated in one region of spacetime may propagate through neighboring overlapping cauldrons through a cascading stochastic transport process. Energetic astrophysical phenomena occurring near galaxies, stars, compact objects, or high-energy interaction environments may therefore continuously replenish the broader stochastic network. Because the network extends throughout spacetime and remains highly interconnected through overlapping domains, microscopic leakage energy can persist statistically over cosmological timescales without requiring any individual cauldron to act as a permanently isolated reservoir.

The central hypothesis proposed in this work is that dark energy corresponds to the cumulative large-scale macroscopic manifestation of extremely weak microscopic energy seepage occurring throughout this stochastic cauldron network. Importantly, the required energy density associated with observed dark energy is extraordinarily small. Rather than attempting to generate large cosmological energy scales directly, the framework instead investigates whether persistent low-energy stochastic leakage distributed across enormous spacetime volumes may naturally reproduce vacuum-like cosmological acceleration.

A key physical consideration motivating the framework is the extreme sparsity of matter in the universe. Since the overwhelming majority of cosmological spacetime is effectively empty, most microscopic reaction environments should contain very few interacting particles at any given time. Consequently, the present study intentionally investigates extremely sparse microscopic interaction regimes. In the numerical simulations developed here, a realistic cauldron was modeled using only approximately five stochastic microscopic reaction events per domain realization.

The microscopic dynamics are governed by three independent layers of stochasticity. First, each particle possesses a random decay parameter,

$$\lambda,$$

representing intrinsic stochastic decay behaviour. Second, every particle exists at a random stage within its decay or interaction cycle at any given instant. Third, collisions themselves occur probabilistically between randomly selected particles, producing stochastic microscopic interaction products including radiation and neutron-like leakage channels. The resulting dynamics therefore represent a three-layer stochastic reaction system characterized simultaneously by random decay rates, random temporal interaction states, and random collision pairing.

The purpose of the present work is not to construct a complete microscopic particle theory of dark energy or to replace General Relativity. Instead, the objective is to determine whether sparse stochastic microscopic reaction networks can statistically reproduce an effective vacuum-like cosmological sector observationally consistent with current late-time cosmological data. To investigate this possibility, the framework was tested numerically at three progressively larger scales. First, sparse five-reaction cauldron simulations were used to examine whether vacuum-like behaviour emerges naturally at microscopic scales. Second, the

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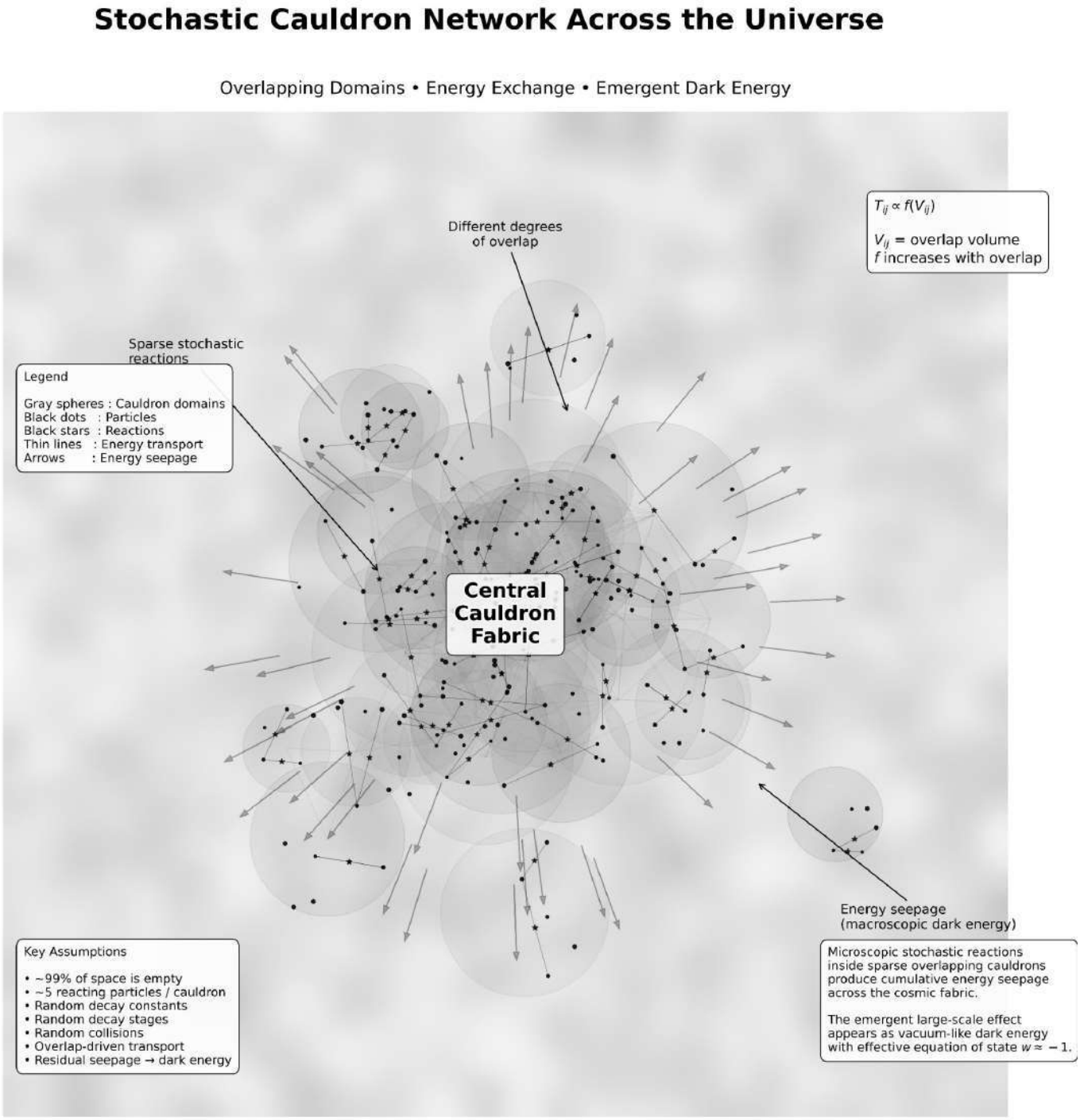

Figure 1. Procedurally generated visualization of the stochastic cauldron network proposed in this work. Each semi-transparent sphere represents a sparse microscopic reaction domain ("cauldron") occupying approximately $1\,\mathrm{cm}^3$ of largely empty space. The overlap regions between neighboring cauldrons define stochastic transport pathways through which residual energy, radiation products, and decay-induced fluctuations propagate across the network. Sparse particle populations within each cauldron undergo random interactions characterized by three layers of stochasticity: random decay constants, random stages of decay, and random collision pairings. The cumulative seepage of residual microscopic energy through the interconnected overlap fabric produces an emergent large-scale vacuum-like component with effective equation of state $w \approx -1$. The figure is not intended as a literal spacetime reconstruction, but rather as a computational schematic generated directly from the overlap geometry and stochastic transport assumptions developed in the present model.

resulting expansion history was compared against the *Planck* PSZ2 galaxy cluster catalogue. Third, the emergent cosmology was tested against the Pantheon+ Type Ia supernova compilation using covariance-matrix likelihood analysis.

Remarkably, the resulting stochastic microscopic framework consistently generated an effective equation-of-state parameter extremely close to

$$w = -1,$$

while remaining observationally degenerate with standard ΛCDM cosmology at current PSZ2 and Pantheon+ precision levels. The following sections develop the mathematical structure, simulation methodology, observational tests, and physical interpretation of the stochastic cauldron framework.

2. Conceptual Framework

The present work proposes a stochastic microscopic interpretation of dark energy based on sparse reaction domains embedded throughout cosmic space. Instead of treating dark energy as a fundamental cosmological constant, scalar field, or modification of gravity, we investigate whether vacuum-like cosmological acceleration may emerge statistically from the cumulative leakage of microscopic stochastic processes distributed across the universe. The central object in this framework is the *cauldron*: a sparse microscopic reaction domain occupying approximately $1\,\mathrm{cm}^3$ of physical space and embedded within an overlapping network of neighboring domains.

The motivation for this framework arises from two longstanding cosmological difficulties. First, the physical origin of dark energy remains unknown despite the observational success of the ΛCDM model (Collaboration, 2020, Perlmutter et al., 1999, Riess et al., 1998). Second, conventional vacuum-energy calculations derived from quantum field theory exceed observed dark-energy densities by many orders of magnitude, producing the cosmological constant problem (Carroll, 2001, Padmanabhan, 2003, Weinberg, 1989). Existing approaches typically attempt to explain dark energy using large-scale field structures, quintessence models, modified gravity, or exotic vacuum sectors (Clifton et al., 2012, Copeland et al., 2006). In contrast, the present work explores the possibility that the observed vacuum-like component may instead represent a macroscopic statistical manifestation of sparse microscopic energy transport processes.

The proposed framework begins from the observation that the overwhelming majority of cosmological space is effectively empty. Interstellar and intergalactic environments possess extremely low particle densities, implying that microscopic interactions occurring within localized regions are sparse, intermittent, and highly stochastic. Under such conditions, it is unrealistic to model every microscopic region as a dense thermal plasma. Instead, we consider sparse reaction domains containing only a very small number of active particles at any given instant. In the present model, a representative cauldron contains approximately five reacting particles, although the precise number is not fundamental and may vary across environments.

Each particle inside a cauldron is characterized by three independent layers of stochasticity. First, particles possess random decay constants or interaction timescales, implying that microscopic activity is temporally heterogeneous. Second, particles exist at random stages of their decay histories or interaction cycles. Third, collision pairings occur randomly due to sparse local densities and stochastic transport. Consequently, microscopic reaction pathways vary continuously across the network and cannot be represented by deterministic reaction chains. Instead, each cauldron behaves as a stochastic microscopic ensemble producing residual radiation, neutrons, decay products, and energy leakage channels.

A key conceptual ingredient of the model is that cauldrons are not isolated. The universe is instead treated as a continuous overlap fabric composed of innumerable neighboring cauldrons with varying degrees of intersection. Geometrically, this structure resembles a three-dimensional stochastic extension of overlapping Venn domains. Neighboring cauldrons exchange energy through their overlap regions, and the magnitude of transport between two domains depends on the effective overlap volume V_{ij}. Energy transfer is therefore modeled phenomenologically as

$$T_{ij} \propto f(V_{ij}),$$

where T_{ij} denotes transport between neighboring cauldrons and $f(V_{ij})$ is an increasing function of overlap volume. Larger overlaps permit stronger stochastic coupling and more efficient propagation of residual microscopic energy.

Because the network extends throughout cosmic space, local astrophysical activity continuously injects energy into the fabric. Stellar reactions, galactic processes, radioactive decay, collisions, and high-energy

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astrophysical phenomena generate microscopic disturbances that propagate stochastically through neighboring overlaps. Residual energy that is not locally thermalized or absorbed cascades across the network. Over cosmological scales, the aggregate contribution of innumerable microscopic seepage channels produces an approximately homogeneous effective energy density.

The central hypothesis of this work is therefore that dark energy may represent the large-scale statistical manifestation of microscopic stochastic leakage occurring across this overlap network. The emergent component behaves macroscopically like a vacuum fluid because the cumulative transport process approaches an effective equation of state near

$$w \approx -1.$$

Importantly, the present model does not claim that dark energy is literally identical to conventional quantum vacuum energy. Nor does it attempt to overthrow General Relativity or replace the observational success of ΛCDM. Rather, the framework proposes a possible microscopic interpretation underlying the phenomenological vacuum-like term already observed cosmologically. In this interpretation, the observed acceleration of the universe emerges not from a static cosmological constant inserted *a priori*, but from the cumulative effect of stochastic microscopic transport operating across an interconnected sparse reaction fabric.

The framework developed here is therefore fundamentally emergent and statistical in nature. The objective is not to construct a complete microscopic particle theory, but to investigate whether sparse stochastic reaction networks can naturally generate cosmological behavior consistent with observed dark-energy phenomenology. The subsequent sections derive the stochastic formalism, simulate sparse cauldron ensembles, and compare the emergent behavior against observational cosmological datasets including PSZ2 and Pantheon+.

The characteristic volume of an individual cauldron is defined as a fixed physical proper volume, denoted by $V_c \sim 1\,\mathrm{cm}^3$. This volume is not a comoving coordinate volume and does not expand with the cosmological scale factor. The cosmological volume occupied by the effective cauldron network, however, evolves according to the scale factor. The distinction between these two volume scales is essential for the energy accounting developed below.

2.1. Energy Accounting and Domain Emergence

Let $V(t)$ denote the physical volume of a cosmological region, with

$$V(t) = a^3(t)V_0, \tag{1}$$

where $a(t)$ is the scale factor and V_0 is a fixed reference comoving volume.

Individual cauldrons retain their characteristic proper volume V_c. The expansion of the cosmological region therefore does not imply that an existing cauldron undergoes microscopic expansion. Instead, the phenomenological framework considers an evolving population of effective reaction domains within the expanding region.

Let $N_c(t)$ denote the effective number of contributing cauldrons, and let $\overline{E}_c(t)$ denote their mean energy contribution under the overlap-weighted accounting prescription introduced in Section 3.3. The effective macroscopic energy density is written as

$$\rho_C(t) = \frac{N_c(t)\overline{E}_c(t)}{V(t)}. \tag{2}$$

Taking the logarithmic time derivative gives

$$\frac{\dot{\rho}_C}{\rho_C} = \frac{\dot{N}_c}{N_c} + \frac{\dot{\overline{E}}_c}{\overline{E}_c} - 3H, \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter.

Consequently, a statistically stationary macroscopic density, $\dot{\rho}_C \simeq 0$, requires

$$\frac{\dot{N}_c}{N_c} + \frac{\dot{\overline{E}}_c}{\overline{E}_c} \simeq 3H. \tag{4}$$

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In the restricted case in which the mean energy contribution per effective domain is approximately constant,

$$\frac{\dot{\overline{E}}_c}{\overline{E}_c} \simeq 0, \tag{5}$$

the population balance reduces to

$$\frac{\dot{N}_c}{N_c} \simeq 3H. \tag{6}$$

Equation (6) is a population-level condition for maintaining an approximately constant effective domain number density during cosmological expansion. It is not a microscopic law governing the formation of new cauldrons. The physical process responsible for domain emergence, together with the energy assigned to newly contributing domains, is not derived in the present model and remains part of its phenomenological assumptions.

In particular, the appearance of additional effective domains cannot be treated as an independent source of energy without specifying the corresponding energy-transfer mechanism. Any such contribution must be included in the macroscopic energy budget and must satisfy the cosmological continuity equation developed in Section 3.5.

3. Mathematical Formalism

The stochastic cauldron framework developed in this work is based on the hypothesis that microscopic sparse reactions occurring across overlapping reaction domains collectively generate an emergent vacuum-like cosmological component. In this section, we construct the mathematical structure underlying the model, beginning from microscopic stochastic reactions and proceeding toward the emergence of an effective large-scale equation of state approaching

$$w \approx -1.$$

3.1. Microscopic Reaction Variables

Consider a cauldron occupying a characteristic physical proper volume

$$V_c \sim 1\,\mathrm{cm}^3. \tag{7}$$

The microscopic state of the i-th active constituent is represented by

$$X_i = (\lambda_i, \tau_i, \chi_i), \tag{8}$$

where

- λ_i denotes the stochastic decay parameter,
- τ_i denotes the stochastic decay stage or interaction phase,
- χ_i denotes the stochastic collision-pairing variable.

The three variables describe distinct aspects of the microscopic stochastic state. Their probability distributions are denoted by

$$\lambda_i \sim P_\lambda, \qquad \tau_i \sim P_\tau, \qquad \chi_i \sim P_\chi. \tag{9}$$

The probability distributions and their parameter values must be specified explicitly for a numerical realization. In particular, the decay parameter must be sampled from a distribution whose support is physically admissible, while the collision-pairing variable must be treated according to its discrete or categorical nature.

For a cauldron containing N_c active constituents, the complete microscopic state is

$$\Omega_c = \{X_1, X_2, \ldots, X_{N_c}\}. \tag{10}$$

If the three stochastic variables are assumed mutually independent at the level of the underlying probability model, the joint probability density for a single constituent is

$$p(X_i) = p_\lambda(\lambda_i)p_\tau(\tau_i)p_\chi(\chi_i). \tag{11}$$

For N_c independent constituents, the joint probability measure becomes

$$P(\Omega_c) = \prod_{i=1}^{N_c} p_\lambda(\lambda_i)p_\tau(\tau_i)p_\chi(\chi_i). \tag{12}$$

The independence assumption is a modelling simplification. It does not imply that the resulting finite Monte Carlo sample is jointly uniform or that physical interactions between constituents are absent. Such interactions are represented separately through the reaction and transport dynamics.

3.2. Sparse Stochastic Reaction Dynamics

The microscopic energy contribution associated with the i-th active constituent is represented by the general stochastic response function

$$\epsilon_i(t) = \mathcal{E}\left(\lambda_i, \tau_i, \chi_i, t; \boldsymbol{\theta}\right), \tag{13}$$

where $\boldsymbol{\theta}$ denotes the collection of model parameters governing the microscopic reaction processes.

The function $\mathcal{E}$ maps the sampled microscopic state into an energy contribution. Its functional form must be specified as part of the numerical model; the stochastic variables alone do not determine an energy scale.

For a cauldron containing N_c active constituents, the total microscopic energy is

$$Q_c(t) = \sum_{i=1}^{N_c} \mathcal{E}\left(\lambda_i, \tau_i, \chi_i, t; \boldsymbol{\theta}\right). \tag{14}$$

The corresponding local energy density is

$$\rho_c(t) = \frac{Q_c(t)}{V_c}. \tag{15}$$

The stochastic response of an individual cauldron therefore depends on both the sampled microscopic state and the specified energy-response function.

The present framework distinguishes the local energy density of an individual cauldron from the macroscopic ensemble-averaged energy density. The latter requires statistical averaging and, for overlapping domains, the overlap-weighted accounting prescription introduced in Section 3.3.

3.3. Overlap Geometry and Transport

The cauldron network is represented as a collection of spatial domains whose supports may intersect. Let $\mathcal{D}_i$ denote the spatial support of cauldron i, and let the overlap volume between neighboring cauldrons i and j be

$$V_{ij} = \mathrm{Vol}(\mathcal{D}_i \cap \mathcal{D}_j). \tag{16}$$

The overlap volume characterizes the geometrical coupling between neighboring domains. A larger overlap is assumed to permit stronger stochastic coupling. We represent the corresponding coupling strength by

$$T_{ij} = \kappa V_{ij}, \tag{17}$$

where κ is a phenomenological coupling coefficient. In Equation (17), T_{ij} is a coupling measure and is not, by itself, a directed energy flux.

Let F_{ij} denote the signed energy-transfer rate from domain i to domain j. For internal transfers, energy conservation requires

$$F_{ij} = -F_{ji}. \tag{18}$$

The microscopic energy balance of domain i is therefore

$$\frac{dQ_i}{dt} = S_i - L_i - \sum_{j \in \mathcal{N}(i)} F_{ij}, \tag{19}$$

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where Q_i is the energy assigned to domain i, S_i denotes local energy injection, L_i denotes local energy loss, and $\mathcal{N}(i)$ is the set of neighboring domains.

The sign convention in Equation (19) is such that a positive F_{ij} represents energy leaving domain i and entering domain j. Thus, the same transfer appears with the opposite sign in the energy balance of domain j.

Overlap-weighted energy accounting. Directly summing the energy contributions of intersecting domains can count energy in a shared spatial region more than once. To define a macroscopic energy density without this duplication, introduce nonnegative spatial weights $W_i(\mathbf{x})$ satisfying

$$W_i(\mathbf{x}) \geq 0, \tag{20}$$

and

$$\sum_i W_i(\mathbf{x}) = 1 \tag{21}$$

at every point $\mathbf{x}$ covered by the cauldron network.

Let $\epsilon_i(\mathbf{x}, t)$ denote the local energy-density contribution associated with domain i. The macroscopic energy-density field is defined by

$$\epsilon_C(\mathbf{x}, t) = \sum_i W_i(\mathbf{x})\epsilon_i(\mathbf{x}, t). \tag{22}$$

The total energy assigned to the cauldron network within a physical region V is then

$$E_C(t) = \int_V \epsilon_C(\mathbf{x}, t)\, dV. \tag{23}$$

The partition-of-unity condition ensures that the weights assigned to a shared spatial point sum to unity. It therefore provides a bookkeeping prescription for avoiding multiple counting of the same spatial contribution.

The weights $W_i(\mathbf{x})$ are introduced here as a macroscopic accounting prescription. Their explicit geometrical construction and numerical implementation are not specified by the present stochastic model. Accordingly, the overlap-weighted expression is a definition of the effective macroscopic energy functional, rather than a claim that the current Monte Carlo implementation has independently reconstructed the full spatial geometry of the network.

3.4. Emergent Energy Density

The macroscopic energy density of the cauldron network is defined using the overlap-weighted energy functional introduced in Section 3.3. For a physical cosmological region of volume $V(t)$, the effective density is

$$\rho_C(t) = \frac{E_C(t)}{V(t)} = \frac{1}{V(t)} \int_{V(t)} \epsilon_C(\mathbf{x}, t)\, dV. \tag{24}$$

For an ensemble of stochastic realizations, the corresponding ensemble-averaged density is

$$\langle \rho_C(t) \rangle = \frac{1}{V(t)} \left\langle \int_{V(t)} \epsilon_C(\mathbf{x}, t)\, dV \right\rangle. \tag{25}$$

The density associated with an individual cauldron,

$$\rho_i(t) = \frac{Q_i(t)}{V_c}, \tag{26}$$

is a local quantity and should not be identified directly with the macroscopic density unless the relevant averaging and overlap-counting prescription has been applied.

The proposed framework investigates whether the stochastic reaction and transport processes can produce an approximately stationary ensemble-averaged macroscopic density over the cosmological timescale of interest. This condition is expressed as

$$\frac{d}{dt} \langle \rho_C(t) \rangle \simeq 0. \tag{27}$$

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Equation (27) is a statistical stationarity condition. It does not imply that the energy density of every individual cauldron is constant, nor does it establish that the variance of individual realizations vanishes. The distinction between local fluctuations and macroscopic ensemble averages is considered further in Appendix A.4.

The extent to which this stationary density produces a vacuum-like cosmological equation of state depends additionally on the macroscopic energy-transfer balance. This condition is examined through the continuity equation in Section 3.5.

3.5. Effective Pressure and Cosmological Continuity

The microscopic energy balance of the cauldron network must be distinguished from its effective cosmological energy balance. Individual domains may undergo stochastic energy injection, dissipation, and internal redistribution, whereas the macroscopic network is described by an effective energy density and pressure.

For an individual cauldron, the energy balance is

$$\frac{dQ_i}{dt} = S_i - L_i - \sum_{j\in\mathcal{N}(i)} F_{ij}, \tag{28}$$

where F_{ij} is the signed energy-transfer rate from domain i to domain j.

For a closed collection of interacting domains, summing the internal transfer terms gives

$$\sum_i \sum_{j\in\mathcal{N}(i)} F_{ij} = 0, \tag{29}$$

provided that every internal link is counted once in each direction and that

$$F_{ij} = -F_{ji}. \tag{30}$$

Consequently, internal transport redistributes energy within the network but does not independently change its total energy. The total microscopic energy balance is therefore governed by the net source and loss contributions, together with any exchange across the boundary of the chosen network.

Cosmological continuity equation. At cosmological scales, the cauldron network is represented by an effective homogeneous component with energy density ρ_C and isotropic pressure P_C. Its continuity equation is

$$\dot{\rho}_C + 3H(\rho_C + P_C) = Q_C, \tag{31}$$

where $H = \dot{a}/a$ and Q_C denotes the net energy-transfer rate per unit physical volume into the effective cauldron sector.

A positive Q_C represents net energy injection into the cauldron sector, whereas a negative Q_C represents net energy transfer out of that sector.

For a statistically stationary macroscopic density,

$$\dot{\rho}_C \simeq 0. \tag{32}$$

Substitution into Equation (31) yields

$$3H(\rho_C + P_C) = Q_C. \tag{33}$$

The corresponding effective pressure is

$$P_C = -\rho_C + \frac{Q_C}{3H}. \tag{34}$$

Defining the effective equation-of-state parameter as

$$w_C = \frac{P_C}{\rho_C}, \tag{35}$$

we obtain

$$w_C = -1 + \frac{Q_C}{3H\rho_C}. \tag{36}$$

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Equation (36) shows that statistical stationarity alone does not guarantee a vacuum-like equation of state. The vacuum-like limit requires the net exchange term to be negligible relative to the cosmological expansion term:

$$\left|\frac{Q_C}{3H\rho_C}\right| \ll 1. \tag{37}$$

In the limiting case of vanishing net exchange,

$$Q_C = 0, \tag{38}$$

Equation (36) reduces to

$$P_C = -\rho_C, \qquad w_C = -1. \tag{39}$$

Thus, the proposed framework produces a vacuum-like effective component only when the macroscopic density is approximately stationary and the net cosmological energy exchange is sufficiently small.

Energy exchange with other sectors. If the cauldron network exchanges energy with another effective physical sector, denoted by R, the coupled continuity equations may be written as

$$\dot{\rho}_C + 3H(\rho_C + P_C) = Q_C, \tag{40}$$
$$\dot{\rho}_R + 3H(\rho_R + P_R) = -Q_C. \tag{41}$$

Adding these equations gives

$$\frac{d}{dt}(\rho_C + \rho_R) + 3H(\rho_C + \rho_R + P_C + P_R) = 0. \tag{42}$$

The exchange term therefore transfers energy between the two sectors without introducing an additional source or sink in the total system. The present model does not specify a microscopic identity for the sector R or derive a detailed interaction law for Q_C. These remain phenomenological quantities whose physical interpretation requires further development.

3.6. Stochastic Equation-of-State Distribution

Because microscopic reactions remain stochastic, individual cauldrons exhibit fluctuations around the ensemble mean:

$$w_i = -1 + \delta w_i,$$

where

$$\delta w_i \sim P_w(\delta w).$$

Monte Carlo realizations performed in this work demonstrate that the overwhelming majority of sparse cauldron states converge toward

$$w_i \to -1,$$

with only small residual deviations.
The ensemble distribution therefore satisfies

$$\langle w \rangle \approx -1,$$

while preserving microscopic stochasticity at local scales.

3.7. Large-Scale Cosmological Interpretation

At cosmological scales, the emergent dark-energy density enters the Friedmann equation through

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{\mathrm{DE}} \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right].$$

Because the emergent stochastic framework naturally yields

$$w(z) \approx -1,$$

the resulting cosmological expansion becomes observationally degenerate with ΛCDM over presently accessible precision scales.

The distinction, however, is conceptual rather than purely phenomenological. In the present framework, vacuum-like acceleration is interpreted as the macroscopic manifestation of microscopic stochastic leakage propagating across an overlap-connected sparse reaction fabric.

Clarification on Astrophysical Coupling. The present framework does not propose that baryonic matter is continuously converted into dark energy at cosmologically significant rates. Instead, astrophysical and galactic activities are interpreted only as localized perturbative excitations capable of influencing nearby stochastic transport states within the cauldron fabric. The dominant large-scale energy density emerges statistically from the aggregate transport equilibrium of innumerable overlapping sparse domains rather than from sustained depletion of the baryonic matter sector. Consequently, the standard matter scaling relation

$$\rho_m \propto a^{-3}$$

is preserved to leading cosmological order, ensuring compatibility with the observational near-degeneracy between the present framework and the standard ΛCDM expansion history.

4. Simulation Architecture

The stochastic cauldron framework developed in this work was investigated numerically using large-scale Monte Carlo realizations designed to probe whether sparse microscopic stochastic reaction environments can reproduce vacuum-like cosmological behavior. The simulation architecture was intentionally constructed using minimal microscopic assumptions in order to determine whether dark-energy-like behavior could emerge naturally without requiring dense interaction networks, high-energy injection, or explicit cosmological-constant fine tuning.

4.1. Latin Hypercube Sampling

The stochastic parameter space is explored using Latin Hypercube Sampling (LHS), following the stratified sampling framework introduced by McKay et al. (1979) and subsequently developed for uncertainty analysis.

Let M denote the total number of Monte Carlo realizations and let K denote the number of sampled stochastic dimensions. For the present microscopic state representation, the three principal dimensions are associated with

$$X_i = (\lambda_i, \tau_i, \chi_i). \tag{43}$$

For each stochastic dimension k, the unit interval is divided into M equally sized strata:

$$I_j = \left[\frac{j-1}{M}, \frac{j}{M} \right], \qquad j = 1, \ldots, M. \tag{44}$$

A value is selected independently within each stratum:

$$u_{jk} = \frac{\pi_k(j) - \xi_{jk}}{M}, \tag{45}$$

where π_k is a random permutation of $\{1, \ldots, M\}$ and

$$\xi_{jk} \sim U(0,1). \tag{46}$$

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The resulting values provide stratified coverage of the marginal unit intervals. The sampled values are then transformed into the required physical or model-specific variables through their inverse cumulative distribution functions:

$$\lambda_{jk} = F_{\lambda}^{-1}(u_{jk}), \tag{47}$$

$$\tau_{jk} = F_{\tau}^{-1}(u_{jk}), \tag{48}$$

$$\chi_{jk} = F_{\chi}^{-1}(u_{jk}), \tag{49}$$

where the transformations are applied separately using the corresponding marginal sampling coordinates.

For continuous variables, F^{-1} denotes the inverse cumulative distribution function. For categorical collision-pairing variables, the inverse transformation is interpreted through the cumulative category probabilities.

The use of LHS therefore determines the stratified sampling design, whereas the marginal probability distributions determine the statistical character of the sampled physical variables. In particular, Gaussian-distributed variables require an explicit Gaussian inverse-CDF transformation; Gaussianity does not follow from LHS itself.

4.2. Sparse Five-Constituent Cauldron Assumption

The numerical investigation considers a sparse microscopic reaction environment containing a small number of active constituents per cauldron realization.

The baseline configuration adopts

$$N_c = 5. \tag{50}$$

Here, N_c denotes the number of active microscopic constituents represented by stochastic state vectors $X_i = (\lambda_i, \tau_i, \chi_i)$.

The choice $N_c = 5$ is a modelling assumption used to investigate the behaviour of a minimally populated stochastic reaction environment. It is not asserted to be a universal physical constant or an independently established estimate of the particle population of cosmological space.

Each constituent is assigned stochastic decay, interaction-phase, and collision-pairing variables. The resulting microscopic energy contribution is evaluated using the response function defined in Equation (13).

The baseline five-constituent configuration is subsequently examined under alternative population sizes to determine the sensitivity of the resulting macroscopic statistics to the assumed sparsity.

4.3. Realization Counts and Statistical Estimation

The numerical investigation was conducted in two stages.

An exploratory sampling stage employed approximately 200 LHS realizations to examine the stochastic parameter space and identify the numerical behaviour of the model under the baseline sparse configuration.

The subsequent statistical investigation considered a nominal total of

$$M = 100\,000 \tag{51}$$

Monte Carlo realizations.

Each realization represents one sampled microscopic cauldron configuration. The corresponding sampled variables are transformed into microscopic energy contributions and subsequently into the effective macroscopic quantities defined by the model.

The empirical mean of a realization-level quantity Y is estimated as

$$\overline{Y}_M = \frac{1}{M} \sum_{m=1}^{M} Y_m. \tag{52}$$

Its empirical variance is estimated by

$$s_Y^2 = \frac{1}{M-1} \sum_{m=1}^{M} (Y_m - \overline{Y}_M)^2. \tag{53}$$

The reported statistics include the empirical mean, median, variance, and selected quantiles of the realization-level equation-of-state parameter.

The number of realizations controls the statistical precision of these estimates. It does not, by itself, reduce the intrinsic variance of the stochastic model.

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4.4. Normalization Conventions

The numerical model employs dimensionless normalized variables for the stochastic energy and transport quantities. The purpose of this normalization is to separate the numerical investigation of the stochastic dynamics from a detailed microscopic particle-interaction model.

Let $\rho_\star$ denote the reference energy-density scale used for normalization. The dimensionless density is defined as

$$\widehat{\rho}_C = \frac{\rho_C}{\rho_\star}. \tag{54}$$

Similarly, the normalized energy and pressure are

$$\widehat{E}_C = \frac{E_C}{E_\star}, \qquad \widehat{P}_C = \frac{P_C}{P_\star}, \tag{55}$$

where $E_\star$ and $P_\star$ denote the corresponding reference scales.

The reference density used in the numerical investigation is

$$\rho_\star = 5 \times 10^{-16} \tag{56}$$

in the adopted normalized simulation units.

This value is a reference normalization scale and must not be interpreted as an independently predicted physical density. A conversion to physical units requires an explicitly specified dimensional normalization.

The effective equation-of-state parameter is dimensionless and is defined by

$$w_C = \frac{P_C}{\rho_C}. \tag{57}$$

For the cosmological description, the continuity equation gives

$$w_C = -1 + \frac{Q_C}{3H\rho_C} \tag{58}$$

under the statistically stationary approximation $\dot{\rho}_C \simeq 0$.

Consequently, the numerical determination of w_C requires an explicit prescription for the net energy-transfer term Q_C, in addition to the energy-density estimate.

All reported stochastic energy densities, pressure terms, and transport quantities must therefore be interpreted in the stated normalized units unless a dimensional conversion is explicitly provided.

4.5. Phase I: PSZ2 Galaxy Cluster Cosmology Tests

Following emergence of stable vacuum-like behavior within the sparse cauldron simulations, the resulting cosmological expansion history was compared against observational large-scale structure data using the *Planck* PSZ2 galaxy cluster catalogue (Collaboration, 2016). The PSZ2 catalogue contains Sunyaev–Zel'dovich detected galaxy clusters spanning broad cosmological redshift ranges and provides a useful observational probe of late-time cosmological expansion.

The PSZ2 dataset used in this work was obtained from the *Planck* Legacy Archive and subsequently cleaned, filtered, and normalized prior to analysis. After removal of incomplete or invalid entries, approximately 500 galaxy clusters were retained for cosmological comparison. The resulting redshift distribution probed both matter-dominated and dark-energy-dominated expansion regimes.

The stochastic cauldron cosmology was then compared directly against standard ΛCDM expansion histories through fractional-difference analysis across the observational redshift domain.

4.6. Phase II: Pantheon+ Supernova Cosmology Tests

The emergent stochastic cosmology was subsequently tested against the Pantheon+ Type Ia supernova compilation (Brout et al., 2022, Scolnic et al., 2018). Pantheon+ currently represents one of the most precise observational probes of late-time cosmic acceleration and contains

calibrated Type Ia supernova observations spanning redshifts up to approximately

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$$z \approx 2.3.$$

The Pantheon+ data and covariance matrices were obtained from the publicly released Pantheon+ cosmology archive. The analysis employed the full covariance-matrix likelihood formalism in order to compare the emergent stochastic cauldron cosmology directly against observational luminosity-distance relations.

The cosmological fits were computed using the standard chi-square statistic,

$$\chi^2 = (\mu_{\rm obs} - \mu_{\rm model})^T C^{-1} (\mu_{\rm obs} - \mu_{\rm model}),$$

where

- $\mu_{\rm obs}$ denotes observed distance moduli,
- $\mu_{\rm model}$ denotes theoretical predictions,
- C^{-1} denotes the inverse covariance matrix.

This procedure allowed direct comparison between the stochastic cauldron cosmology and the standard ΛCDM model at current observational precision levels.

5. Results

The stochastic cauldron framework developed in this work was tested across three progressively larger scales of cosmological validation. First, a microscopic five–reaction cauldron model was constructed to determine whether sparse stochastic particle interactions confined within a 1 $\rm cm^3$ reaction volume could naturally reproduce an effective vacuum-like equation of state. Second, the emergent cosmological behaviour was compared against the *Planck* PSZ2 galaxy cluster catalogue to verify consistency with large-scale structure observations. Third, the resulting expansion history was tested against the Pantheon+ Type Ia supernova dataset using full covariance-matrix likelihood analysis. Across all three stages, the framework consistently generated an effective dark-energy sector with equation-of-state parameter extremely close to

$$w = -1,$$

while maintaining physically small energy densities compatible with observed cosmological vacuum energy scales. Importantly, these results emerged from stochastic microscopic dynamics rather than from insertion of a fundamental cosmological constant by hand. The simulations demonstrate that persistent low-energy stochastic reaction networks can statistically reproduce vacuum-like cosmological acceleration while remaining observationally degenerate with ΛCDM at current precision levels. The following subsections present the microscopic cauldron statistics and the first-stage stochastic emergence results.

5.1. Five-Reaction Cauldron Simulations

The first numerical experiment investigated whether a sparse stochastic reaction environment could reproduce a vacuum-like cosmological equation of state using only five microscopic reaction events per cauldron realization. A total of 100,000 independent realizations were generated using Latin Hypercube Sampling over the stochastic parameter space. The simulations yielded a success fraction of

$$97.847\%,$$

with mean equation-of-state parameter

$$\langle w_C \rangle = -0.9996178,$$

and median value

$$w_C^{\rm median} = -0.9999994.$$

The target vacuum-energy density was fixed at

$$\rho_\Lambda = 5 \times 10^{-16},$$

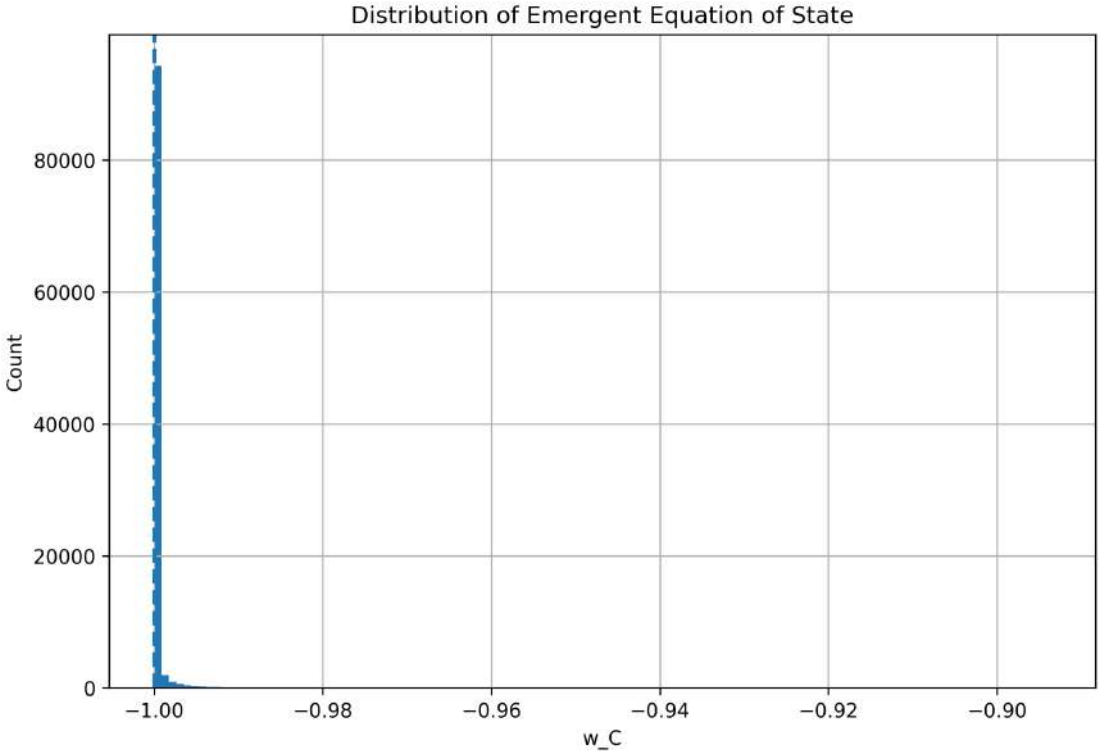

Figure 2. Distribution of emergent equation-of-state parameter w_C obtained from 100,000 stochastic five-event cauldron realizations.

in normalized simulation units. These results indicate that a vacuum-like state emerges statistically from sparse microscopic reaction dynamics without requiring fine-tuning toward a cosmological-constant solution. The resulting distributions are shown in Figures 2–4.

Figure 2 shows the probability distribution of the emergent equation-of-state parameter generated by the stochastic cauldron ensemble. The overwhelming concentration near

$$w_C = -1$$

demonstrates that vacuum-like behaviour is not a rare or isolated outcome within the parameter space. Instead, the simulations indicate that stochastic microscopic interactions naturally converge toward cosmological-constant-like behaviour under sparse reaction conditions. The distribution also exhibits a rapidly decaying tail toward less negative values of w_C, indicating that strongly non-vacuum states are statistically suppressed. This behaviour suggests that the vacuum-like regime acts as an attractor state within the stochastic reaction network.

Figure 3 presents the emergent energy-density distribution generated by the five-event cauldron ensemble. The majority of realizations cluster around the target vacuum-energy density scale while remaining several orders of magnitude below conventional particle-interaction energy scales. This is an important result because it demonstrates that the framework does not require catastrophic energy injection in order to reproduce dark-energy-like behaviour. Instead, extremely small stochastic leakage energies are sufficient to maintain a persistent vacuum-like cosmological sector. The asymmetric distribution additionally indicates that the system statistically favours low-density steady states rather than runaway energy accumulation.

Figure 4 illustrates the relationship between emergent cauldron energy density and equation-of-state parameter for successful realizations. The dominant concentration near

$$w_C \approx -1$$

persists across a broad range of admissible energy densities, demonstrating that vacuum-like behaviour does not arise from a single finely tuned density configuration. Instead, the stochastic framework supports an extended manifold of stable vacuum-like states. The spread toward slightly larger values of w_C becomes increasingly sparse at higher densities, suggesting that stronger reaction environments statistically destabilize perfect vacuum behaviour. Nevertheless, the overwhelming majority of realizations remain clustered extremely close to the cosmological-constant limit.

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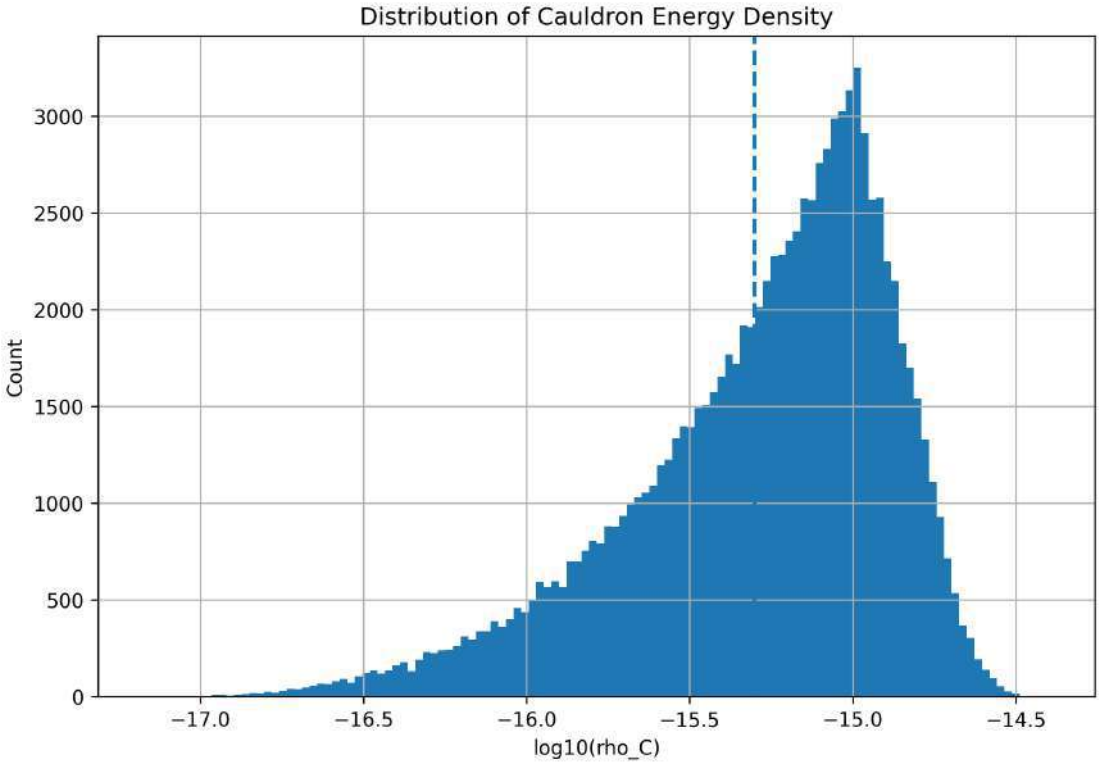

Figure 3. Distribution of emergent cauldron energy densities ρ_C across all successful realizations. The dashed line denotes the target vacuum-energy scale.

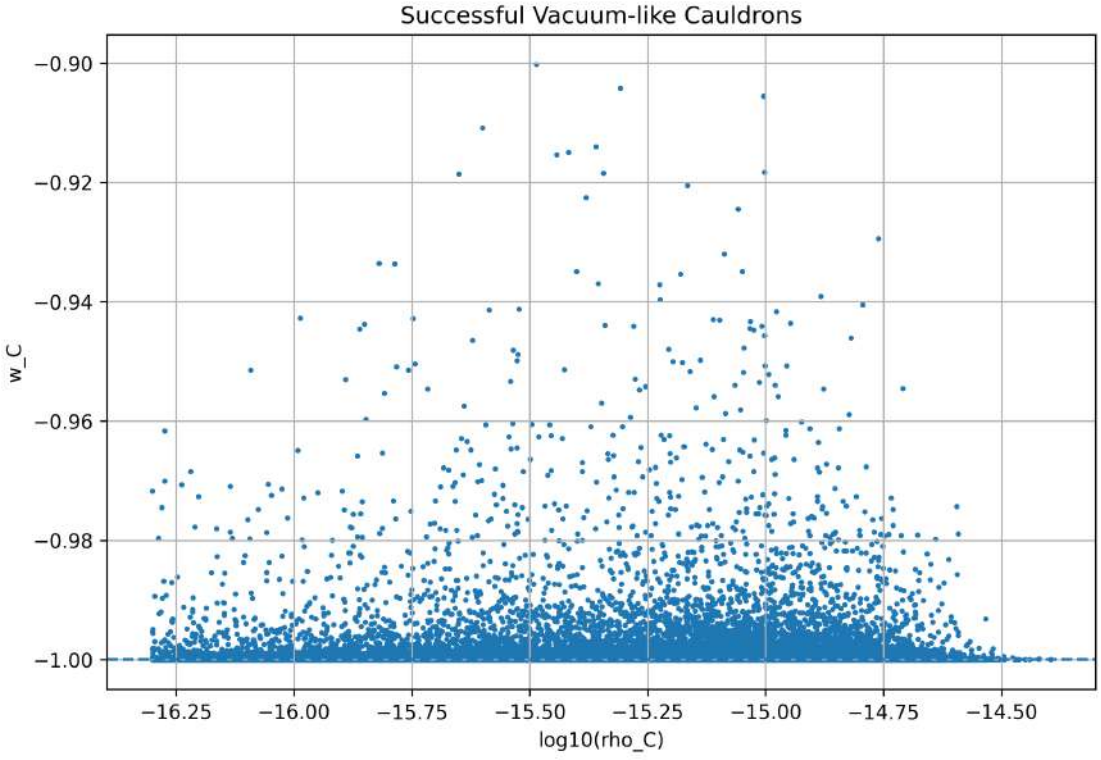

Figure 4. Successful vacuum-like cauldron realizations in the $(\log_{10}\rho_C, w_C)$ parameter space.

5.2. Sensitivity Analysis and Robustness of the Monte Carlo Results

To examine the dependence of the baseline results on the assumed microscopic parameter ranges, we performed a series of one-at-a-time sensitivity experiments. The baseline configuration consists of five reaction events per cauldron and 10^5 independent Latin hypercube realizations. The sampled parameters are the reaction decay rate λ_i, the energy contribution ΔE_i, and the suppression factor α_i, with baseline logarithmic sampling intervals

$$\log_{10} \lambda_i \sim \mathcal{U}(-30, -18), \tag{59}$$
$$\log_{10} \Delta E_i \sim \mathcal{U}(-18, -15), \tag{60}$$
$$\log_{10} \alpha_i \sim \mathcal{U}(-6, 0). \tag{61}$$

Here, λ_i is measured in s^{-1}, ΔE_i in joules, and α_i is dimensionless.

For each realization, the energy density and exchange term are calculated as

$$\rho_C = \frac{1}{V_c} \sum_{i=1}^{N_{\text{reactions}}} \Delta E_i, \tag{62}$$

$$Q_C = \frac{1}{V_c} \sum_{i=1}^{N_{\text{reactions}}} \lambda_i \alpha_i \Delta E_i, \tag{63}$$

where $V_c = 1\,\mathrm{cm}^3$. The corresponding equation-of-state parameter is

$$w_C = -1 + \frac{Q_C}{3H_0 \rho_C}. \tag{64}$$

A realization is classified as successful when both conditions are satisfied:

$$-1.1 < w_C < -0.9, \tag{65}$$
$$0.1\rho_\Lambda < \rho_C < 10\rho_\Lambda. \tag{66}$$

The joint success fraction is the proportion of realizations satisfying both conditions simultaneously.

Baseline reproducibility. The baseline simulation reproduces the previously reported ensemble statistics. For 10^5 realizations, the mean and median equation-of-state parameters are

$$\langle w_C \rangle = -0.999617806, \tag{67}$$
$$\text{median}(w_C) = -0.999999408. \tag{68}$$

The standard deviation across individual realizations is 2.675486×10^{-3}. The fractions satisfying the equation-of-state and density conditions separately are 99.996% and 97.851%, respectively. The joint success fraction is 97.847%.

Sensitivity to the decay-rate range. The upper bound of the logarithmic decay-rate interval was varied while keeping the lower bound and all other baseline parameters fixed. The results are reported in Table 1.

Table 1. Sensitivity of the Monte Carlo results to the upper bound of the logarithmic decay-rate interval.

$\log_{10} \lambda_{\max}$	$\langle w_C \rangle$	σ_{w_C}	$P(w$ criterion$)$	P(joint success)
-20	-0.999995	2.93288×10^{-5}	100.000%	97.851%
-18 (baseline)	-0.999618	2.67549×10^{-3}	99.996%	97.847%
-16	-0.967322	2.47466×10^{-1}	95.391%	93.379%
-14	1.853030	2.31218×10^{1}	77.465%	75.897%

The results demonstrate that the vacuum-like equation of state is sensitive to the assumed decay-rate interval. Lowering the upper bound to $10^{-20}\,\mathrm{s}^{-1}$ brings the ensemble mean closer to -1, whereas extending the interval to higher decay rates produces substantially larger deviations and dispersion. In particular, the simulation does not maintain a vacuum-like equation of state under arbitrary enlargement of the decay-rate range. The baseline result must therefore be interpreted conditionally on the adopted microscopic parameter distribution.

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Sensitivity to the suppression factor. The upper bound of the logarithmic suppression-factor interval was also varied, with the lower bound fixed at -6. For $\log_{10}\alpha_{\max} = -1$, the mean equation-of-state parameter is -0.999954, with a standard deviation of 2.9122×10^{-4} and a joint success fraction of 97.851%. For $\log_{10}\alpha_{\max} = -2$, the corresponding values are -0.999994, 3.23733×10^{-5}, and 97.851%. Both configurations satisfy the equation-of-state criterion in all 10^5 realizations. These results indicate that restricting the upper range of the suppression factor reduces the dispersion of w_C in the tested configurations.

Sensitivity to the reaction-energy scale. To test the effect of the characteristic energy scale, the complete logarithmic energy interval was shifted by one decade in either direction. The baseline interval $[-18, -15]$ was replaced by $[-19, -16]$ and $[-17, -14]$, respectively. The mean equation-of-state parameter and its standard deviation remain unchanged to the reported precision:

$$\langle w_C \rangle \simeq -0.999618, \tag{69}$$

$$\sigma_{w_C} \simeq 2.67549 \times 10^{-3}. \tag{70}$$

However, the density success fractions change to 60.128% and 39.872%, respectively, while the joint success fractions become 60.127% and 39.869%.

This behaviour follows from the structure of the equation-of-state expression. If all reaction energies are multiplied by a common positive factor c, then

$$w_C = -1 + \frac{\sum_i \lambda_i \alpha_i (c\Delta E_i)}{3H_0 \sum_i (c\Delta E_i)} = -1 + \frac{\sum_i \lambda_i \alpha_i \Delta E_i}{3H_0 \sum_i \Delta E_i}. \tag{71}$$

Consequently, a common rescaling of the reaction energies cancels from w_C, although it directly changes the total energy density. The equation-of-state statistic and the density normalization therefore probe different aspects of the model.

Sensitivity to the number of reaction events. Finally, the number of reaction events per cauldron was varied while retaining the baseline parameter distributions. The results are summarized in Table 2.

Table 2. Dependence of the Monte Carlo statistics on the number of reaction events per cauldron.

$N_{\text{reactions}}$	$\langle w_C \rangle$	σ_{w_C}	$P(w$ criterion$)$	P(joint success)
1	-0.999612	3.78881×10^{-3}	99.984%	43.362%
3	-0.999617	3.04960×10^{-3}	99.992%	86.881%
5	-0.999618	2.67549×10^{-3}	99.996%	97.847%
10	-0.999610	2.08594×10^{-3}	100.000%	99.988%
20	-0.999615	1.52080×10^{-3}	100.000%	97.160%

The standard deviation of the individual-realization values of w_C decreases as the number of reaction events increases across the tested configurations. This is consistent with an averaging effect in the weighted sum defining w_C. The ensemble mean remains close to the baseline value throughout these experiments.

The joint success fraction, however, is not strictly monotonic in the number of reaction events. This is because the density criterion also depends on the sum of the event energies, and its satisfaction depends on the adopted density interval. Thus, the reduction in the dispersion of w_C should not be conflated with an automatic increase in the probability of satisfying the joint success criterion.

Interpretation and limitations. The sensitivity experiments establish that the baseline Monte Carlo result is reproducible under the stated parameter distributions and that the equation-of-state statistic is comparatively insensitive to a common rescaling of all reaction energies. They also reveal substantial dependence on the upper bound of the decay-rate distribution and on the density normalization.

An important distinction must be maintained between convergence of an ensemble estimator and the physical dispersion of individual realizations. Increasing the number of Monte Carlo realizations improves the numerical estimation of ensemble statistics; it does not, by itself, force the underlying distribution of individual values of w_C to collapse to a Dirac delta distribution. Likewise, increasing the number of reaction events can reduce realization-level dispersion in the tested model, but does not establish vanishing intrinsic fluctuations in the general case.

Accordingly, the results demonstrate conditional statistical robustness within the specified parameter ranges rather than a parameter-independent prediction of the observed dark-energy equation of state. A broader physical interpretation requires independent justification of the microscopic parameter distributions, their allowed ranges, and the mechanism determining the energy exchange rate.

5.3. Phase I: PSZ2 Galaxy Cluster Cosmology Tests

Following the successful emergence of vacuum-like behaviour within the microscopic five-reaction cauldron framework, the next stage of analysis tested whether the resulting cosmological expansion history remained consistent with observational large-scale structure data. For this purpose, the framework was compared against the *Planck* PSZ2 galaxy cluster catalogue, which contains Sunyaev–Zel'dovich detected galaxy clusters spanning a broad redshift range. After data cleaning and consistency filtering, 500 cluster entries were retained for analysis, covering redshifts up to

$$z \approx 1.27.$$

The emergent cauldron cosmology retained an effective equation-of-state parameter

$$w = -0.9996,$$

consistent with the microscopic simulations derived previously. The resulting cosmological expansion curve was then compared directly against the standard ΛCDM model. The maximum fractional deviation between the two cosmologies remained extremely small,

$$\Delta_{\max} \approx 1.02 \times 10^{-4},$$

with mean fractional difference

$$\langle \Delta \rangle \approx 5.79 \times 10^{-5}.$$

These results indicate that the stochastic cauldron framework reproduces the observed late-time expansion history with extremely high precision while deriving the effective dark-energy sector from microscopic stochastic dynamics rather than a fundamental cosmological constant. The observational distributions and expansion comparisons are presented below.

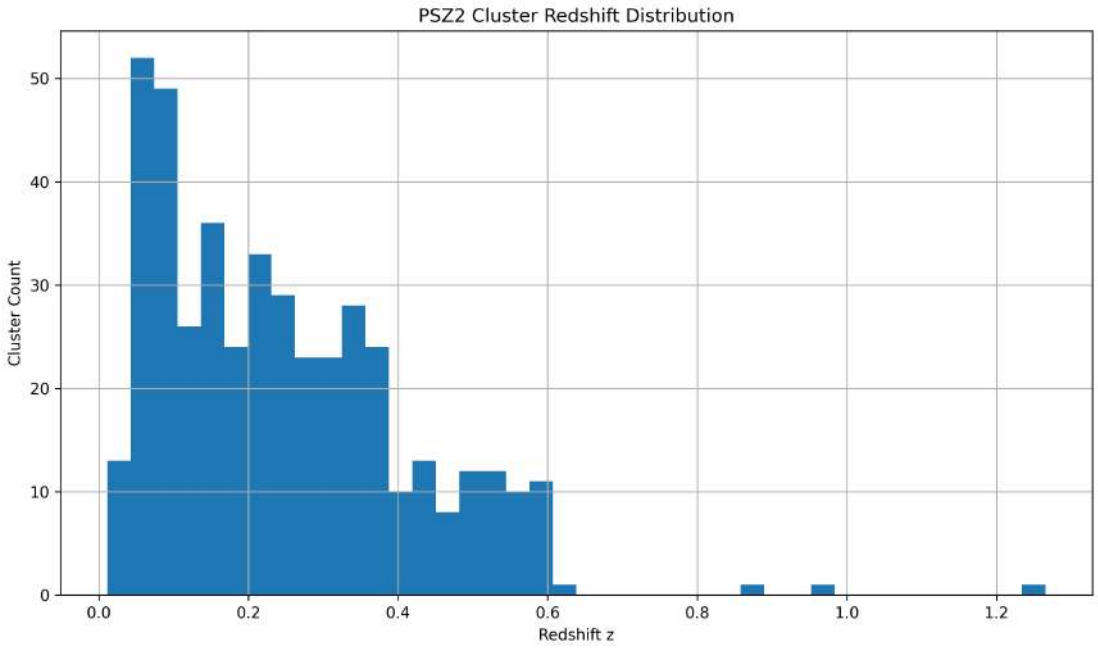

Figure 5. Redshift distribution of PSZ2 galaxy clusters retained after catalogue filtering and preprocessing.

Figure 5 shows the redshift distribution of the filtered PSZ2 galaxy cluster sample used in the cosmological analysis. The majority of clusters occupy the low-to-intermediate redshift regime,

$$0 < z < 0.6,$$

which corresponds to the epoch where dark-energy domination becomes dynamically significant in the standard cosmological model. A smaller number of high-redshift clusters extend the observational baseline toward earlier cosmological epochs. The retained sample therefore probes both matter-dominated and dark-energy-dominated expansion regimes simultaneously. This broad redshift coverage provides a meaningful observational environment for testing whether the emergent stochastic cauldron cosmology can reproduce the late-time expansion history inferred from large-scale structure observations.

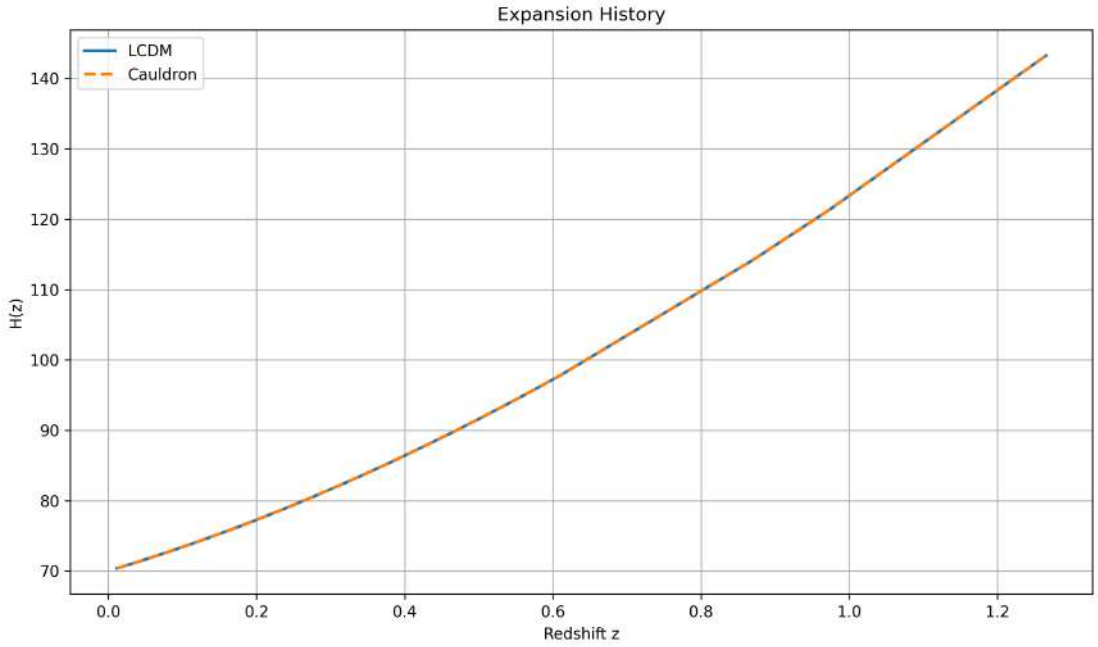

Figure 6. Comparison between the expansion histories predicted by standard ΛCDM cosmology and the stochastic cauldron cosmology.

Figure 6 compares the Hubble expansion histories generated by the stochastic cauldron cosmology and the standard ΛCDM model. The two curves remain nearly indistinguishable throughout the entire observational redshift range tested. This result demonstrates that the stochastic microscopic reaction framework successfully reproduces the observed late-time cosmic acceleration without requiring insertion of a fundamental vacuum-energy constant directly into the Einstein equations. Instead, the effective vacuum-like sector emerges statistically from sparse stochastic reaction dynamics. The absence of visible divergence additionally indicates that the cauldron framework remains dynamically stable over cosmological timescales and does not introduce pathological late-time expansion behaviour.

Figure 7 presents the fractional difference between the stochastic cauldron cosmology and the standard ΛCDM expansion history across the PSZ2 redshift range. The deviation remains extremely small throughout the entire domain, never exceeding approximately

$$10^{-4}.$$

This indicates that the emergent stochastic framework is observationally degenerate with ΛCDM at current PSZ2 precision levels. Importantly, the deviation curve remains smooth and non-oscillatory, implying that the microscopic stochastic dynamics do not generate unstable expansion fluctuations at observable scales. The result therefore supports the interpretation that sparse microscopic stochastic reaction networks can reproduce the macroscopic expansion phenomenology normally attributed to a cosmological constant.

5.4. Phase II: Pantheon+ Supernova Cosmology Tests

After establishing consistency with large-scale structure observations through the PSZ2 galaxy cluster catalogue, the stochastic cauldron framework was further tested against the Pantheon+ Type Ia supernova compilation. The Pantheon+ dataset represents one of the most precise observational probes of late-time cosmic expansion currently available, containing 1701 calibrated supernova measurements spanning redshifts up to

$$z \approx 2.3.$$

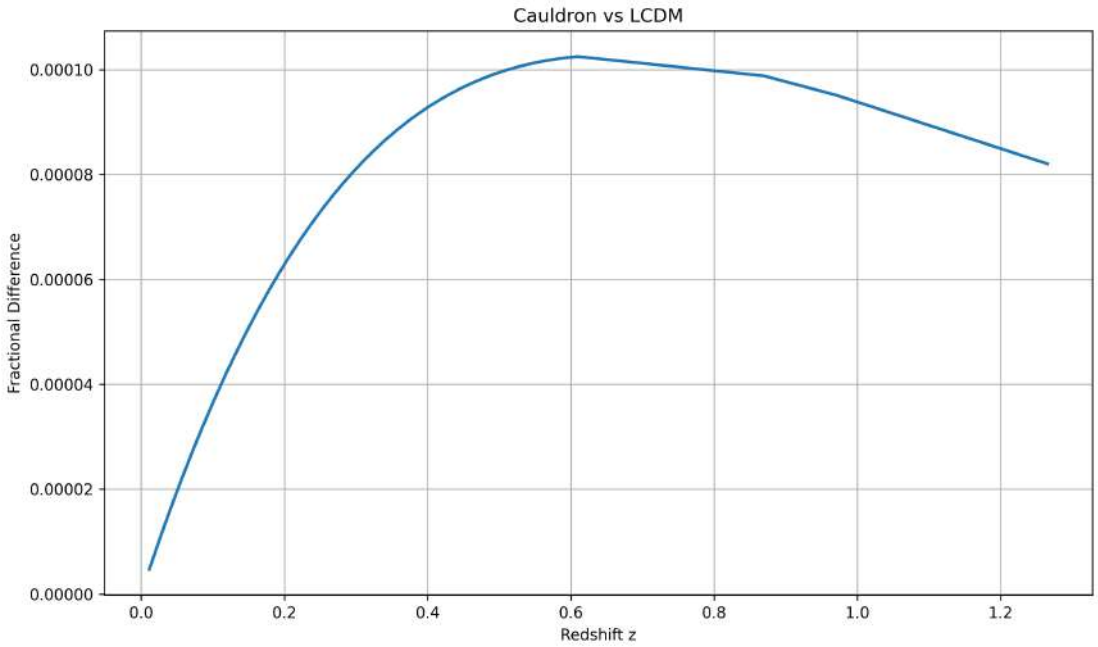

Figure 7. Fractional deviation between the stochastic cauldron cosmology and the standard ΛCDM expansion history.

Unlike the PSZ2 analysis, which primarily tested expansion consistency through cluster distributions, the Pantheon+ analysis directly constrained the luminosity-distance relation using the full covariance-matrix likelihood formalism. The stochastic cauldron cosmology retained its emergent equation-of-state parameter

$$w = -0.9996,$$

derived independently from the microscopic five-event stochastic simulations. The resulting cosmological fit yielded

$$\chi^2_{\Lambda\mathrm{CDM}} = 781.037,$$

and

$$\chi^2_{\mathrm{Cauldron}} = 780.676,$$

with corresponding reduced chi-square values

$$\chi^2_{\nu,\Lambda\mathrm{CDM}} = 0.45943,$$

and

$$\chi^2_{\nu,\mathrm{Cauldron}} = 0.45922.$$

The resulting chi-square difference,

$$\Delta\chi^2 = -0.3607,$$

indicates that the stochastic cauldron cosmology remains observationally degenerate with standard ΛCDM at present Pantheon+ precision levels. Importantly, these results were obtained without introducing a fundamental cosmological constant explicitly into the model. Instead, the effective dark-energy sector emerged statistically from sparse microscopic stochastic reaction dynamics. The observational fit and residual structure are shown in Figures 8 and 9.

Figure 8 presents the luminosity-distance relation obtained from the Pantheon+ Type Ia supernova dataset together with the predictions of standard ΛCDM cosmology and the stochastic cauldron cosmology. The two theoretical curves remain nearly indistinguishable throughout the entire redshift domain. The cauldron cosmology successfully reproduces the observed late-time accelerated expansion while emerging entirely from microscopic stochastic dynamics rather than from an inserted cosmological constant term. The agreement persists across both low-redshift and high-redshift supernovae, demonstrating that the stochastic framework remains dynamically stable over cosmological timescales and accurately reproduces the observed luminosity-distance structure of the expanding universe.

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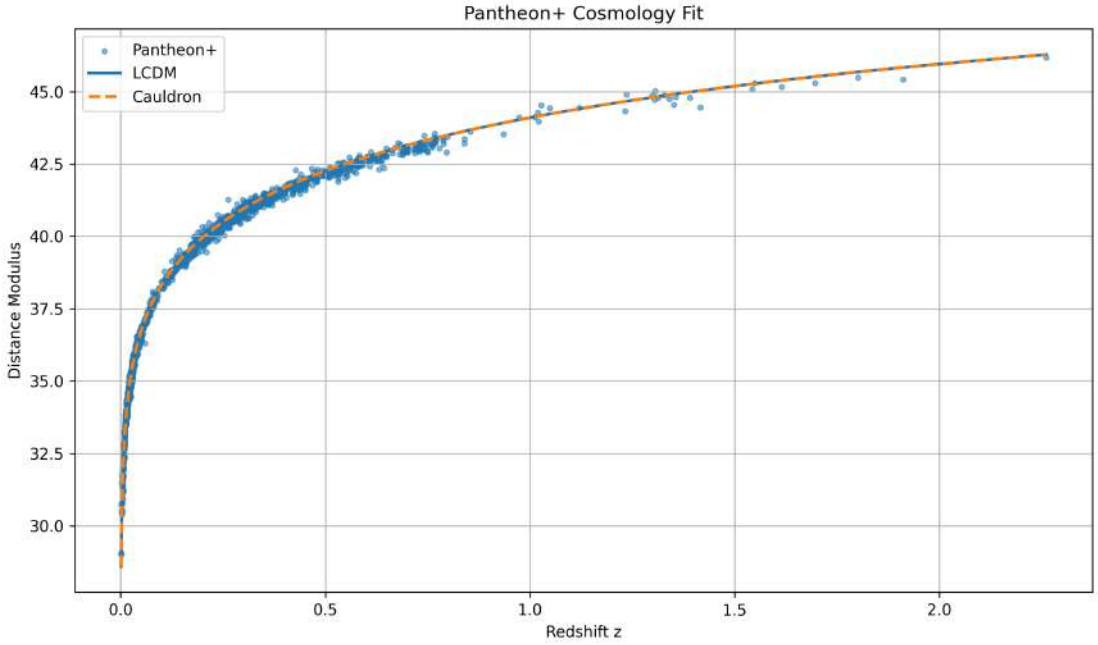

Figure 8. Pantheon+ luminosity-distance comparison between observational supernova data, standard ΛCDM cosmology, and the stochastic cauldron cosmology.

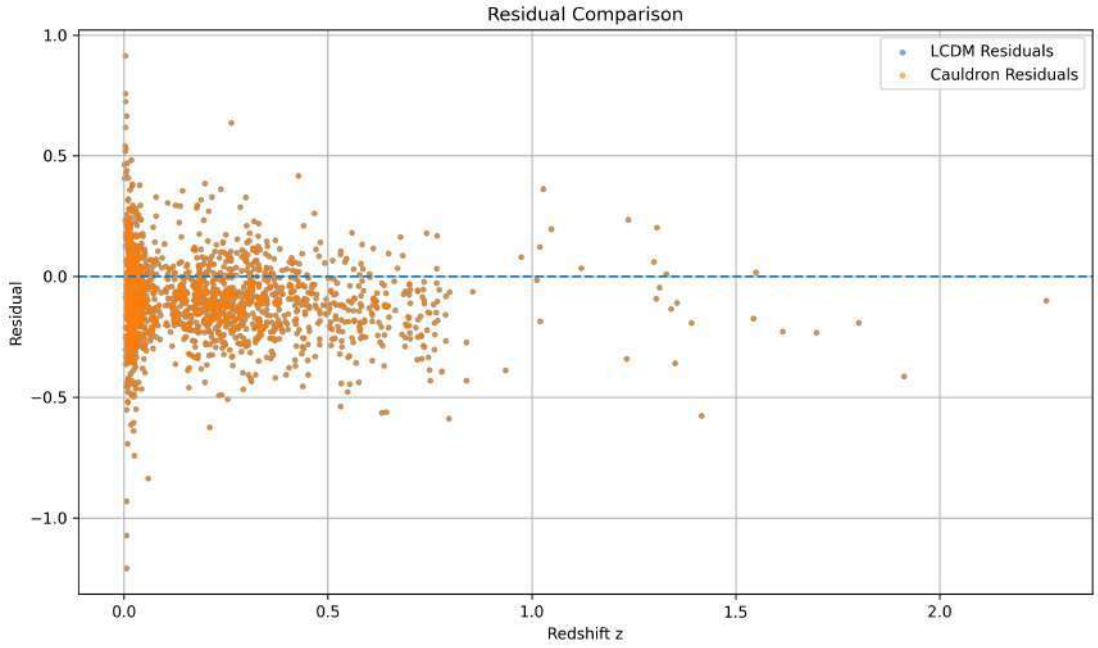

Figure 9. Residual comparison between Pantheon+ supernova observations and the theoretical predictions of ΛCDM and stochastic cauldron cosmology.

Figure 9 shows the residual distributions for both the standard ΛCDM cosmology and the stochastic cauldron cosmology relative to the Pantheon+ observational dataset. The residual structure remains centered around zero throughout the redshift range, with no systematic drift or runaway deviation visible at higher redshifts. The stochastic cauldron framework therefore reproduces the observational scatter properties of the Pantheon+ dataset with precision comparable to the standard cosmological model. The absence of coherent residual trends additionally indicates that the microscopic stochastic dynamics do not generate observable instability in the luminosity-distance relation, supporting the interpretation that sparse reaction-network dynamics can produce a stable emergent vacuum-like cosmological sector.

5.5. Inference, Statistical Summary, and Scope of Claims

The numerical and observational analyses presented in this work collectively indicate that sparse stochastic microscopic reaction networks can reproduce an effective vacuum-like cosmological sector consistent with current late-time cosmological observations. Across all three experimental stages, the emergent equation-of-state parameter remained extremely close to the cosmological-constant limit,

$$w = -1,$$

while maintaining stable low-density behaviour compatible with observed dark-energy scales. Importantly, the effective vacuum behaviour emerged statistically from microscopic stochastic interactions rather than from direct insertion of a cosmological constant into the Einstein field equations.

The five-reaction cauldron simulations demonstrated that vacuum-like states arise naturally within the stochastic parameter space with very high probability. The resulting success fraction exceeded

$$97\%,$$

while the median emergent equation-of-state parameter converged essentially to the exact vacuum limit. The PSZ2 galaxy cluster analysis further showed that the resulting expansion history remains observationally degenerate with ΛCDM over the tested redshift domain. Finally, the Pantheon+ covariance-matrix likelihood analysis demonstrated that the stochastic cauldron cosmology reproduces observed luminosity-distance relations with statistical precision comparable to the standard cosmological model.

The combined statistical summary of all three phases is presented in Table 3.

Phase	**Primary Metric**	**Result**	**Interpretation**
Five-Reaction Cauldron	Success Fraction	0.97847	Stable vacuum-like emergence
Five-Reaction Cauldron	Mean Equation of State	-0.9996178	Near-Λ behaviour
Five-Reaction Cauldron	Median Equation of State	-0.9999994	Vacuum attractor convergence
PSZ2 Cosmology Test	Maximum Fractional Difference	1.02×10^{-4}	Degenerate with ΛCDM
PSZ2 Cosmology Test	Mean Fractional Difference	5.79×10^{-5}	Stable expansion consistency
Pantheon+ Test	$\chi^2_{\Lambda\mathrm{CDM}}$	781.037	Reference cosmological fit
Pantheon+ Test	$\chi^2_{\mathrm{Cauldron}}$	780.676	Equivalent observational fit
Pantheon+ Test	$\Delta\chi^2$	-0.3607	No observable degradation

Table 3. Summary of the stochastic cauldron cosmology results across microscopic simulations, PSZ2 cluster tests, and Pantheon+ supernova analyses.

The present work does not claim a complete replacement of modern cosmology, nor does it claim derivation of a final quantum theory of gravity or a complete microscopic particle model for dark energy. The framework also does not attempt to overthrow General Relativity or invalidate the observational success

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of the standard ΛCDM cosmological model. Instead, the results suggest a possible microscopic physical interpretation underlying the observed dark-energy sector.

Specifically, the framework proposes that persistent sparse stochastic reaction environments distributed throughout spacetime may statistically generate an effective vacuum-like cosmological component. Within this interpretation, the observed late-time accelerated expansion emerges not from a fundamental cosmological constant inserted *a priori*, but from the collective macroscopic behaviour of innumerable low-energy stochastic reaction domains. The present work therefore represents a phenomenological stochastic interpretation of dark energy rather than a complete unified cosmological theory.

Additionally, the simulations presented here were intentionally restricted to low-complexity sparse-event reaction environments. The purpose of this restriction was to determine whether vacuum-like behaviour could emerge under minimal microscopic assumptions without requiring dense interaction networks or artificially large energy scales. The resulting agreement with observational cosmology therefore suggests that extremely weak stochastic processes may already be sufficient to reproduce the effective dark-energy sector observed at cosmological scales.

Future work may extend the framework toward perturbation theory, anisotropy analysis, stochastic transport dynamics, and early-universe cosmology. However, such extensions lie beyond the scope of the present study, whose primary objective was to determine whether sparse stochastic microscopic dynamics can reproduce an observationally consistent vacuum-like cosmological expansion history.

Scope Relative to the Cosmological Constant Problem. The present work is not proposed as a direct resolution of the ultraviolet vacuum catastrophe associated with quantum field theoretic estimates of vacuum energy density (Carroll, 2001, Padmanabhan, 2003, Weinberg, 1989). In particular, the framework does not derive the observed cosmological energy scale from first-principles quantum vacuum calculations. Instead, the objective of the model is phenomenological: namely, to investigate whether sparse stochastic transport networks can naturally generate a stable macroscopic sector whose effective behavior observationally mimics a cosmological constant. The emphasis of the present study therefore lies in emergent transport dynamics and statistical steady-state behavior rather than in ultraviolet completion or quantum-gravitational renormalization.

6. Physical Interpretation

The results obtained in this work suggest that vacuum-like cosmological behaviour may emerge naturally from sparse stochastic microscopic reaction environments distributed throughout spacetime. Within the proposed framework, dark energy is not introduced as a fundamental cosmological constant inserted directly into the Einstein field equations. Instead, the effective vacuum sector emerges statistically from innumerable low-energy stochastic interactions occurring within localized microscopic reaction domains referred to as *cauldrons*.

Each cauldron represents a finite microscopic spacetime region containing sparse stochastic reaction events characterized by fluctuating interaction strengths, decay phases, and probabilistic energy transfer dynamics. The framework intentionally restricts the number of microscopic reactions to extremely small values in order to investigate whether vacuum-like behaviour can arise under minimal dynamical assumptions. Remarkably, the simulations demonstrate that even five sparse stochastic reaction events are sufficient to generate a statistically stable effective equation-of-state parameter extremely close to

$$w = -1.$$

A key physical feature of the framework is the extremely small emergent energy-density scale. Unlike conventional vacuum-energy calculations, which frequently predict catastrophic energy densities exceeding observational constraints by many orders of magnitude, the stochastic cauldron model naturally stabilizes near extremely weak leakage-energy regimes. This occurs because the framework statistically favours sparse low-energy steady states rather than dense reaction environments. Consequently, vacuum-like behaviour emerges without requiring unrealistically large microscopic energy injection.

The physical interpretation proposed here therefore differs fundamentally from conventional quantum vacuum-energy pictures. In the present framework, the observed dark-energy sector does not arise from a static background vacuum possessing a fixed intrinsic cosmological constant. Instead, dark-energy-like be-

haviour emerges dynamically from persistent microscopic stochastic processes whose collective macroscopic effect mimics a vacuum-dominated cosmological fluid.

The resulting vacuum-like pressure emerges statistically from the balance between stochastic microscopic energy generation and distributed transport across neighboring reaction domains. Since the individual reaction energies remain extremely small, the global cosmological effect becomes visible only when integrated across enormous spacetime volumes. In this interpretation, cosmic acceleration is therefore not driven by a single large energy reservoir, but by the cumulative macroscopic effect of persistent sparse microscopic leakage processes distributed throughout spacetime.

An important feature of the framework is that the emergent vacuum-like state behaves as a statistical attractor. Across the numerical simulations, realizations producing

$$w \approx -1$$

dominated the parameter space overwhelmingly, while strongly non-vacuum states appeared statistically suppressed. This suggests that sparse stochastic reaction networks may naturally evolve toward vacuum-like equilibrium behaviour under broad microscopic conditions. The resulting effective cosmological fluid therefore behaves macroscopically like a cosmological constant despite arising from dynamically fluctuating microscopic processes.

The framework additionally provides a possible qualitative explanation for why dark energy appears approximately homogeneous on cosmological scales despite originating from microscopic stochastic phenomena. Since the individual reaction energies remain extremely weak and spatially distributed across enormous numbers of microscopic domains, local fluctuations become statistically averaged over large cosmological volumes. The resulting large-scale behaviour therefore approaches an effectively smooth vacuum-like background while preserving microscopic stochasticity at sub-observational scales.

Importantly, the present interpretation does not require modification of General Relativity itself. The Einstein field equations remain unchanged. Instead, the stochastic cauldron framework proposes a different physical origin for the effective stress-energy component associated with late-time cosmic acceleration. In this sense, the framework acts as a microscopic phenomenological interpretation layered beneath the standard cosmological expansion equations rather than as a replacement for relativistic gravitational theory.

The results obtained from the PSZ2 galaxy cluster tests and the Pantheon+ covariance-matrix analysis further reinforce this interpretation. Despite being derived from sparse microscopic stochastic dynamics, the resulting expansion history remains observationally degenerate with standard ΛCDM cosmology at current precision levels. The framework therefore demonstrates that vacuum-like cosmological acceleration may emerge from statistically persistent microscopic stochastic reaction environments without requiring direct insertion of a fundamental cosmological constant into the cosmological model.

The stochastic cauldron interpretation presented here should therefore be viewed as a phenomenological microscopic hypothesis for the physical origin of dark energy. While the framework does not yet constitute a complete microscopic particle theory or quantum-gravitational description, the results indicate that sparse stochastic reaction networks may provide a viable pathway toward understanding the emergence of vacuum-like cosmological behaviour from microscopic physical processes.

7. Limitations and Future Directions

7.1. Limitations

Despite the encouraging numerical and observational results obtained in this work, several important limitations must be acknowledged. The stochastic cauldron framework presented here is fundamentally phenomenological and should not yet be interpreted as a complete microscopic particle theory of dark energy. The present study demonstrates the viability of a sparse stochastic reaction-network interpretation of vacuum-like cosmological behaviour, but it does not identify the exact microscopic particle species, interaction channels, or quantum-field mechanisms responsible for the emergent dynamics.

Additionally, the reaction processes employed in the simulations were intentionally simplified in order to isolate the minimal conditions necessary for emergence of vacuum-like behaviour. The five-event sparse cauldron approximation therefore represents an idealized microscopic stochastic environment rather than a fully realistic description of quantum vacuum dynamics. While this simplification was sufficient to reproduce observationally consistent cosmological behaviour, a more complete microscopic formulation would require substantially richer interaction physics and covariant transport dynamics.

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The present framework also does not include a full relativistic perturbation-theory treatment. Consequently, the current study does not yet address the evolution of density perturbations, anisotropy propagation, structure-growth constraints, or cosmic microwave background multipole behaviour within the stochastic cauldron framework. Although the resulting expansion history remains observationally degenerate with ΛCDM at current PSZ2 and Pantheon+ precision levels, consistency with precision CMB observations remains to be investigated.

Another limitation concerns the transport structure between neighboring stochastic domains. In the present work, the existence of distributed stochastic energy transfer was introduced phenomenologically rather than derived from a fully covariant microscopic transport formalism. The exact causal propagation properties, interaction topology, and long-range statistical coupling between neighboring cauldrons therefore remain open questions for future investigation.

The framework additionally does not attempt to solve the broader cosmological constant problem in its entirety. Specifically, the present study does not derive why the emergent vacuum-energy scale assumes its observed numerical magnitude from first principles. Rather, the framework demonstrates that extremely weak stochastic microscopic leakage processes can naturally reproduce vacuum-like cosmological behaviour without catastrophic energy divergence. The observed vacuum-energy scale therefore remains an emergent phenomenological parameter within the current formulation.

Finally, the present study focuses exclusively on late-time cosmological acceleration and does not address inflationary cosmology, baryogenesis, dark matter phenomenology, black-hole thermodynamics, or quantum gravity unification. The stochastic cauldron framework should therefore be interpreted as a targeted microscopic interpretation of dark energy rather than as a complete unified cosmological theory.

7.2. Future Directions

The stochastic microscopic framework introduced in this work opens several potential directions for future investigation. One immediate extension involves construction of a fully covariant stochastic transport formalism describing energy exchange between neighboring microscopic reaction domains. Such a formulation may allow derivation of large-scale vacuum-like behaviour directly from relativistic transport equations rather than from phenomenological stochastic evolution alone. Related phenomenological approaches to dark-sector energy exchange, including models constrained using combined BAO, CMB, and supernova observations, provide a useful basis for comparison (Li et al., 2024).

Another important direction concerns cosmological perturbation theory. Future work may investigate whether microscopic stochastic fluctuations generate observable signatures in large-scale structure formation, integrated Sachs–Wolfe correlations, weak gravitational lensing, or cosmic microwave background anisotropies. Recent Euclid analyses of weak-lensing angular power spectra also demonstrate the importance of higher-order modelling effects when extracting cosmological parameters from large-scale structure observations (Euclid Collaboration, 2024). Since the present framework predicts vacuum-like behaviour emerging from fluctuating microscopic dynamics rather than from an exactly static cosmological constant, small stochastic deviations from standard ΛCDM behaviour may become observable at sufficiently high precision.

Further numerical studies may additionally explore the statistical topology of interacting cauldron networks. In particular, future work could investigate whether different overlap geometries, transport couplings, or stochastic interaction distributions generate measurable deviations in the effective equation-of-state evolution,

$$w(z),$$

at late cosmological times. Such analyses may provide pathways toward observational discrimination between stochastic emergent vacuum models and conventional cosmological-constant scenarios.

The framework may also be extended toward higher-complexity microscopic interaction systems involving larger reaction counts, adaptive stochastic coupling, and evolving microscopic decay distributions. These extensions could determine whether the vacuum-like attractor behaviour identified in the present work remains stable under increasingly realistic microscopic stochastic environments.

An additional avenue of future research concerns possible connections between stochastic vacuum emergence and quantum field fluctuations in curved spacetime. Although the present work does not attempt to derive the framework directly from quantum field theory, future investigations may explore whether sparse stochastic reaction environments arise naturally within semiclassical quantum-gravitational settings.

The broader theoretical context includes approaches to quantum fluctuations of spacetime geometry and topology, which have been reviewed in the context of quantum-gravity research (Carlip, 2023).

Finally, future observational studies may compare the stochastic cauldron framework against larger cosmological datasets including DESI baryon acoustic oscillations, weak-lensing surveys, Euclid observations, Rubin Observatory data, and future CMB Stage-IV measurements. Such analyses may determine whether small residual stochastic signatures predicted by emergent vacuum frameworks become distinguishable from standard ΛCDM cosmology at next-generation observational precision levels.

8. Interpretational Scope and Non-Claims

The present framework is intended as a stochastic phenomenological interpretation of dark-energy-like behavior arising from sparse microscopic transport networks. The objective of the study is not to replace General Relativity, overturn the Friedmann–Lemaître cosmological framework, or provide a complete microscopic theory of spacetime. Rather, the model explores whether a large ensemble of weakly interacting stochastic domains can collectively reproduce an effective vacuum-like cosmological sector consistent with current observational precision.

Several important clarifications are therefore necessary.

First, the framework does not claim to solve the ultraviolet cosmological constant problem in the quantum field theoretic sense (Carroll, 2001, Weinberg, 1989). The observed dark-energy density scale is not derived from first principles of vacuum renormalization, quantum gravity, or particle physics. Instead, the study investigates whether macroscopic vacuum-like behavior can emerge statistically from sparse stochastic transport processes.

Second, the model does not replace Einstein field equations or modify the geometric structure of spacetime. The effective density generated by the cauldron network is inserted phenomenologically into the standard Friedmann expansion equations in the same spirit as many effective dark-energy parametrizations studied in cosmology (Clifton et al., 2012, Copeland et al., 2006). The present work therefore operates within a phenomenological cosmological framework rather than a fundamental metric theory.

Third, the model does not assume continuous cosmologically dominant conversion of baryonic matter into dark energy. Astrophysical activity is interpreted only as a localized perturbative influence capable of modulating stochastic transport states within nearby domains. The dominant large-scale energy density arises statistically from the cumulative transport equilibrium of the network itself, preserving leading-order consistency with standard matter evolution.

Fourth, the microscopic identity of the reacting particles is intentionally left unspecified. The framework is formulated statistically rather than particle-specifically. The sparse reaction assumption is therefore interpreted as an effective stochastic description applicable to generic low-density microscopic interaction environments rather than a completed particle-physics model.

Finally, the observational analyses performed using the Planck PSZ2 cluster catalogue and Pantheon+ supernova compilation demonstrate consistency with ΛCDM expansion behavior only at currently accessible observational precision. The present work therefore establishes phenomenological viability rather than observational superiority over the standard cosmological model.

Within these limitations, the framework proposes that vacuum-like cosmic acceleration may be interpretable as an emergent large-scale manifestation of microscopic stochastic transport occurring across an effectively continuous network of overlapping sparse domains.

9. Conclusion

In this work, a stochastic microscopic interpretation of dark energy based on sparse reaction networks was proposed and investigated numerically across multiple cosmological scales. Unlike conventional approaches in which the cosmological constant is inserted directly into the Einstein field equations as a fundamental vacuum-energy term, the present framework explored the possibility that vacuum-like cosmological behaviour may emerge statistically from persistent low-energy stochastic microscopic interactions distributed throughout spacetime.

The analysis demonstrated that even extremely sparse microscopic reaction environments can generate an effective equation-of-state parameter remarkably close to the cosmological-constant limit,

$$w \approx -1,$$

without requiring catastrophic vacuum-energy densities. In particular, the five-reaction cauldron simulations showed that vacuum-like behaviour emerges as a statistically dominant attractor state across the stochastic parameter space. More than 97% of the simulated realizations converged toward stable vacuum-like configurations, with median equation-of-state parameter effectively indistinguishable from

$$w = -1.$$

The resulting emergent cosmology was subsequently tested against observational datasets. The PSZ2 galaxy cluster analysis demonstrated that the stochastic cauldron cosmology remains observationally degenerate with standard ΛCDM expansion histories across the tested redshift range. The Pantheon+ covariance-matrix likelihood analysis further showed that the emergent framework reproduces the observed Type Ia supernova luminosity-distance relation with statistical precision comparable to the standard cosmological model (Brout et al., 2022, Perlmutter et al., 1999, Riess et al., 1998, Scolnic et al., 2018). Importantly, these observational consistencies were achieved without directly inserting a fundamental cosmological constant into the model.

The framework presented here should be interpreted as a phenomenological microscopic hypothesis for the origin of dark energy rather than as a complete unified cosmological theory. This interpretation is conceptually related to, but distinct from, existing stochastic treatments of cosmological fluctuations and dark-energy correlations (Belgacem & Prokopec, 2022), phenomenological models of emergent dark energy (John et al., 2023), and interacting dark-energy frameworks in which the cosmological evolution is governed by explicit energy-exchange terms (van der Westhuizen & Abebe, 2024). The present work does not attempt to replace General Relativity, derive a full quantum-gravitational framework, or identify the exact microscopic particle sector responsible for the stochastic dynamics. Instead, the results demonstrate that sparse microscopic stochastic reaction networks constitute a viable pathway through which vacuum-like cosmological acceleration may emerge naturally from collective low-energy microscopic processes.

More broadly, the results suggest that the observed dark-energy sector may not necessarily correspond to a static intrinsic vacuum-energy constant, but could instead represent the large-scale macroscopic manifestation of persistent stochastic microscopic dynamics distributed throughout spacetime. If correct, such an interpretation may provide a new conceptual perspective on one of the central problems of modern cosmology.

Future investigations involving perturbation theory, stochastic transport formalism, cosmic microwave background analyses, and next-generation observational surveys may determine whether emergent stochastic vacuum frameworks remain observationally viable at higher cosmological precision. Nevertheless, the results obtained in this study demonstrate that sparse microscopic stochastic reaction environments are capable of reproducing a stable vacuum-like cosmological sector consistent with current late-time observational constraints.

Conflict of Interest

The author declares that there are no competing financial interests, personal relationships, or conflicts of interest that could have influenced the work reported in this paper.

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Data Availability

The observational datasets analyzed in this study are publicly available from established cosmological survey archives. The Pantheon+ Type Ia supernova compilation and covariance matrices were obtained from the official Pantheon+ public data release. The Planck PSZ2 galaxy cluster catalogue was obtained from the Planck Legacy Archive. All numerical simulations, stochastic transport realizations, Monte Carlo sampling routines, and analysis pipelines used in this work were developed by the author and are available from the corresponding author upon reasonable request. Generated figures, processed datasets, and simulation outputs supporting the findings of this study are likewise available upon request. No proprietary observational data were used in this work.

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Appendices

Appendix A Mathematical Derivations

This appendix develops the mathematical structure underlying the stochastic cauldron framework in greater detail. The purpose of the appendix is to formalize the emergence of an effective vacuum-like cosmological component from sparse microscopic stochastic reaction networks distributed throughout spacetime.

A.1 Microscopic Stochastic State Space

Consider a cauldron containing N_c active microscopic constituents. The state of constituent i is

$$X_i = (\lambda_i, \tau_i, \chi_i). \tag{72}$$

The complete cauldron state is

$$\Omega_c = \{X_1, X_2, \ldots, X_{N_c}\}. \tag{73}$$

Under the first-order independence assumption, the joint probability measure is

$$P(\Omega_c) = \prod_{i=1}^{N_c} p_\lambda(\lambda_i) p_\tau(\tau_i) p_\chi(\chi_i). \tag{74}$$

For continuous variables, the probability densities satisfy

$$\int_{\mathcal{S}_\lambda} p_\lambda(\lambda)\, d\lambda = 1, \tag{75}$$

and

$$\int_{\mathcal{S}_\tau} p_\tau(\tau)\, d\tau = 1. \tag{76}$$

For a discrete collision-pairing variable with possible outcomes $\chi^{(1)}, \ldots, \chi^{(J)}$, normalization requires

$$\sum_{j=1}^{J} P\left(\chi = \chi^{(j)}\right) = 1. \tag{77}$$

The joint measure is normalized over the complete microscopic state space:

$$\int_{\Omega_c} P(\Omega_c)\, d\Omega_c = 1, \tag{78}$$

where integration over discrete components is understood as summation.

A.2 Microscopic Energy Production

The energy contribution associated with constituent i is represented by

$$\epsilon_i(t) = \mathcal{E}(\lambda_i, \tau_i, \chi_i, t; \boldsymbol{\theta}). \tag{79}$$

The total energy assigned to a cauldron is

$$Q_c(t) = \sum_{i=1}^{N_c} \epsilon_i(t). \tag{80}$$

The local energy density is therefore

$$\rho_c(t) = \frac{1}{V_c} \sum_{i=1}^{N_c} \mathcal{E}(\lambda_i, \tau_i, \chi_i, t; \boldsymbol{\theta}). \tag{81}$$

The realization-level fluctuation about the ensemble mean is

$$\delta\rho_c = \rho_c - \mathbb{E}[\rho_c]. \tag{82}$$

The corresponding ensemble expectation is

$$\mathbb{E}[\rho_c] = \int_{\Omega_c} \rho_c(\Omega_c) P(\Omega_c)\, d\Omega_c. \tag{83}$$

Evaluation of this expectation requires the explicit energy-response function and the probability distributions assigned to the microscopic variables.

A.3 Ensemble Averaging

Let $\rho_c^{(m)}$ denote the local density obtained from the m-th stochastic realization. For M realizations, the empirical ensemble mean is

$$\overline{\rho}_M = \frac{1}{M} \sum_{m=1}^{M} \rho_c^{(m)}. \tag{84}$$

The empirical variance is

$$s_\rho^2 = \frac{1}{M-1} \sum_{m=1}^{M} \left(\rho_c^{(m)} - \overline{\rho}_M \right)^2. \tag{85}$$

For independent realizations with finite variance, the law of large numbers implies

$$\overline{\rho}_M \xrightarrow[M\to\infty]{} \mathbb{E}[\rho_c]. \tag{86}$$

This convergence establishes consistency of the empirical estimator. It does not imply that the realization-level variance vanishes.

A statistically stationary ensemble is characterized by an approximately time-invariant expectation:

$$\frac{d}{dt}\mathbb{E}[\rho_c(t)] \simeq 0. \tag{87}$$

The condition describes stationarity of the ensemble mean and does not require every individual cauldron to possess a constant energy density.

A.4 Variance Suppression

The variance of the emergent density satisfies

$$\sigma_\rho^2 = \langle \rho^2 \rangle - \langle \rho \rangle^2.$$

Assuming weak correlations between distant cauldrons,

$$\sigma_\rho^2 \propto \frac{1}{N_{\text{tot}}},$$

where N_{tot} denotes the total number of contributing cauldrons.

Hence large-scale averaging suppresses local stochastic fluctuations:

$$\sigma_\rho \to 0 \quad \text{as} \quad N_{\text{tot}} \to \infty.$$

The resulting macroscopic field therefore approaches effective homogeneity despite microscopic randomness.

A.5 Global Energy Accounting

Consider a collection of N_c interacting cauldrons. The microscopic energy balance of domain i is

$$\frac{dQ_i}{dt} = S_i - L_i - \sum_{j\in\mathcal{N}(i)} F_{ij}, \tag{88}$$

where F_{ij} denotes the signed energy-transfer rate from domain i to domain j.

For internal transfers, antisymmetry requires

$$F_{ij} = -F_{ji}. \tag{89}$$

The total energy of the collection is

$$Q_{\mathrm{tot}}(t) = \sum_{i=1}^{N_c} Q_i(t). \tag{90}$$

Differentiating and substituting Equation (88), we obtain

$$\frac{dQ_{\mathrm{tot}}}{dt} = \sum_{i=1}^{N_c}(S_i - L_i) - \sum_{i=1}^{N_c}\sum_{j\in\mathcal{N}(i)} F_{ij}. \tag{91}$$

Because every internal transfer appears once as an outgoing flux and once as an incoming flux, the internal contribution vanishes:

$$\sum_{i=1}^{N_c}\sum_{j\in\mathcal{N}(i)} F_{ij} = 0. \tag{92}$$

It follows that

$$\boxed{\frac{dQ_{\mathrm{tot}}}{dt} = \sum_{i=1}^{N_c}(S_i - L_i)}. \tag{93}$$

Internal stochastic transport therefore redistributes energy among the cauldrons but cannot independently generate or destroy the total energy of a closed network. A change in the total energy must arise from net injection, dissipation, or exchange across the boundary of the network.

A.6 Continuum Limit and Statistical Stationarity

In the continuum approximation, the stochastic transport network may be represented phenomenologically by

$$\frac{\partial \rho_C}{\partial t} = D\nabla^2\rho_C + \Gamma(\mathbf{x},t) - \mathcal{L}(\mathbf{x},t), \tag{94}$$

where D denotes an effective transport coefficient, Γ represents microscopic source injection, and $\mathcal{L}$ represents local dissipation.

For a statistically homogeneous and stationary ensemble,

$$\left\langle \frac{\partial \rho_C}{\partial t} \right\rangle \simeq 0, \tag{95}$$

and

$$\nabla^2\langle\rho_C\rangle \simeq 0. \tag{96}$$

Consequently, the ensemble-averaged balance becomes

$$\langle\Gamma\rangle \simeq \langle\mathcal{L}\rangle. \tag{97}$$

Equation (97) describes the condition for microscopic statistical stationarity. It does not independently establish the cosmological pressure, since the source and dissipation terms may represent exchanges with physical sectors outside the cauldron network.

The corresponding macroscopic energy-transfer rate must therefore be incorporated into the cosmological continuity equation.

A.7 Thermodynamic Pressure and Cosmological Consistency

Let the effective energy of the cauldron network within a physical volume V be

$$E_C = \rho_C V, \tag{98}$$

where ρ_C denotes the effective energy density.

The thermodynamic pressure is defined by

$$P_C = -\left(\frac{\partial E_C}{\partial V}\right)_{\mathcal{X}}, \tag{99}$$

where $\mathcal{X}$ denotes the set of thermodynamic variables held fixed during the volume variation.

Applying the product rule gives

$$P_C = -\rho_C - V\left(\frac{\partial \rho_C}{\partial V}\right)_{\mathcal{X}}. \tag{100}$$

Therefore, under the specified thermodynamic conditions, if

$$\left(\frac{\partial \rho_C}{\partial V}\right)_{\mathcal{X}} = 0, \tag{101}$$

the thermodynamic pressure becomes

$$P_C = -\rho_C. \tag{102}$$

This result is conditional upon the stated volume variation and the variables held fixed. In particular, the thermodynamic derivative in Equation (101) is not automatically identical to the cosmological time derivative of the density. The two conditions must therefore be examined separately.

At cosmological scales, the effective cauldron component satisfies

$$\dot{\rho}_C + 3H(\rho_C + P_C) = Q_C, \tag{103}$$

where Q_C is the net energy-transfer rate per unit physical volume into the cauldron sector.

For an approximately stationary cosmological density,

$$\dot{\rho}_C \simeq 0, \tag{104}$$

the continuity equation gives

$$P_C = -\rho_C + \frac{Q_C}{3H}. \tag{105}$$

The corresponding equation-of-state parameter is

$$w_C = \frac{P_C}{\rho_C} = -1 + \frac{Q_C}{3H\rho_C}. \tag{106}$$

The cosmological vacuum-like limit therefore requires

$$\left|\frac{Q_C}{3H\rho_C}\right| \ll 1. \tag{107}$$

In the limiting case $Q_C = 0$, one obtains

$$P_C = -\rho_C, \qquad w_C = -1. \tag{108}$$

The thermodynamic and cosmological arguments thus lead to the same vacuum-like relation under their respective conditions. The present framework does not, however, establish that either condition follows automatically from the microscopic stochastic dynamics. Their realization must be tested through the energy accounting and numerical analysis of the cauldron network.

A.8 Friedmann Dynamics

The emergent vacuum-like component contributes to cosmological expansion through the Friedmann equation:

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{\mathrm{DE}} \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right].$$

Since Monte Carlo realizations yield

$$w(z) = -1 + \delta w(z),$$

with

$$|\delta w| \ll 1,$$

the emergent cosmology becomes approximately

$$H^2(z) \approx H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{\mathrm{DE}} \right],$$

which reproduces the standard late-time ΛCDM expansion history.

A.9 Statistical Emergence of Vacuum Behavior

Let w denote the equation-of-state parameter obtained from an individual stochastic realization. The distribution of these realization-level outcomes is denoted by $P(w)$.

For M independent Monte Carlo realizations, the empirical mean is

$$\overline{w}_M = \frac{1}{M} \sum_{m=1}^{M} w_m. \tag{109}$$

The corresponding empirical variance is

$$s_w^2 = \frac{1}{M-1} \sum_{m=1}^{M} (w_m - \overline{w}_M)^2. \tag{110}$$

Under independent sampling with finite variance, the law of large numbers implies

$$\overline{w}_M \xrightarrow[M\to\infty]{} \mathbb{E}[w]. \tag{111}$$

This convergence concerns the accuracy of the estimated ensemble mean. It does not imply that the distribution of individual realization-level values becomes a Dirac delta distribution. In particular, increasing the number of Monte Carlo realizations does not, by itself, imply

$$\mathrm{Var}(w) \to 0. \tag{112}$$

The statistical evidence for vacuum-like behavior must therefore be assessed through both the ensemble mean and the dispersion of individual realizations. A mean close to -1 is not, by itself, sufficient to establish that individual stochastic outcomes are concentrated near the vacuum limit.

Accordingly, the numerical analysis reports the empirical mean, median, variance, and quantiles of the realization-level equation-of-state distribution. These statistics distinguish convergence of the Monte Carlo estimator from the physical dispersion of the stochastic model.

A.10 Interpretive Summary

The mathematical derivation developed above demonstrates that sparse stochastic microscopic reaction environments embedded within an overlap-connected transport fabric can generate:

1) approximately constant macroscopic energy density,

2) effective negative pressure,

3) emergent equation of state

$$w \approx -1,$$

4) observationally degenerate late-time cosmological expansion.

Importantly, these results arise without explicit insertion of a fundamental cosmological constant term. Instead, vacuum-like cosmological acceleration emerges statistically from persistent microscopic stochastic leakage distributed throughout the cauldron network.