Pulsars and Millisecond Pulsars IV: From Subroutine Validation to Population Synthesis in NBODY6++GPU

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Abstract

Direct N-body codes such as NBODY6++GPU can solve the dynamical evolution of globular clusters with great accuracy, yet historically uses simple models: spin period, spin-down rate, and magnetic field strength are never updated once a neutron star forms. Earlier works in this series identified this gap, proposed a seven-scenario framework (S1–S7) covering the full range of pulsar evolutionary channels, and demonstrated the absence of pulsar tracking in an existing cluster simulation. Here we bring together three subsequent stages that solve this gap. A pulsar evolution subroutine was first tested in isolation across all seven scenarios, with two independently written codes agreeing to better than 0.1% over more than two thousand compared values, and reproducing accretion-driven spin-up for the first time within this framework. The validated framework was then embedded directly into NBODY6++GPU, and test integrations spanning timescales from under a Megayear up to a Gigayear show that the resulting period-period derivative distribution already resembles the pattern seen in real globular cluster pulsar populations. Finally, an early production test followed ten neutron stars self-consistently through the integrated code over roughly 28 Myr, uncovering that most of them pass through at least one episode of backward evolution, a behaviour that only becomes visible when spin parameters are sampled at every simulation step rather than inferred afterwards. Taken together, these results show that pulsar physics can now evolve as an integral part of cluster dynamics rather than a separate bookkeeping exercise, setting the stage for a full production run at $N = 250{,}000$.

Keywords: *Pulsars, Millisecond Pulsars, N-body simulations, NBODY6++GPU, Spin Evolution, Magnetic Fields, Globular Clusters.*

1. Introduction

Globular clusters are unusually efficient factories for millisecond pulsars: their dense cores drive close stellar encounters that assemble and disrupt binaries far more often than in the Galactic field, and the resulting mass transfer episodes can spin a neutron star up from a period of seconds to a few milliseconds (Alpar et al., 1982, Bhattacharya & van den Heuvel, 1991). Several hundred of such pulsars are now known across the Galactic globular cluster system (Bagchi et al., 2025), with new discoveries continuing to emerge from targeted surveys of individual clusters (Chen et al., 2023, Dai et al., 2020, Padmanabh et al., 2024). Direct N-body integrators are, in principle, the ideal tool for following this process, since they track every stellar encounter, binary formation event, and mass-transfer episode self-consistently. In practice, however, codes such as NBODY6++GPU (Wang et al., 2015, 2016) treat neutron stars as inert masses after formation: nothing in the code records how their spin or magnetic field responds to the dynamical history that the star actually experiences. Earlier papers in this series laid the groundwork for closing this gap. A statistical census of observed cluster pulsars exposed how little of their diversity current simulations could explain (Rah et al., 2024); a classification of neutron star evolution into seven physically distinct scenarios, spanning isolated spin-down, wide binaries, accretion-driven recycling, and dynamical encounters, provided

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a concrete target for implementation (Rah et al., 2025a); and a worked example within an existing cluster simulation confirmed that no trace of this physics was present in the code output at all (Rah et al., 2025b).

The present paper reports on three further steps that, taken together, turn this framework into a working part of NBODY6++GPU. The first step (Rah et al., 2025a) tested the seven scenarios as isolated field stars, checking each one against an analytical solution or an observationally motivated benchmark and cross-checking two independently written implementations against each other. The second step carried the validated prescriptions into the full N-body code itself, adding the bookkeeping needed to track spin and field evolution for every neutron star at every timestep, and confirming that short and long test runs produce spin-down and recycling behaviour consistent with the standalone results. The third step used the newly integrated code to follow an early population of neutron stars all the way through a short cluster simulation, providing the first direct look at how spin parameters respond to a live dynamical environment rather than an idealised one. Section 2 summarises the standalone validation and its main quantitative results. Section 3 describes the integration into NBODY6++GPU and the qualitative agreement it produces with the observed pulsar population. Section 4 presents the first population-tracking results, including a diagram that traces ten neutron stars through their full observable history. Section 5 closes with the implications of these results for the forthcoming full-scale production run.

2. Standalone Validation of the Pulsar Subroutine

Before any pulsar physics is allowed to run inside a live cluster simulation, the codes implementing it must first be shown to behave correctly on their own. The subroutine developed for this purpose was tested outside NBODY6++GPU (Wang et al., 2015, 2016) entirely, under seven monitored configurations chosen to isolate the seven specific physical processes intended: an isolated young pulsar spinning down under magnetic braking (S1); an isolated millisecond pulsar evolving under a very weak torque (S2); a low-mass X-ray binary in which accretion competes with and eventually overcomes magnetic braking (S3); a millisecond pulsar embedded in the kind of dense, high-velocity-dispersion environment expected in a cluster core (S4); a millisecond pulsar with a wide, dynamically inert companion (S5); and two compact-binary configurations, neutron star–neutron star and neutron star–black hole, evolved under dipole spin-down alone while their orbital coupling is left for future work (S6, S7), consistent with the rich phenomenology of binary neutron star dynamics documented in globular clusters (Sigurdsson & Phinney, 1995). Table 1 summarises the initial conditions and dominant physics of each case.

Table 1. The seven standalone test scenarios used to validate the pulsar subroutine, with representative initial conditions and the physical process each one isolates.

ID	Configuration	B_0 (G)	Dominant physics
S1	Young isolated pulsar	1×10^{12}	Dipole spin-down + field decay
S2	Isolated MSP	5×10^{8}	Weak-torque spin-down
S3	Accreting LMXB	3×10^{8}	Accretion-driven spin-up
S4	MSP in a dense core	3×10^{8}	Environmental torque (subdominant)
S5	MSP with a wide companion	4×10^{8}	Negligible tidal coupling
S6	NS–NS compact binary	5×10^{11}	Dipole spin-down; GW deferred
S7	NS–BH compact binary	5×10^{11}	Dipole spin-down; GW deferred

Each scenario was evolved for 100 Myr in two entirely independent codes, and the two were compared point by point at every Myr across the magnetic field, period, and period derivative. Across all seven scenarios (101 timesteps and three physical quantities per scenario) this comparison spans more than two thousand matched values, and the two implementations agree to within a few parts in ten thousand for the period and period derivative, and to within one part in a million for the magnetic field, well inside the precision expected of two independently coded integrators.

Beyond this numerical cross-check, each scenario was also compared against an independent physical expectation. S1 reproduces the closed-form dipole spin-down solution (Ostriker & Gunn, 1969) to the same sub-percent precision once a redundant field-decay term, of the kind discussed for accreting and isolated neutron stars alike (Konar & Bhattacharya, 1997, Pons et al., 2009), identified during testing was removed. S2 settles into a characteristic spin-down age near one Gyr, matching the population of millisecond pulsars seen in 47 Tucanae (Baumgardt & Vasiliev, 2021, Lorimer & Kramer, 2004). S4 and S5 confirm that neither a dense cluster environment (Verbunt & Freire, 2014) nor a wide, tidally coupled binary companion (Bildsten & Cutler, 1992) measurably perturbs the dipole-dominated evolution, with environmental torques

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contributing a few hundredths of a percent of the total at most. S6 and S7 spin down identically under the currently implemented physics, as expected, since their gravitational-wave (Peters & Mathews, 1963) and tidal terms are intentionally deferred rather than active.

The most demanding test is S3, where the subroutine must not merely avoid breaking but actively reproduce a qualitatively different behaviour: sustained spin-up rather than spin-down. Early attempts produced spin-up on a timescale of about a year, seven orders of magnitude faster than physically reasonable, tracing back to an accretion-torque coefficient that had not been brought into a consistent system of units. Once corrected to match the standard Ghosh & Lamb prescription (Ghosh & Lamb, 1979), the same scenario settles into a spin-up episode lasting roughly a hundred Myr and asymptotes to a period near the neutron-star breakup limit of one millisecond, with the accretion torque exceeding the magnetic braking torque by a factor of order ten throughout the active phase, consistent with the timescales expected for recycling in low-mass X-ray binaries (Tauris & Savonije, 1999). This is, to our knowledge, the first time this transition has been reproduced from first-principles torque balance within this validation framework, and it establishes that the subroutine is capable of driving a neutron star through the full recycling process rather than only slowing one down.

Taken together, the seven scenarios show that the framework reproduce every qualitatively distinct regime it is meant to capture, agrees with independent analytical and observational benchmarks wherever those exist, and does so identically across two separately written implementations. This is the baseline of confidence required before the same physics is allowed to run, unsupervised, inside a full NBODY6++GPU cluster integration (Spurzem & Kamlah, 2023), which is the subject of the next section.

3. Integration into NBODY6++GPU

Passing standalone validation is a necessary but not sufficient condition: the same physics must then survive being embedded inside a live, million-particle-capable code (Kamlah et al., 2022) without breaking either the subroutine or the surrounding integrator. Other groups have pursued parallel efforts to embed simplified pulsar spin-up prescriptions into direct N-body and related codes (Song et al., 2025, Ye et al., 2019). Here, this required adding a dedicated data block to NBODY6++GPU to carry each neutron star's spin period, magnetic field, birth conditions, and current scenario alongside the usual dynamical variables, and hooking the subroutine into the points in the code where a neutron star forms or enters a mass-transferring binary, following the existing stellar and binary evolution prescriptions already built into the code (Hurley et al., 2000, 2002). The framework now records a full spin history, position, velocity, and scenario label for every tracked object at each simulation output step, giving direct access to quantities that were previously invisible to the code entirely.

Two regimes were used to check that the integration preserves the standalone behaviour once the subroutine is exposed to a real dynamical environment rather than fixed test conditions. Short integrations of order a hundred Myr reproduce the spin-down tracks expected for young and weakly evolving pulsars, with wide binaries (separations of order a few AU) and moderate cluster core densities ($\rho_c \sim 10^5\ M_\odot\,\mathrm{pc}^{-3}$, Rah et al. 2025b) leaving spin evolution essentially unchanged, exactly as found in the standalone tests. Longer integrations extending toward a Gyr allow the recycling channel to operate for the first time inside a dynamical simulation, together with dynamical exchanges and, in a small number of cases, compact-binary coalescence (Chattopadhyay et al., 2020). The resulting period-period derivative distribution, built up self-consistently from the live cluster rather than imposed by hand, already falls into the same regions of parameter space as the observed young-pulsar and millisecond-pulsar populations in real globular clusters, without any parameter having been tuned to make this happen. The integrated code is now stable up to particle numbers well beyond what is needed for the next stage of this project (Kamlah et al., 2022, Spurzem & Kamlah, 2023), clearing the way for the population-tracking results presented in Section 4.

4. First Population-Tracking Results

With validation complete at both the standalone and the integrated level, an early test run provides the first direct look at how the subroutine behaves for a real, if small, population of neutron stars evolving inside a live cluster. The simulation used $N = 5{,}000$ particles with a core radius $r_c \approx 0.47$ pc and tidal radius $r_t \approx 52$ pc, placing it firmly in the globular cluster regime, with stellar masses drawn from a standard initial mass function (Kroupa, 2001), and was evolved for a total of 28.13 Myr of physical time, during

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which ten neutron stars appeared and were tracked continuously, together accounting for several hundred recorded output steps. Only three of the seven scenarios (Rah et al., 2025a) were triggered under these conditions: isolated spin-down (S1), wide-binary evolution (S5), and impulsive merger-or-kick events (S7). The remaining four require either a longer integration time, higher stellar densities than reached in this test model, or specific companion types not yet present; that this fourth and only fully successful iteration of the integrated test series (N = 5,000, 28 Myr), markedly shorter and smaller than the standalone 100 Myr scenario tests of Section 2, cannot yet supply; this is expected, and is exactly what the forthcoming production run at $N = 250{,}000$ is designed to remedy.

The ten objects fall naturally into seven distinct evolutionary sequences depending on which scenarios they pass through and in what order (Table 2). At one extreme, three neutron stars never leave the isolated spin-down regime over their much shorter individual observable windows (roughly 0.6 Myr for one object and under 0.1 Myr for the other two, since they appear only late in the run); at the other, one object cycles through five separate scenario transitions within only a few Myr, moving from isolation into a binary, through an impulsive encounter, back into a binary, and finally settling again, a stochastic sequence of encounters rather than a designed outcome. The full set of trajectories spans spin periods from about five to a hundred milliseconds and surface fields from roughly 1.7×10^8 to 10^9 G, placing every tracked object in the recycled or mildly recycled regime and, crucially, above the pulsar death line throughout the simulation: none of the ten objects is ever driven into a state where radio emission would be expected to switch off.

Table 2. The seven scenario sequences identified among the ten neutron stars of the test run, ordered from the most dynamically active to the most quiescent.

Group	Scenario sequence	Objects
1	S1→S5→S7→S5→S1→S5	1
2	S1→S5→S7→S5 (or →S1)	2
3	S1→S5→S7→S1	1
4	S1→S5	1
5	S1→S7→S5→S7→S1	1
6	S7→S1	1
7	S1 only	3

The most notable outcome of this test is not any single trajectory but a pattern that cuts across most of them: a majority of the ten neutron stars pass through at least one episode that moves them *backward* along the usual recycling path rather than forward (Figure 1). In some cases this shows up directly in the spin parameters, as when a single impulsive encounter pushes an already partially recycled object back toward longer periods within one output step. In other cases it shows up as a reversion in scenario label itself, when a binary is disrupted and the neutron star returns permanently to isolated spin-down after only a brief accretion episode. In still other cases the assigned S-scenario label never changes at all, yet the underlying spin parameters jump abruptly in response to a dynamical encounter too weak to cross any classification threshold, a level of detail that would be invisible to any method that assigns spin evolution after the fact rather than tracking it at every timestep. None of this behaviour was anticipated when the scenarios were first defined, and it could only be found by sampling spin parameters as frequently as the dynamical simulation itself is sampled, which is precisely what the integration described in Section 3 makes possible for the first time.

5. Discussion and Conclusions

Across the three stages summarised here, pulsar physics has moved from an absent feature of NBODY6++GPU (Wang et al., 2015, 2016) to a working, tested component of it. The seven standalone scenarios established that the underlying physical prescriptions are correct and mutually consistent between two independently written codes; the integration into the full code showed that this correctness survives contact with a live, dynamically evolving cluster rather than only idealised test conditions; and the first population-tracking run demonstrated that ten neutron stars can be followed continuously through a real simulation, occupying exactly the recycled and mildly recycled region of parameter space expected for globular cluster pulsars and remaining active radio emitters throughout. Taken together, these results mean that a neutron star's spin and magnetic history are no longer a missing dimension of an N-body cluster simulation, but a quantity computed alongside its position and velocity at every timestep.

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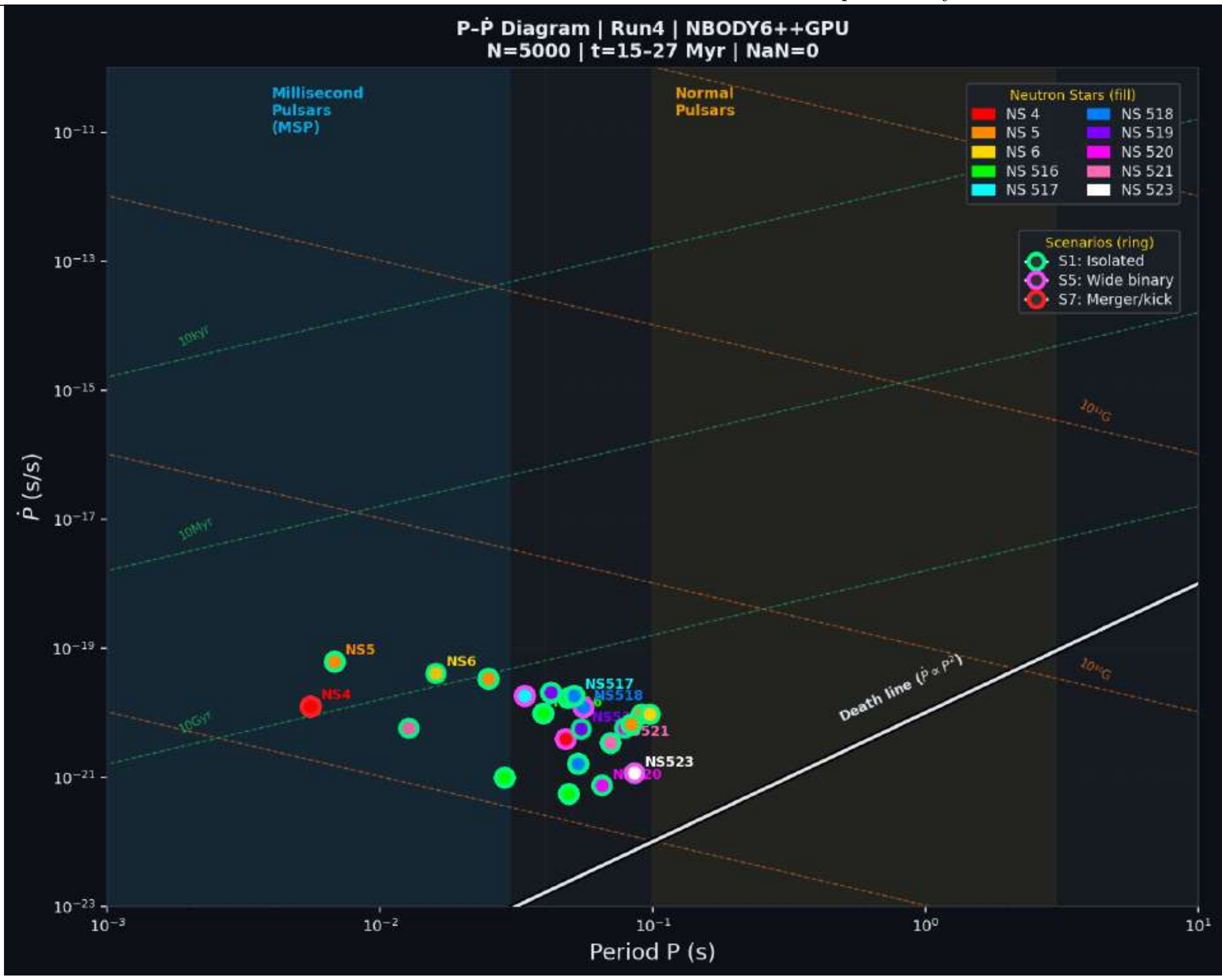

Figure 1. Period-period derivative diagram for all ten neutron stars tracked in the test run (N = 5,000, $t \approx$ 15–27 Myr). Each point is one recorded output step; fill colour identifies the neutron star and ring colour marks the active scenario at that step (S1: isolated spin-down; S5: wide binary; S7: merger or kick). Diagonal dashed lines show loci of constant characteristic age (green) and constant surface field (orange); the solid line is the pulsar death line. All ten objects remain in the recycled and mildly recycled regime throughout the simulation and stay above the death line at every tracked step, while several trajectories visibly double back on themselves, the direct graphical signature of the regression episodes described above.

The most unexpected outcome of this early test is not that the subroutine works, but what it reveals once it does: a majority of the tracked neutron stars pass through at least one episode in which they move backward along the usual recycling sequence, whether through a sudden change in spin parameters, a reversion to a simpler evolutionary state after a binary is disrupted, or a spin-parameter jump too small to register as a change in scenario at all. None of these behaviours were built into the subroutine by design; they emerge only because spin evolution is now sampled at the same cadence as the dynamical simulation itself, rather than reconstructed afterward from a coarser record. This suggests that some fraction of the isolated and partially recycled pulsars observed in real clusters may carry a disrupted or interrupted recycling history that a purely forward-evolving model would never produce, a possibility that only becomes visible through this kind of fully integrated tracking.

The natural next step is to repeat this exercise at a scale where all seven scenarios, not only three of them, have a realistic chance to occur. The present test used a comparatively small and short-lived cluster, which is why the accreting, tidally synchronised, and magnetar-like channels were never triggered; a longer, denser environment is needed to populate them. We are therefore preparing a production run at N = 250,000 particles extending to roughly 100 Myr of cluster evolution, an environment expected to be dense enough and long-lived enough for every one of the seven scenarios to appear and be tracked in full, and large enough to support a statistically meaningful comparison with the observed pulsar populations of real globular clusters. This production run will form the basis of the population-synthesis results that complete the next stage of this dissertation project.

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